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Essays on Financial Networks and Systemic Risk

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A mis padres y hermanos.
A Mario.

I would like to express my special thanks to my supervisors Professor Domenico delli Gatti. Moreover, special thanks are also due to Prof. Ester Faia e Prof. Gianmarco Ottaviano.

Abstract

The last global financial crisis clearly illustrated the crucial role of interbank linkages in channeling and amplifying shocks hitting the system and, therefore, in the emergence of systemic risk.

In this thesis, we present theoretical and empirical methodologies for analysing the potential for systemic risk in a interconnected banking network.

The dissertation comprehends two essays on financial networks and systemic risk and is organised in two chapters. In chapter I, we analyse and model some complex interactions and feedback relationships within a financial network, with the objective of delving into the linkages between fragility in the real economy and in the banking system. For this purpose, we provide a qualitative and quantitative description of leverage dynamics.

In chapter II, we exploit an original dataset on 15 European banks classified as G-SIBs by the BIS to assess whether expansion in foreign markets increases their riskiness, and through which channels that eventually happens.

Sommario

L'ultima crisi finanziaria ha evidenziato il ruolo decisivo delle connessioni nel mercato interbancario come canale e strumento amplificatore dei shock finanziari, e di conseguenza del rischio sistemico.
In questa tesi presentiamo delle metodologie teoriche ed empiriche per analizzare il potenziale rischio sistemico in una rete bancaria interconnessa.
La tesi comprende due saggi sulle reti finanziarie e il rischio sistemico ed è organizzata in due capitoli. Nel capitolo I analizziamo e modelliamo alcune delle complesse interazioni all'interno di una rete finanziaria, con l'obiettivo di approfondire nella interrelazione fra la fragilità dell'economia reale e quella del sistema bancario. A questo scopo, forniamo una descrizione qualitativa e quantitativa delle dinamiche della leva finanziaria.
Nel capitolo II, sfruttiamo un set originale di dati su 15 banche europee classificate come G-SIB per valutare se l'espansione nei mercati esteri aumenta la loro rischiosità, e attraverso quali canali si materializza.

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Chapter 1

Leverage Dynamics in overlapping portfolios: a network approach to systemic risk

In this paper we analyse and model some complex interactions and feedback relationships within a financial network, with the objective of delving into the linkages between fragility in the real economy and in the banking system. Our aim is to address the potential spillover effects of external events on the vulnerability of the financial system by providing a qualitative and quantitative description of leverage dynamics. We explore financial contagion applying a network approach combined with a balance-sheet amplification mechanism.

Keywords: Systemic Risk, Financial Networks, Leverage.

1.1 Introduction

A shock on external asset prices affects banks' leverage, inducing fully-leveraged financial intermediaries to actively manage their balance sheets so as to maintain leverage at the target level. In a financial system of interconnected financial institutions, this behaviour do may have systemic effects.
We identify overlapping portfolios as the source of financial contagion and the balance sheet management mechanism as the primary cause of amplification effects.

The procyclical leverage policy pursued by financial institutions translates into scenarios of varying systemic leverage in response to changing levels of market volatility. Individualistic financial institutions increase or reduce leverage depending on whether the market volatility is low or high, causing systemic leverage trends that turn out destabilizing from a dynamical system point of view (de Haas and Peters, 2004). On a systemic level, collective leveraging and deleveraging of financial institutions can lead to asset market cycles.

The motivation behind our interest in the analysis of the dynamics unleashed by events external to the interbank market is the possible application of our analytic framework to the last sovereign crisis in Europe, which has generated reasonable concerns about the trigger effects of sovereign risk in the propagation of financial distress in a highly connected financial system.

As shown in previous literature (Darraq Paries and Faia, 2012; Baglioni and Cherubini, 2013), there is a positive correlation between sovereign and bank risk. The aim of our analysis is to explore how sovereign risk can spread to the banking system[1]. Our model tries to capture the previous connection in order to evaluate the magnitude of the spillover effects of a deterioration in sovereign financial robustness -or of a credit event affecting sovereign debt- on the banking system. For this purpose, we delve into the dynamics of leverage within the financial network given an exogenous shock in sovereign debt prices.

In analyzing the interactions and feedback relationships within the financial network, we follow an analytical and quantitative approach to overlap and contagion. We develop a network approach to analyse the potential spillover effects of external financial disruptions on the interbank market.
One of the novelties of our work if that we provide explicit analytical foundation to the functioning of the overlap financial structure and of leverage dynamics. The results of

[1] We omit the dynamics of shock propagation among sovereigns, which has already been covered in previous work

this analysis reveal some behavioural patterns observed during the crisis. The conclusions derived are then tested by simulating leverage dynamics within a heterogeneous financial network. To illustrate the amplification mechanism of financial distress that took place during the crisis, we construct a model of financial contagion based on bank balance sheet identities and behavioural assumptions of deleveraging. Our simulation results confirm the importance of leverage in the origination of endogenous financial processes, ultimately leading to cascade effects. These results also evidence that while asset price fluctuations have a short-term impact on leverage ratios, the convergence to target leverage ratios and, therefore, to the systemic equilibrium might be achieved only in the long term.

1.1.1 Some empirical facts

There has been a transmission of risk from the banking system to governments during the last financial crisis, and then back from sovereign obligors to banks during the successive sovereign crisis.

By the time of the financial turmoil in 2008, a significant fraction of external assets was related to the real side of the economy (e.g., asset-backed, real estate and mortgage-backed securities). When this kind of assets was hit by a fundamental shock and the interbank market of highly leveraged financial institutions froze, governments performed bailout measures in order to avoid the collapse of the financial system. As a consequence, the perceived default probability of banks was reduced[2].
Afterwards, as public needs for funding increased, banks boosted lending to governments. Given the deterioration of public finance, Governments increased debt issuances. Being sovereign debt classified as a safe asset, banks undertook purchases of that type of debt in order to expand Tier 1 capital, as part of their strategy to meet capital requirements.

Somehow, a change in the composition of assets in the banks' balance sheet took place: banks' preferences shifted from assets related to real economy -in particular real estate- to sovereign debt.

We consider a system of heterogeneous banks with linked balance sheets and positions in external assets whose dynamics are described within a network. A bi-dimensional (internal-external) structure could be identified, in the sense that there exists an interbank network, where internal securities are negotiated, that also interacts with a network external to the

[2]Gary et al. (2006) and Gapen et al. (2005) have previously provided the methodology for valuing the bailout modelled as a put option to be included in the asset side of the balance sheet of financial intermediaries and in the liability side of governments' balance sheet. (Baglioni and Cherubini, 2013).

bank sector (the real economy, specifically the public sector -sovereigns-).

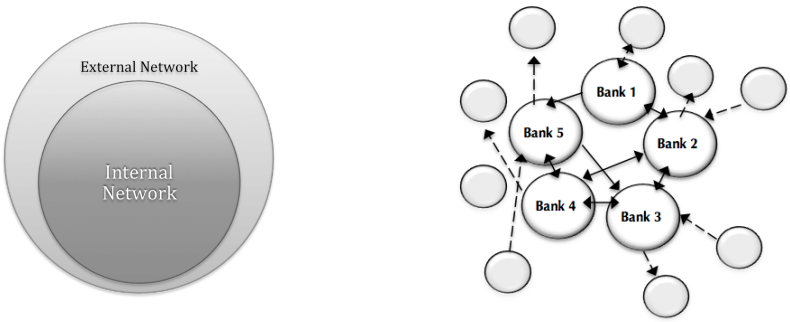


Figure 1.1: Illustration example of a stylised financial system

Accordingly to this configuration, it could be said that governments in the "internal network" became an important counterparty of the "external network" as obligors in the first instance, and then as creditors.

Following the bailout measures, the sovereign risk perceived by the markets increased, triggering a rise of sovereign CDS spreads and the consequent fall in prices of sovereign debt. As shown in previous literature (Adrian and Shin, 20110), price shocks can activate a balance-sheet amplification mechanism; in the particular case of the European sovereign debt crises, the decrease in prices initially induced a contraction of highly-leveraged balance sheets -banks started selling assets-, amplifying the effects of the initial shock.
Nevertheless, some empirical studies (Battistini, Pagano and Simonelli, 2014) have also revealed that in some countries of the Eurozone, and especially in its periphery, bank's sovereign exposures responded positively to increases in yield, which suggests distorted incentives in periphery banks' behaviour. This would be the case if governments implement financial repression or banks behave as value investors.

1.1.2 Systemic Risk and leverage.

In this paper we focus on the contribution of sovereign risk, i.e., a shock in external assets, to the propagation of financial distress in a banking network. We identify leverage as the systemic risk component of overlapping portfolios as well.

Systemic risk is commonly identified with the default of financial institutions (Poledna, Thurner, Farmer and Geanokoplos, 2014). Other definitions refer to the risk of financial instability "so widespread that it impairs the functioning of a financial system to the point where economic growth and welfare suffer materially" (ECB, 2010), or to "widespread failures and losses of financial institutions that impose externalities on the rest of the economy" (Achayra, Pedersen, Philippon and Richardson, 2010). Other authors focus on more specific triggering mechanisms when defining systemic risk: feedback behaviour (Kapadia, Drehman, Elliot and Sterne, 2009), contagion (Moussa, 2011), correlates exposures (Achayra, Pedersen, Philippon and Richardson, 2009), asset bubbles (Reinhart and Rogoff, 2009)...

By systemic risk here we mean the vulnerability of the financial system to events triggering contagion. The event causing the systemic episode will be a shock in the price of the sovereign debt, our external asset. Credit asset prices incorporate the probability of default. A decrease in sovereign debt prices can therefore be associated with an increase of the probability of sovereign default and, consequently, of credit risk.

We model the behaviour of the systemic risk component -leverage- in a framework of market dynamics amplifying the reaction triggered by an exogenous shock in sovereign debt prices.

In the literature several types of financial distress propagation and amplification mechanisms can be identified, from the ones present in the traditional models of the financial accelerator relying on the demand-side of credit channel (Bernanke, Gertler, 1989; Kiyotaki and Moore, 1997) to the financial intermediaries' balance sheet dynamics (Adrian and Shin, 2010a; Geanakoplos, 2009).

When analysing the financial fragility in our model, we are identifying leverage as the propagating factor of the financial distress triggered by the deterioration of sovereigns' financial robustness, i.e., a negative shock in external asset price, and overlapping portfolios as the structure that enables contagion within the banking network, that's to say, the contagion mechanism.

We identify system stability with either the absence of defaults or the compliance with a target leverage. Therefore, for the purposes of this paper defaults and/or violations of this maximum leverage ratio (in the sense of $\lambda_{i,t} > \lambda^T$) are consider indicators of systemic risk. We could consider for further research quantitative proxies of systemic risk, such as the fractions of defaults or the fraction of banks whose actual leverage exceeds the target. It would be sensible to do that when performing simulations for N banks, with $N > 2$.

On the purpose of exploring the relationship between systemic risk and the market procyclicality induced by the amplification mechanism, our research is developed in a context of active management of marked-to-market balance sheets carried out by financial institutions in response to fluctuations in prices and, therefore, measured risk.

Bank's leverage, defined as its asset-to-equity ratio and expressed as a function of obligors' leverage, is considered the propagation channel of a price shock. In our model the mechanism works as follows: a negative shock to external asset prices lead to an increase in leverage. Leverage-targeting banks will shrink their balance sheets by selling assets and repaying part of the external debts. This active balance sheet adjustment may reinforce the business cycle and can lead to a self-reinforcing amplification mechanism if procyclical leverage is allowed (Adrian and Shin, 2010). This approach is aligned with the idea of endogenous crises: a high-leveraged market becomes more vulnerable to small fluctuations. In our model, the variation in the value of external securities holdings resulting from the balance sheet management triggers the positive feedback.

In a banking network where agents are connected through overlapping balance sheets, shocks to external asset prices not only induces an immediate variation on the leverage of those agents holding that asset. Furthermore, spillover effects arise when the value of the internal assets, i.e., other bank's obligations, is affected due to the direct exposure of the obligor to the shocks in prices; namely, the propagation of the shock within the financial network -contagion- occurs through direct and indirect channels.[3].

[3]Spillover effects across countries could be considered as banks may not only be exposed to the sovereign risk of its own country, but also to the sovereign risk of other countries.However, this is beyond the scope of our research.

1.1.3 The financial sector and a network approach.

Recently, there has been a growing interest in network analysis applied to economic and financial theory.
There is an increasing concern to improve the identification and understanding of systemic risk and contagion mechanisms. Many authors have analyzed how the structure of the financial system can affect the emergence of systemic risk and its propagation (F. Allen, A. Babus, E. Carletti, 2010; Vitali, Battiston, Gallegati, 2013; Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi, 2012; Battiston, Delli Gatti, Gallegati, Greenwald and Stiglitz, 2011).

In order to study the contagion mechanism and the resiliency of the interbank market, we adopt a network approach to represent the banking system.
How networks propagate shocks depends on how they are constructed[4].

In our model, we design a static internal network[5], defining from the beginning the network matrix representing the interbank market consisting of heterogeneous financial institutions. It would be interesting to capture certain dynamics of the network under consideration by implementing an endogenous mechanism of link formation, based on a preferential attachment mechanism, for instance[6]. This will be useful to model certain processes such as the loss of density in the "internal network", due to a shrinking interbank market, but it's beyond the scope of this paper.
Nevertheless, we allow for dynamics regarding the interconnections between the internal and external network by designing an external assets demand function[7].

[4]In random networks, edges connecting vertices are generated randomly (Erdos and Renyi, 1950). Scale-free networks have a power-law degree distribution of links (Barabasi and Albert, 1999).

[5]Whereas banks actively manage their balance sheets through variation in external asset and liabilities holdings, the internal network is static in the sense that interbank debt remains constant over time. This is one of the assumptions our analysis is based on.

[6]If this would be the case, the dynamics governing the interaction among agents could be described by an endogenous fitness mechanism. The fitness of a node -the node's inherent competitive factor- would be susceptible to affect the network's evolution. The probability of a node to connect to another would be supplied with the node's -agent- financial robustness (the inverse of the leverage expressed as the assets-to-equity ratio). The introduction of the fitness property entails that the exponent in the power-law degree evolution formula is different among nodes, giving the opportunity to new-coming nodes to dominate the system.

[7]There has been also a transformation of the external network, given the changes in the type of external assets held by banks, but the study of the dynamics in this specific case is set aside in this paper.

1.1.4 Baseline and summary assumptions.

We will extend the analysis already developed in previous literature (Tasca and Battiston, 2013) by building a model that captures the borrower-lender relationships within the financial network and the dynamics of the market in a context of heterogeneity among agents. After deriving a leverage formula capturing all these interactions, we will proceed to estimate the response of leverage dynamics in the banking system to an exogenous shock in external asset prices, given different market conditions. These conditions will be captured by factors such as agents' initial leverage and target leverage levels, the depth of cross-holdings among financial institutions, etc.

Our model differs from existing models analysing systemic risk within a context of balance sheet amplification mechanism in a number of ways. We firstly introduce heterogeneity by considering a financial system represented as a **network of heterogeneous and interacting agents**. We also present a simple agent-based model of this heterogeneous financial system.
The aim is to capture the fact that micro-level agents' interactions lead to macro-level complex dynamics.

On the purpose of analysing the propagation mechanism of financial distress, different and sequential approaches will be followed.

At first, we delve into the (accounting) dependence of one agent's leverage on counterparties' leverage and external asset prices, in order to get explicitly analytical foundation to overlap and contagion issues. In doing so, we follow two different approaches regarding the valuation of securities traded within the interbank network:

- According to the approach generally adopted in empirical finance, we initially consider the book value of interbank liabilities -as a proxy for its market value- and the marked-to-market value of internal assets.
- Then, as in finance theory, leverage will be couched in terms of the present market value of both both interbank assets and liabilities.

We will see that the chosen method affects the results of the analysis of the static equilibrium in a leveraged system.
Subsequently, we study the properties of the dynamic equilibrium. We consider different adjustment models capturing the leverage dynamics.

Secondly, we develop an agent-based model that allows for testing the conclusions drawn from the analysis of the static and dynamic equilibrium:

- Different valuations of leverage lead to different potential equilibriums.
- The existence of an equilibrium is ultimately strongly affected by the structure of balance sheets, i.e., by the level of leverage of the system.
- In order to ensure a systemic equilibrium, the choice of leverage ratios by macro-prudential authorities should be based on the actual structure.

and for supplying the limitations of that analysis, by enabling:

- The assessment of some relationships among variables, that can be ambiguous when working with linearised systems.
- The evaluation of effects of macro-prudential policies.
- The analysis of the dependence of the existence of an equilibrium on market structures and conditions (network structure, banks' capitalization...).

In this context, an adjustment process capturing bank's decisions about external asset holdings is incorporated[8]. Our simulations capture the fact that an exogenous shock to external asset prices triggers balance sheet dynamics impacting leverage and leading to a contagion process.

Balance sheet dynamics impacting leverage will be triggered by an exogenous shock to external asset prices, leading to a contagion process that will be simulated. According to different leverage scenarios, a *Balance Sheet Managem ent process (BSM)* will be activated, allowing for *financial amplification*.

In order to extend the existing research and to come through a more realistic model when considering specifically sovereign debt, some assumptions should be done:

- As said previously, we are not considering an homogeneous financial system, but heterogeneous players with different balance sheets structures[9]. For instance, the initial exposure to assets related to real economy differ among banks.
- We allow for heterogeneity in terms of the external asset[10].

[8] We disregard the effect of trading movements on prices and any consequent endogenous price process.

[9] Even if banks in this heterogeneous network could be grouped by homogeneous parameters, creating a sort of market microstructures.

[10] Actually, sovereign debt exhibits heterogeneity in prices (return) and volatility. Additionally, it cannot be asserted that external assets are uncorrelated and have the same initial values. From an empirical point of view, correlation in sovereign debt has been observed in European peripheral countries, and also the price at which the debt is issued diverges among countries.

- The asset market is considered complete, as banks can use financial instruments in order to mitigate the credit risk.
- Notional value of interbank debts will remain constant over time: $b_{i,t} = b_i$.[11]
- Equity remains "sticky". Equity behaves like the pre-determined variable and the asset size in the balance sheet will be endogenously chosen according to the degree of leverage allowable given the market conditions (Adrian and Shin, 2011).
- Regarding the leverage targets, we analytically study the case of both heterogeneous and homogeneous targets. The latter approach would be consistent with the introduction of a leverage ratio according to Basel III regulatory framework.
  A bank's degree of reaction to asset-price changes is measured by the level of compliance with own target leverage or capital requirements. We assess how leverage dynamics are affected by the rate of adjustment to the target leverage in certain scenarios in order to establish a connection with the part of the literature supporting the suitability of imposing counter-cyclical capital requirements.

[11] Our interest is focused on the spillover effects of the sovereigns in the banking sector. We consider interbank relationships as transmission channels, but we are not going in depth into the dynamics.

1.2 The model

1.2.1 The interbank network.

We consider a financial system represented by a finite set of N individual banks interconnected through financial contracts on the interbank market.
For this population of banks we define a network (N, g), where g is the set of all directed links among heterogeneous banks. The index i indicates each individual bank, which can be either a lender or a borrower. The index j indicates the counterparty of each bank in a trading relationship within the interbank network.
A link between two nodes is denoted by g_{ij}, with $g_{ij} = 1$ if there exists an edge from i to j, or otherwise, $g_{ij} = 0$ if there is no edge. The graph (N,g) is associated with a weight matrix, W (column-stochastic matrix), where $w_{i,j} > 0$ iff $g_{ij} = 1$, $w_{i,j}$ being $w_{i,j}$ the exposure of agent i to agent j ($\sum_j w_{i,j} = 1$).
By exposure we mean the quantity percentage of debt issued by bank j and held by bank i. Node "i" will be exposed to node "j" from a lender prospective, as long as the value of the liabilities of agent j to i (i's claims) depend on j's ability to meet her obligation.

As in Eisenberg and Noe (2001), we assume that all nominal claims will be nonnegative and that no node has a nominal claim against itself. In our design, this is equivalent to specify that the weight matrix is non-negative and that all of the diagonal elements of the matrix are equal to 0:

$$w_{i,j} \geq 0; w_{i,i} = 0 \quad \forall i, j \in N$$

We define $N_i(g) = \{j \in N | g_{i,j} \neq 0\}$ as the neighbourhood of i, that is, as the set of banks with whom bank i has a direct link representing a credit relationship in the network. The cardinality of this set is given by the degree of node i, expressed as:

$$d_i = |N_i(g)|, d_i \leq N - 1$$

We could also define:

- The "out-degree" of node i as the number of out-going edges representing bank i's lending to counterparties within the interbank network:

$$\bar{d}_i = |\bar{N}_i(g)|, \bar{d}_i \leq N - 1, \quad \bar{N}_i(g) = \{j \in N | w_{i,j} \neq 0\}$$

- The "in-degree" of node i as the number of of in-going edges representing bank i's borrowing from counterparties within the interbank network:

$$\underline{d_i} = |\underline{N_i}(g)|, \quad \underline{N_i}(g) = \{j \in N | w_{i,j} \neq 0\}$$

We are not neccessary imposing a fully connected network; we could eventually allow for incomplete networks. According to Allen and Gale (2000), the incompleteness of the network would allow for financial fragility. They showed that financial contagion through credit linkages among banks depends on the topology of the network: the higher the interconnectedness, the lower the systemic risk. Nevertheless, other works have shown that the role of diversification can be ambiguous in the presence of amplification mechanisms (Battiston et al., 2011; Gai, Haldane and Kapadia, 2011).

We consider a static network[12], and banks are endowed with bilateral claims, defined ex-ante.

1.2.2 A bank's balance sheet.

As said before, banks interact not only with other banks within the interbank market, but also with agents external to the interbank market. Therefore, the stylized balance sheet of a bank at time t[13] can be represented as follows[14]:

Bank i

Assets	**Liabilities**
$\sum_{k=1}^{K} n_{ik,t} p_{k,t}$	$d_{i,t}$
	$b_{i,t}$
$\sum_{j=1}^{\bar{d}_i} \omega_{ij} \frac{b_{j,t}}{(R_t+\xi_{j,t}\lambda_{j,t})^{\bar{t}}}$	$e_{i,t}$

Where:

$n_{ik,t}$	bank i's holdings of external asset "k" at time t
$p_{k,t}$	external asset price at time t
$d_{i,t}$	bank i's external liabilities at time t
$b_{i,t}$	bank i's interbank liabilities (book value) at time t
$e_{i,t}$	bank i's equity at time t
$\sum_{j} \omega_{ij} \frac{b_{j,t}}{(R_t+\xi_{j,t}\lambda_{j,t})^{\bar{t}}}$	bank i's holdings of interbank obligations (market value) at time t

[12]An alternative model of dynamic networks is included in appendix B.

[13]The assumptions outlined in Section 1.1.4 do not apply yet.

[14]Instead of considering a classification of the components of the balance sheet according to maturity -short and long-term assets and liabilities-, we will categorize securities depending on the sector of the counterparties involved in the trading.
Liquidity is not specified in this stylised balance sheet structure.

From now on, we assume that there is only one type of interbank debt[15], with same seniority and maturity, $\hat{t} = 1$.

We make a distinction within assets and liabilities according to the sector of the counterparties entering a financial relationship:

- internal assets and liabilities, traded within the interbank market.
- external assets and liabilities, traded between the bank and an agent in the rest of the economy (in the private or public sector).

Defining bank i's external assets as the sum of bank i's claims against agents in the real sector -private or public (sovereigns)-, the market value of external assets held by bank i at time t is:

$$\sum_{k=1}^{K} n_{ik,t} p_{k,t}$$

Defining bank i's internal assets as the bank i's claims against other banks, the market value of internal assets held by bank i at time t is[16]:

$$\sum_{j=1}^{\bar{d}_i} \omega_{ij} \frac{b_{j,t}}{R_t + \xi_{j,t}\lambda_{j,t}}$$

where

$\omega_{i,j}$ the exposure of bank i to j, defined previously
b_j the nominal value of bank j's total debt
$\frac{1}{R+\xi_j\lambda_i}$ the discount factor used in the computation of the present value of $\hat{t}$-years ($\hat{t} = 1$) maturity obligations
$R = 1 + r_f$, r_f being the market risk-free rate

[15]The market value of debt is usually more difficult to obtain directly, since many firms (banks) may hold non-traded debt, which would be specified in book value terms but not market value terms. A simple way to convert book value debt into market value debt is to treat the entire debt on the books as one coupon bond, with a coupon set equal to the interest expenses on all the debt and the maturity set equal to the face-value weighted average maturity of the debt, and then to value this coupon bond at the current cost of debt for the bank.

[16]Hereafter, for the sake of simplicity upper and lower bounds of the summations will be omitted. It remains understood that, in the summation for external assets, the index ranges from 1 to K, and that, in the summation for counterparties, it varies within 1and N-1.

When considering the marked-to-market value of interbank claims, the present market value is expressed as the discount value of future cashflows. The discount rate incorporates a risk premium, which is generally defined as the return in excess of the risk-free rate of return an investment is expected to yield. That return in excess is the credit spread associated with the issuer of the obligations.

In line with a large body of theoretical and empirical literature, the rate of return on future payments depends on the debtor credit worthiness, among other factors (Tasca and Battiston, 2011). In our specification, and similarly to the approach followed by Tasca and Battiston (2012), two factors explain the internal rate of return of banking obligations: the risk free rate and leverage, as an indicator of financial fragility.

Furthermore, the definition of the credit spread as a function of leverage and the market perception of the obligor's risk is consistent with the findings in literature related to the relationship between credit spreads and leverage.

In an efficient market, spreads reflect both the issuer's current risk and investor's expectations about the potential risk behaviour over time. Empirical works have tried to explain credit spreads using firm leverage and different proxies for asset volatility (Krishnan, Ritchken and Thomson, 2005; Campbell and Taskler, 2003; Avramov, Jostova and Philipov, 2007). In particular, Collin-Dufresne and Goldstein (2001) argue that credit spread changes are determined by changes in leverage, among other state variables such as volatility and interest rates, and that the sensitivity to changes in leverage also tends to increase when leverage does. After modelling leverage as mean reverting and simulating credit spreads closed to the observed in the market, the authors conclude that the appropriate credit spread (on corporate bonds) should reflect not only current information about a firm's leverage but also investors' expectations about future leverage.

Existing models of default risk predict a relationship between credit spreads and leverage (Merton, 1974). An increase in leverage is expected to rise the probability of default and hence the credit spread on outstanding debt obligations. Therefore, credit spreads reflect information about a firm's default probability, which depends on current leverage and investors' expectations of future leverage, according to the structural model of credit risk derived by Black and Scholes (1973) and Merton (1974).

We have defined the return on bank j's obligations as the aggregation of the risk-free rate r_f and a risk premium or credit spread $\xi_j \lambda_j$, that depends on the market perception of

the bank's risk ξ_j and is proportional to her leverage λ_j[17]. The rate of return on bank j's obligations towards bank i, $r_j = r_f + \xi_j \lambda_j$, increases with the level of leverage (is decreasing with financial robustness).
We define the leverage of a bank as the ratio of total assets to equity, which in turn is defined as the difference between the value of the bank's portfolio of claims and its liabilities[18].

1.2.3 Leverage.

Rajan and Zigales (2005) describe different leverage measures, each of which having its own limitations. The authors explain that the most common used expression of leverage is the ratio of total short and long-term liabilities to total assets. This measure can be understood as a proxy of shareholders' leftovers in case of liquidation. The main disadvantage of this measure is that it does not allow for capturing default risk.
An alternative measure is provided by the ratio of debt to total assets. This one focusses more on financial leverage, but a potential caveat of this measure is that, even if it prevents non-debt liabilities to influence the capital structure, specific assets can be financed by non-debt liabilities. Aditionally, leverage measured as the ratio of market value of debt to market value of assets of a firm does not seem to be the best choice when the objective is to identify the determinant of optimal leverage (Banerjee, Heshmati and Wihlborg, 2000). Other possible measure of leverage is debt divided by net assets, defining net assets as total assets less accounts payables and other liabilities. Rajan and Zingales (1995) argue that the debt to capital ratio captures better the effects of past financing decisions. In this case capital is defined as long-term debt plus equity. The last considered measure can determine whether a firm can meet its fixed payments and is the interest coverage ratio, defined as EBIT to interest expense.

It is also common to measure financial leverage as the book value of debt relative to the market value of equity. The measure assumes that market prices are efficient estimates of the value of equity. However, the book value of debt, which always equals its value at origination, may not be a good proxy for market value of debt as market value can moves away from book value. The last recent crisis has evidenced this.

Leverage defined as the ratio of total assets to equity is commonly used as well (Adrian and Shin, 2010). This is the formulation adopted in this paper. It measures financial robustness and is more intuitive when analysing the relationship between leverage and the balance sheet size. It is also the most consistent with Basel's leverage ratio, that gives the

[17] $\xi_j \in [0, 1]$could be understood as the reactivity of the rate of return to a bank's financial condition.
[18] In Tasca and Battiston (2012), the leverage ratio is defined as a bank's debt-to-assets ratio.

proportion of assets funded by equity capital, which absorbs losses, rather than liabilities such as debt and deposits, which do not. It can be understood as a measure of the losses a bank can sustain before defaulting.
The fact that assets enters explicitly the leverage formulation is convenient for the purposes of this paper, as the scope of our research is assessing the effects of external financial disruptions (shocks in external asset prices) on the interbank market (considering leverage as the propagation factor) by analysing the leverage dynamics that arise from the active management of balance sheets by financial intermediaries who respond to changes in prices.

Given the balance sheet structure considered above, our general expression for leverage is given by:

$$\lambda_{it} = \frac{\sum_k n_{ik,t} p_{k,t} + \sum_j \omega_{ij} \frac{b_{j,t}}{R_t + \xi_{j,t}\lambda_{j,t}}}{\left(\sum_k n_{ik,t} p_{k,t} + \sum_j \omega_{ij} \frac{b_{j,t}}{R_t + \xi_{j,t}\lambda_{j,t}}\right) - (d_{i,t} + b_{i,t})} \tag{1.1}$$

which captures the overlapping structure of the financial system. By specifying the present values of interbank obligations, bank i's leverage is expressed as a function of all the obligors' leverages $\lambda_{j,t}$, $j = 1, ..., \bar{d}_i$:

$$\lambda_{i,t} = \lambda(\lambda_{1,t}, \lambda_{2,t}, ..., \lambda_{\bar{d}_i,t})$$

More precisely, it is the value of one bank's internal asset that depends on the obligors' leverage, i.e., on the strength of the borrowers' balance sheets. The marked-to-market value of one bank's claims against other banks depends on the financial robustness of the latter[19].

1.1 is a system of N non-linear equations in the level of leverage for each and every agent $i = 1, 2, ..., N$, given the exposure to external agents $\sum_k n_{ik} p_k$ and individual liabilities d_i and b_i.
In order to better analyse the effects of price shocks on leverage, these equations are linearised. The ultimate objective is to get a system of equations of the form $\lambda_{i,t} = \lambda(p_{k,t}, \lambda^*)$.

Individual leverage depends directly on counterparties' leverage, but also indirectly on the own level of leverage, as other banks' leverage depend on the former[20]. So individual leverage could be to some extent understood as a function of systemic leverage.

[19]The financial robustness is proxied by the equity ratio, defined as the inverse of the leverage ratio we consider: $1/\lambda_i, \lambda_i \in [1, \infty)$.

[20]We will face systems of leverage functions to be solved simultaneously.

1.2.3.1 Target Leverage

The premise for the following analysis is that the level of leverage of a bank at a certain point in time does not necessary correspond to the bank's target leverage.

Previous literature provides evidence that firms -banks, in our case- follow leverage targets (Graham and Harvey, 2001; Fama and French, 2002; Leary and Roberts, 2005; Flannery and Rangan, 2006 and Lemmon et al., 2008). The results differ regarding the speed of adjustment and the relative importance of targeting behaviour but the finding that firms actively rebalance capital structure in order to close the gap between the current and the targeted leverage appears robust.
There can be several reason for the leverage target to arise, for instance as a consequence of the imposition of macroprudential regulation -leverage ratio- or due to internal risk management leading banks to adopt a constraint based on the Value-at-Risk.

Value -at-Risk[21] has been identified as a driver in financial intermediaries' procyclical leverage policy (Adrian and Shin, 2013). Banks actively shed risks through adjustments on leverage in reaction to changing economic conditions. Even if banks can in principle react to an increase in market risk by either raising more capital or by cutting back their asset exposures, empirical evidence has exposed that they tend to do the latter. Equity is said to be relatively "sticky" in this sense (Adrian and Shin, 2011).
Along the lines of Adrian and Shin (2013), we can assume that leveraged banks adjust their balance sheets so as to maintain their Value-at-Risk, V_i, equal to their equity, e_i, as part of their risk management policy. This behaviour can be approximated by the so-called *Value-at-Risk rule*: $V_i = e_i$, and it is equivalent to targeting a reference leverage.
Being v_i the *unit VaR* or VaR per dollar of assets, and a_i the total assets of bank i, we have from the previous rule that $e_i = v_i * a_i$. This implies that bank i's leverage target satisfies:

$$\lambda_i^T = \frac{1}{v_i}$$

VaR is expected to be low in economic booms and low in busts, while (marked-to-market) leverage is typically high during expansions and low during recessions (Adrian and Shin, 2011).

Empirical evidence suggests, as documented by Adrian and Shin (2010), that marked-to-market financial intermediaries' leverage is procyclical. Leveraged agents manage actively

[21] The Value-at-Risk (VaR) is a quantile measure of the loss distribution defined as the expected maximum loss of a risky asset or portfolio over a defined period for a given confidence interval.

their balance sheets in response to fluctuations in asset prices, so during booms an expansion of balance sheets materialises as financial intermediaries expand both the assets and the liabilities sides. Analogously, during bursts banks shrink their balance sheets by contracting both the assets and the liabilities sides.

We are considering the following amplification mechanism: shocks in external asset prices set off a balance sheet management process that could lead to further feedback effects affecting the value of holdings of external assets, as banks adjust the quantity of externa securities in their balance sheets. The intensity of this process depends on the initial level of leverage.
That's to say, an initial shock in external assets prices can be amplified by asset market dynamics. In this case, the propagation factor will be the leverage and the higher the leverage, the higher the reaction.

Our model assumes that the banks adjust dynamically the actual level of leverage to a target. The distance from target could be defined as the absolute distance between the target leverage and the current leverage at the beginning of the period. We assume that the level at the beginning of one period is equal to the level of leverage at the end of the previous period. The logic in our model works as follows: the shock affecting the level of leverage takes place in one period, the adjustment of leverage will take place either during the following period, if considering full adjustment with a period, or during the following periods, if considering partial adjustment, as we will see.

1.3 The analysis of overlapping portfolios and leverage.

In order to analyse the propagation mechanism of financial distress, we firstly consider static nominal aggregates in banks' balance sheets and study analytically how the overlap structure works.

Distinct approaches are used to study in detail the relations among overlapping portfolios, depending on whether we express the market value of interbank debt by using the book or the marked-to-market value of interbank liabilities.
The determination of the systemic equilibrium is affected by the valuation methodology adopted: different measures of leverage lead to different leverage functions and, consequently, to different equilibriums. The chosen analytic approach will influence the results regarding both comparative statics and dynamic behaviour of leverage.

Given the analytical difficulty of studying the N-banks case, we would consider a complete and regular network and perform first the analysis on a financial system consisting of N-symmetric banks, subsequently moving on to a two-banks case.

1.3.1 Market value of assets, book value of liabilities.

In this section, we consider the marked-to-market value of internal assets and the book value of liabilities when defining the assets-to-equity ratio.

1.3.1.1 N-symmetric banks with regular and complete network.

Risk-taking behaviour of the management of financial institutions might be influenced by imitation of competitor's behaviour. It is relevant to analyse the case of symmetric banks as it allows to capture this imitation pattern.

High leverage can lead to a fast expansion of the asset size in the balance sheet of financial institutions and, therefore, to a maximization of the return on equity in the short-to-medium term (Avgouleas and Cullen, 2014). As leverage maximize shareholders returns, managers of financial institutions are incentivised to imitate competitor strategies as self-interested rational individuals. This behaviour results in agency costs, i.e. risk-taking, that may jeopardise the long-term stability of the financial institution and hence of the whole financial network due to interconnectedness.

When performing the analysis in a model of N-banks with symmetric balance sheets, we assume:

- Only one type of external assets, n_k.
- The amount of the external liabilities is given: $d_i = d$.
- Interbank debt is given: $b_i = b$.
- Interbank exposure is uniform: $\omega_{ij} = \frac{1}{N-1}$.
- Market perception of an agent's risk is uniform: $\xi_i = \xi$.

The balance sheet of bank i is:

Bank i

Assets	**Liabilities**
$p_k n_k$	$d_i = d$
	$b_i = b$
$l_i = l$	$e_i = e$

The leverage of bank i is[22]

$$\lambda_i = \frac{p_k n_k + \frac{1}{N-1}(N-1)\frac{b}{R+\xi\lambda}}{p_k n_k + \frac{1}{N-1}(N-1)\frac{b}{R+\xi\lambda} - (d+b)} \tag{1.2}$$

Rearranging and simplifying we get:

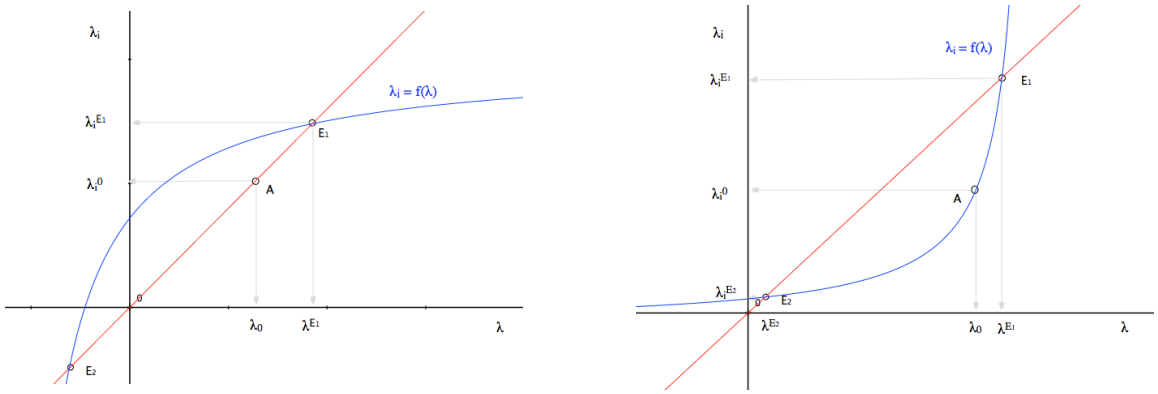
$$\lambda_i = \frac{p_k n_k + \frac{b}{R+\xi\lambda}}{p_k n_k + b\left(\frac{1-(R+\xi\lambda)}{R+\xi\lambda}\right) - d} \tag{1.3}$$

According to equation 1.3, the individual leverage λ_i is a function of "average" leverage λ[23]. It is easy to see that λ_i is increasing in λ as shown by the individual leverage curves in figure 1.2. The partial derivative of one bank's leverage with respect to its obligor's leverage is given by

$$\frac{\partial^2 \lambda_i}{\partial \lambda_j} = \frac{\frac{\xi b_j (d_i + b_i)}{(R+\xi\lambda_j)^2}}{\left[n_{i,k} p_k + \frac{b_j}{R+\xi\lambda_j} - (d_i + b_i)\right]^2}$$

[22]The expression for leverage won't depend on the size of the network.

[23]The analysis of the symmetric case leads to the same result regardless of the number of banks.



(a) Concave leverage curves

(b) Convex leverage curves

Figure 1.2: N-symmetric banks case: accounting equilibrium leverage.

The concavity of the individual leverage curves can not be assessed a priori, as the curvature of the leverage function depends on the particular balance sheet composition.

$$\frac{\partial^2 \lambda_i}{\partial \lambda_j^2} = \frac{\left[\frac{2\xi^2 b_j (d_i+b_i)}{(R+\xi\lambda_j)^3}\right]\left[n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j} - (d_i+b_i)\right]\left[\frac{b_j}{R+\xi\lambda_j} - \left(n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j} - (d_i+b_i)\right)\right]}{\left[n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j} - (d_i+b_i)\right]^4}$$

For the leverage curve to be concave, it must hold $n_{i,k}p_k > d_i + b_i$.

The symmetry condition is

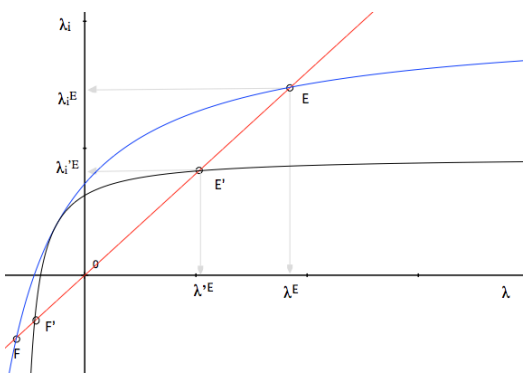
$$\lambda_i = \lambda \tag{1.4}$$

Substituting 1.4 in 1.3 we get the quadratic expression

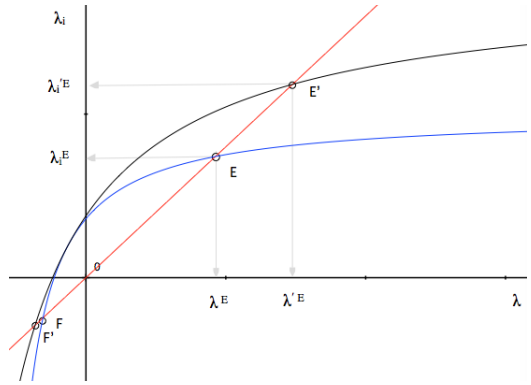
$$\lambda^2 \left[\xi \left(n_k p_k - d - b\right)\right] + \lambda \left[(R-\xi) n_k p_k - R(d+b) + b\right] - (R n_k p_k + b) = 0 \tag{1.5}$$

The solutions of equation 1.5 are the coordinates of points E_1 and E_2, i.e., the points of intersection between the individual leverage curve and the 45-degree line. Of course only the positive solution is economically meaningful. λ^E is the equilibrium leverage, i.e. the individual (and average) leverage in a complete regular network with identical banks.
Point E brings about consistency between the individual and the average leverage.
When λ is smaller than λ^E (see point A on the leverage curve), individual leverage would be grater than average: ${\lambda_i}^0 > \lambda_0$. This is true for each and every bank. Hence average leverage must be bigger than λ_0. Only at point E there is consistency (and symmetry). Furthermore, in the context of banks targeting a reference leverage ${\lambda_i}^T$, ${\lambda_i}^E = {\lambda_i}^T$ must hold to ensure that consistency.

Changes in exogenous variables will make the leverage curve shift and therefore will generate a new symmetric consistent equilibrium (see figure 1.3 and 1.4).

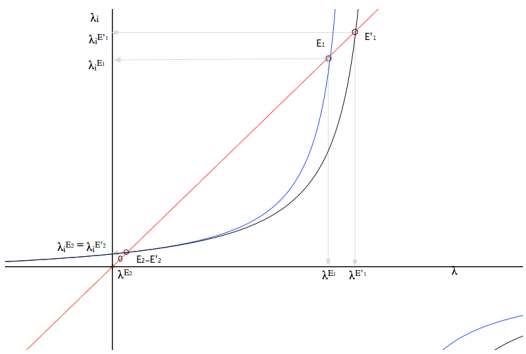


(a) Increase in the value of the external asset

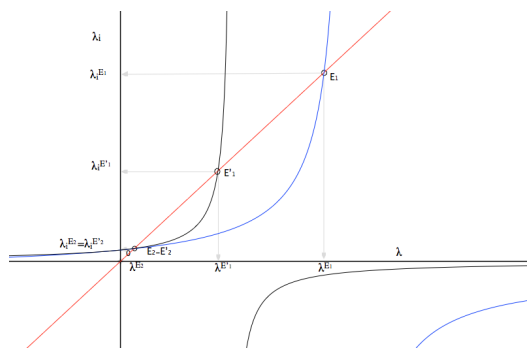


(b) Decrease in the value of the external asset

Figure 1.3: Effects of shocks in exogenous variables, concave curves



(a) Increase in the value of the external asset



(b) Decrease in the value of the external asset

Figure 1.4: Effects of shocks in exogenous variables, convex curves

Expression 1.5 is linearised to analyse the behaviour of the original non-linear equation around a point λ^*. We impose $\lambda^* = \lambda^T$[24]. The final expression

$$\lambda = \frac{(\lambda^*)^2 \left[\xi \left(n_k p_k - d - b\right)\right] + R n_k p_k + b}{2\lambda^* \xi \left(n_k p_k - d - b\right) + \left(R - \xi\right)\left(n_k p_k\right) - R(d+b) + b} \tag{1.6}$$

represents the coordinate of the intersection points on the 45-degree line.

The existence of the solution to equation 1.5 is subject to some conditions, as the expression for the intersection point depends on the specific structure of the balance sheets. Therefore, it's not straightforward to come analytically to sound and general conclusions. For λ to be positive, it must hold that:

$$n_k p_k > f(d, b, \lambda^*) = \frac{2\lambda^*\xi + R}{\xi(2\lambda^* - 1)R} d + \left[\frac{2\lambda^*\xi + R}{\xi(2\lambda^* - 1)R} - 1\right] b > 1$$

[24] As we are considering fully-leveraged banks, we are interesting in understanding the behaviour of leverage near the equilibrium, λ^T.

For the purpose of delving into the static properties and effects of overlaps among bank portfolios, we are interested in studying the behaviour of leverage, as defined in expression 1.6.
First, we measure the sensitivity of individual leverage to changes in the target leverage. We expect bank i's leverage to be a positive function of the target leverage. After calculating the partial derivative of one bank's leverage with respect to this target, we obtain that this derivative is positive only if the market value of external assets is greater than a term proportional to the value of total debt (with the coefficient of proportionality larger than 1).
In addition, leverage is inversely related to external asset prices, i.e., decreases in external assets prices lead to increases in leverage.

1.3.1.2 2-banks framework.

We now proceed to study analytically the mechanism of overlap on a financial system consisting of two banks. As in the previous case, the system of non-linear equations given by 1.1 is linearised, being $i = 1, 2$ now. By solving the system, the intersection of the linearised leverage curves are found and the feasible equilibrium points are identified, therefore.

For the system to be in an accounting-consistent equilibrium, $\lambda_i = {\lambda_i}^E$ must hold in the two-banks case too. As shown in figure 1.16, if the initial individual level of leverage ${\lambda_i}^0$ is different from the equilibrium leverage ${\lambda_i}^E$, there would be no consistency as the counterpartie's accountant leverage ${\lambda'}_{-i}$ and the effective one, λ^0_{-i} will not coincide. So point E is the only feasible point ensuring consistency between individual leverages[25].

Henceforward, we will keep some of the previous assumptions:

- Only one type of external assets, n_k.
- Market perception of an agent's risk is uniform: $\xi_i = \xi$.

[25] When fully-leveraged banks are pursuing a target leverage, the accounting equilibrium is given by an intersection vector of the form ${\lambda_i}^* = \lambda_i^T$.

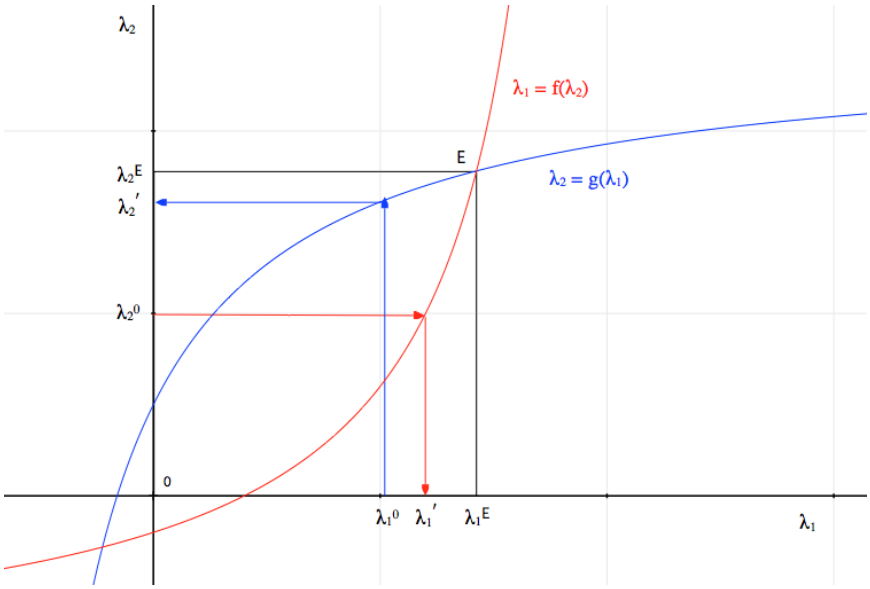


Figure 1.5: 2-banks case: accounting equilibrium.

1.3.1.2.1 2-symmetric banks.

The analysis in this case is equivalent to the one conducted in the a financial network consisting of N-symmetric banks.

We have previously assumed that $\lambda^* = \lambda^T$ and argued that only the condition $\lambda_i = \lambda_i^E$ ensures consistency. In a system consisting of two symmetric banks, it is straightforward to see that in the equilibrium $\lambda^* = \lambda_i^E = \lambda^T$ must hold.

Only if the system is linearised around the point $\lambda_i{}^* = \lambda_i^E = \lambda^T$ is the existence of the equilibrium -given by the intersection of the linearised curves- ensured. When leverage functions are estimated near any other points (See points A and B in figure 1.6), the equilibrium could be achieved at either significantly higher levels of leverage or at negative values for λ_i, that we exclude by assumption.

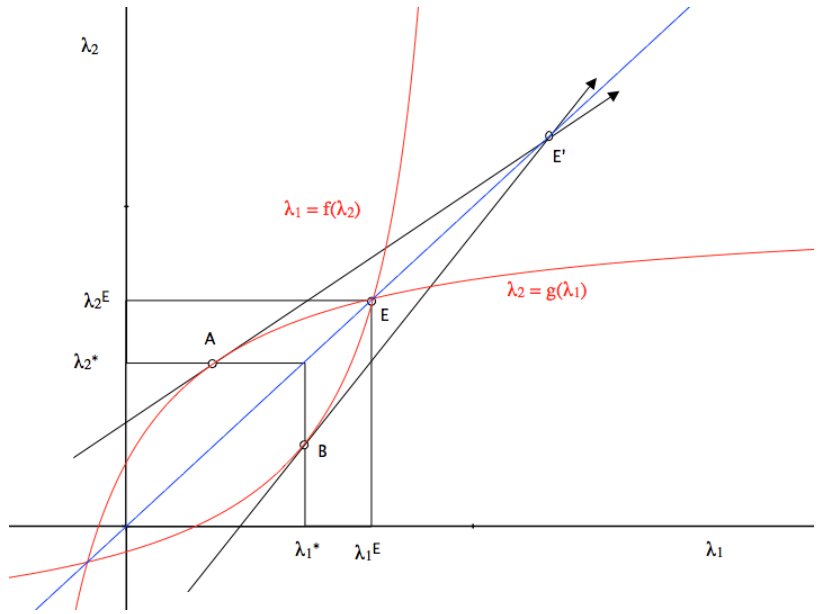


Figure 1.6: Linearisation of leverage functions: consistent equlibrium.

1.3.1.2.2 2-asymmetric banks.

The expression for leverage in the specific case of a banking system formed by two asymmetric banks is[26]:

$$\lambda_i = \frac{n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_2}}{n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j} - (d_i + b_i)} \tag{1.7}$$

As done previously, we study the behaviour of the leverage curves in the two-dimensional coordinate plane.

The sensitivity of individual leverage to changes in the leverage of its obligor is given by the following expression

$$\frac{\partial\lambda_i}{\partial\lambda_j} = \frac{\frac{\xi b_j}{(R+\xi\lambda_j)^2}(d_i + b_i)}{\left(n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j} - (d_i + b_i)\right)^2} > 0 \tag{1.8}$$

Bank i's leverage is a positive function of i's obligor one. Therefore, increases in counterparties' leverage lead to increases in own leverage. Furthermore, this derivative is an expression depending on the debt-to-equity ratio -an alternative measure for leverage[27]-,

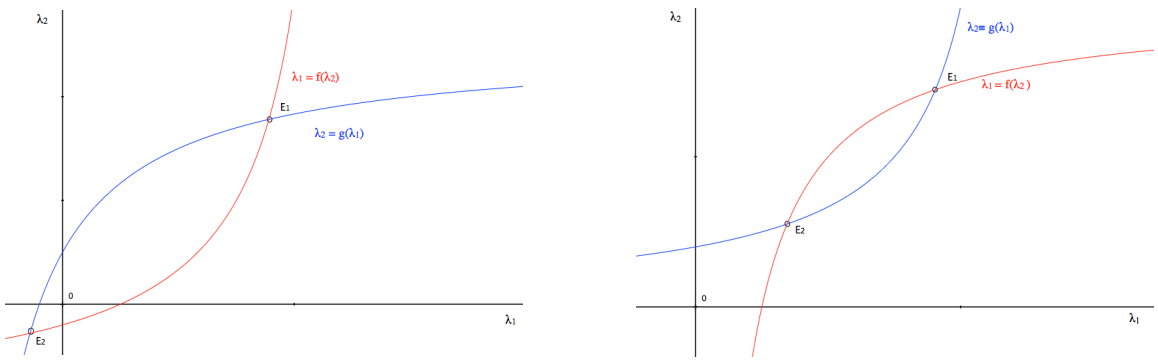
[26] In this particular case, $\sum_j \omega_{ij} = \omega_{12} = \omega_{21} = 1$.

[27] The debt-to-equity ratio is a debt ratio closely related to leveraging that measures the riskiness of a company's (bank) financial structure

and it turns out the higher the own leverage, the higher the response to changes in counterparties' leverage.

From the analysis of the second derivative capturing the curvature of the leverage function, $\frac{\partial^2 \lambda_i}{\partial \lambda_j^2}$, it cannot be deduced a priori whether the change of the rate of change of bank i's leverage is increasing or decreasing in λ_j, as already shown. It would depend on whether the holdings of external assets are smaller than total liabilities -convexity- or not -concavity-. That's to say, on whether the exposure to the interbank network is greater than equity or not.

The representation of the leverage curves in the phase plane could be as follows:



(a) Concave case

(b) Convex case

Figure 1.7: Leverage curves

It should be noticed that, while concavity makes the existence of an intersection point more probable, convexity doesn't. It depends on the specific structure of the balance sheets, as already seen.

The effects of the exposure to a counterparty debt, b_j, on own-leverage responsiveness to counterparties' leverage cannot be assessed a priori, as well. Higher exposures will lead to steeper leverage curves whenever the value of external assets net of internal assets is bigger that the value of total liabilities: $n_{i,k}p_k - \frac{b_j}{R+\xi\lambda_j} > (d_i + b_i)$.
On the contrary, it's straightforward to state that the riskier the counterparty, i.e., the higher ξ, the steeper the slope of the leverage curve. (For in-depth considerations, we refer the reader to appendix A.1.1.)

Regarding the effect of shocks in external asset prices on leverage, captured by

$$\frac{\partial \lambda_i}{\partial p_k} = \frac{-n_{i,k}\left(b_i + d_i\right)}{\left(n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j} - \left(d_i + b_i\right)\right)^2} < 0 \tag{1.9}$$

we deduce that leverage is inversely related to external asset prices. Moreover, the debt-to-equity ratio defines the magnitude of 1.9, meaning that the higher the leverage, the higher the response to changes in external asset prices.
In addition, it's observed that, for banks having more exposure to external than to internal assets, $n_{ik}p_k - \frac{b_j}{R+\xi\lambda_j} > d_i + b_i$, the higher the exposure to the external asset, the higher the response of leverage to shocks in prices.

When solving the system of equations given by 1.7, we face quadratic equations[28] (Appendix A.1.2.). Should the system be linearised, consistency is verified only when linearisation is performed around the point $\lambda_i{}^* = \lambda_i^E = \lambda^T$. Whenever leverage functions are estimate near any other points (See points A and B), inconsistency arises between effective and accounting individual levels of leverage.

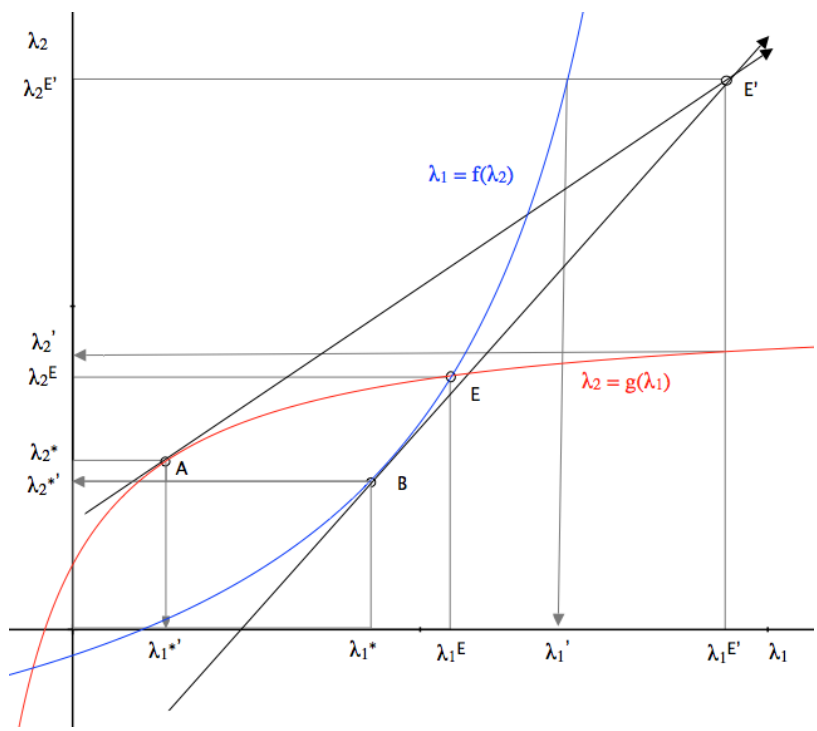


Figure 1.8: Linearisation of leverage functions: consistent equilibrium.

[28]The existence of real solutions depends on the sign of the expression $n_{i,k}p_k - (d_i + b_i)$.

1.3.2 Market value of internal assets and liabilities.

Finance theory mainly focuses on the market value of debt, in contrast with the widespread use of book value in empirical finance. Since most corporate debt is traded over the counter and infrequently, market value of debt is usually more difficult to obtain directly.

In previous sections we have adopted the approach followed in empirical finance, by using book value of interbank liabilities, b_i, as a proxy for market value of interbank debt, and the marked-to-market values of internal assets.
Given that within the interbank market in our model one bank's liabilities are other banks' assets, being able to measure the market value of interbank assets means being capable of assessing the market value of interbank debt. Therefore, in this section leverage will be couched in terms of market value of both interbank assets and liabilities.

The general expression for leverage becomes

$$\lambda_{it} = \frac{\sum_k n_{ik,t} p_{k,t} + \sum_j \omega_{ij} \frac{b_{j,t}}{R+\xi\lambda_{j,t}}}{\left(\sum_k n_{ik,t} p_{k,t} + \sum_j \omega_{ij} \frac{b_{j,t}}{R+\xi\lambda_{j,t}}\right) - \left(d_{i,t} + \frac{b_{i,t}}{R+\xi\lambda_{i,t}}\right)} \tag{1.10}$$

1.3.2.1 N-symmetric banks with regular and complete network.

Keeping the assumptions made in section 1.3.1, the expression for leverage in this case is given by

$$\lambda_i = \frac{n_k p_k + \frac{b}{R+\xi\lambda}}{n_k p_k - d} \tag{1.11}$$

Substituting 1.4 in 1.11 solving for λ we get the following quadratic expression:

$$\lambda^2 \left[\xi \left(n_k p_k - d\right)\right] + \lambda \left[R \left(n_k p_k - d\right) - \xi n_k p_k\right] - \left(R n_k p_k + b\right) = 0 \tag{1.12}$$

that could be linearised following a Taylor approximation around λ^*. The final expression results:

$$\lambda = \frac{(\lambda^*)^2 \left[\xi \left(n_k p_k - d\right)\right] + R n_k p_k + b}{2\lambda^* \xi \left(n_k p_k - d\right) + R \left(n_k p_k - d\right) - \xi n_k p_k} \tag{1.13}$$

As previously explained, the solutions of equation 1.13 are the coordinates of the equilibrium points λ^E bringing out consistency between either the individual and average leverage (N banks), or individual levels of leverage (2 banks). In addition, the existence of an economic-meaningful solution depends on the structure of the balance sheet and will be ensured when the following condition holds

$$n_k p_k > f(d, \lambda^*) = \frac{2\lambda^* \xi + R}{\xi(2\lambda^* - 1) + R} d$$

In this scenario, leverage is increasing in the target level as well, provided that the market value of external assets is greater than the value of external debt, i.e., the bank has to be creditor with respect to the the external network.

1.3.2.2 2-asymmetric banks.

Recalling the expression for leverage given by equation 1.10 and the assumptions previously made regarding the uniqueness of the external asset and the uniformity of risk perception, the leverage of bank i is

$$\lambda_i = \frac{n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j}}{n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j} - \left(d_i + \frac{b_i}{R+\xi\lambda_i}\right)} \tag{1.14}$$

so the banking system of 2-asymmetric banks is represented by a set of non-linear equations

$$\left[\xi\left(N_i - d_i\right) - 1\right]\lambda_i^2 + \left[N_i\left(R-\xi\right) - d_i R\right]\lambda_i - N_i R = 0 \tag{1.15}$$

The system of quadratic expressions given by equation 1.15 is solved using the first order term[29] of the Taylor expansions around a point $\lambda^* = (\lambda_1^*, \lambda_2^*) = (\lambda_1^T, \lambda_2^T)$.
We distinguish between two cases, depending on whether the target leverage is the same for both banks or not.

1.3.2.2.1 Linearisation around λ^*, general case: $\lambda_1^* \neq \lambda_2^*$.

By linearising 1.15, and after some algebra, we have a system of two linear equations with 2 unknowns, λ_1, λ_2, that can be solved for λ_1 and λ_2, respectively:

$$\lambda_1 = f\left(\lambda_2, \lambda^*, p_k\right) = \left(\left[n_{1,k}p_k + \frac{b_2}{R+\xi\lambda_2^*}\right]\left[\xi\left(2\lambda_1^* - 1\right) + R\right] - 2\lambda_1^*\left(\xi d_1 + 1\right) - d_1 R\right)^{-1}$$
$$\left[\lambda_2^* \frac{\xi b_2}{\left(R+\xi\lambda_2^*\right)^2}\left(R - \lambda_1^*\left[\xi\lambda_1^* + \left(R-\xi\right)\right]\right) - \left(\lambda_1^*\right)^2\left[1 + \xi d_1 - \xi\left(n_{1,k}p_k + \frac{b_2}{R+\xi\lambda_2^*}\right)\right] + R\left(n_{1,k}p_k + \frac{b_2}{\left(R+\xi\lambda_2^*\right)}\right)\right.$$
$$\left. + \lambda_2 \frac{\xi b_2}{\left(R+\xi\lambda_2^*\right)^2}\left(R - \lambda_1^*\left[\xi\lambda_1^* + \left(R-\xi\right)\right]\right)\right] \tag{1.16}$$

$$\lambda_2 = g\left(\lambda_1, \lambda^*, p_k\right) = \left(\left[n_{2,k}p_k + \frac{b_1}{R+\xi\lambda_1^*}\right]\left[\xi\left(2\lambda_2^* - 1\right) + R\right] - 2\lambda_2^*\left(\xi d_2 + 1\right) - d_2 R\right)^{-1}$$
$$\left[\lambda_1^* \frac{\xi b_1}{\left(R+\xi\lambda_1^*\right)^2}\left(R - \lambda_2^*\left[\xi\lambda_2^* + \left(R-\xi\right)\right]\right) - \left(\lambda_2^*\right)^2\left[1 + \xi d_2 - \xi\left(n_{2,k}p_k + \frac{b_1}{R+\xi\lambda_1^*}\right)\right] + R\left(n_{2,k}p_k + \frac{b_1}{\left(R+\xi\lambda_1^*\right)}\right)\right.$$
$$\left. + \lambda_1\left[\frac{\xi b_1}{\left(R+\xi\lambda_1^*\right)^2}\left(R - \lambda_2^*\left[\xi\lambda_2^* + \left(R-\xi\right)\right]\right)\right]\right] \tag{1.17}$$

[29] Second order and higher terms will contribute little to the sum.

When solving the previous system, we get expressions of the form $\lambda_1 = F(\lambda^*, p_k)$ and $\lambda_2 = G(\lambda^*, p_k)$. (See Appendix A.2.1 for further details).

1.3.2.2.2 Linearisation around λ^*, general case: $\lambda^* = \lambda_1^* = \lambda_2^*$.

The target leverage for both banks being identical could be equivalent to the imposition of a macroprudential leverage ratio, for instance.
In that case, we get a system of equations $\lambda(\lambda_i, \lambda_j)$ as follows:

$$
\begin{aligned}
&f(\lambda_1, \lambda_2) \approx \\
&\lambda_1 \left(\left[n_{1,k} p_k + \frac{b_2}{R + \xi\lambda^*} \right] [\xi(2\lambda^* - 1) + R] - 2\lambda^* (\xi d_1 + 1) - d_1 R \right) + \lambda_2 \left[\frac{\xi b_2}{(R + \xi\lambda^*)^2} (R - \lambda^* [\xi\lambda^* + (R - \xi)]) \right] \\
&\quad + (\lambda^*)^2 \left[1 + \xi d_1 - \xi \left(n_{1,k} p_k + \frac{b_2}{R + \xi\lambda^*} \right) \right] - \lambda^* \frac{\xi b_2}{(R + \xi\lambda^*)^2} (R - \lambda^* [\xi\lambda^* + (R - \xi)]) \\
&\qquad - R \left(n_{1,k} p_k + \frac{b_2}{(R + \xi\lambda^*)} \right) = 0 \quad (1.18)
\end{aligned}
$$

$$
\begin{aligned}
&g(\lambda_1, \lambda_2) \approx \\
&\lambda_2 \left(\left[n_{2,k} p_k + \frac{b_1}{R + \xi\lambda^*} \right] [\xi(2\lambda^* - 1) + R] - 2\lambda^* (\xi d_2 + 1) - d_2 R \right) + \lambda_1 \left[\frac{\xi b_1}{(R + \xi\lambda^*)^2} (R - \lambda^* [\xi\lambda^* + (R - \xi)]) \right] \\
&\quad + (\lambda^*)^2 \left[1 + \xi d_2 - \xi \left(n_{2,k} p_k + \frac{b_1}{R + \xi\lambda^*} \right) \right] - \lambda^* \frac{\xi b_1}{(R + \xi\lambda^*)^2} (R - \lambda^* [\xi\lambda^* + (R - \xi)]) \\
&\qquad - R \left(n_{2,k} p_k + \frac{b_1}{(R + \xi\lambda_1^*)} \right) = 0 \quad (1.19)
\end{aligned}
$$

, from which we we can deduce expressions of the form $\lambda_i = \lambda(\lambda^*, p_k)$, that are expressed in terms of differences of holdings (of external/internal assets and liabilities) between banks (See appendix A.2.2.).

When studying the behaviour of non-linear leverage curves given by expression 1.14, we find bank i's leverage is a positive function of i's obligor one, and is inversely related to external asset prices, as expected. Additionally, it's immediate to see that the value of already-issued own debt valued at market prices decreases as the financial robustness of the issuer deteriorates. The bigger bank i's leverage, the smaller the present value of i's interbank liabilities, as own leverage enters the expression of the rate at which the value of interbank debt is discounted. Hence, the weight of own liabilities in the balance sheet diminishes[30].

[30] The rate at which the interbank debt in the books has been issued is in any case lower than the one of new issuances given the financial deterioration of the issuer.

However, the sign of the relations mentioned above is ambiguous for expressions of the form $\lambda_i = \lambda_i(p_k, \lambda^*)$ resulting from linearisation. Regarding the response of leverage to changes in counterparties' leverage, additional conditions must hold in order to have the expected results.
For instance, the corresponding derivative is positive provided that the value of total assets is higher than a term proportional to external funds in the case $\lambda_1^* \neq \lambda_2^*$:

$$n_{i,k}p_k + \frac{b_j}{R + \xi\lambda^*} > \frac{2\lambda^*(\xi d_i + 1) + Rd_i}{\xi(2\lambda^* - 1) + R} \tag{1.20}$$

while in the case $\lambda_1^* = \lambda_2^*$, λ^* must additionally lie within the interval $[R, \infty]$. Given the complexity of the linearised equations, the derivatives with respect the target leverage and the external asset price are hard to be assessed analytically.

We can perform additional analysis within the scenario $\lambda_1 = \lambda_2 = \lambda^*$ -equivalent to the application of macro prudential policy-, considering different lending profile of banks and different investment strategies. We refer the reader to appendix A.2.4 for the study of theses cases.

Also, an alternative approach to the analysis of leverage behaviour in the presence of symmetry is provided in Appendix C.

In this paper we contribute to the analysis of the contagion mechanism by delving analytically into the reasons for portfolio overlap and document the explanatory role of exposure to other financial institutions' distress (through leverage, identified as the systemic risk component of financial intermediaries' balance sheets) for amplification mechanisms.
In this section, we have mathematically proven conditions for the existence of overlap among portfolios.

Distinct approaches are used regarding the valuation of interbank assets and liabilities. The common accounting practice has been to carry financial assets or liabilities on balance sheets at the book value. Unlike book value accounting, where the value of an asset is determined by its balance sheet account balance, market value reflects the market price. The revaluation of assets and liabilities allows for immediate recognition of profit and losses on financial instruments, which brings transparency to the balance sheet and reflect changes in market conditions that do not affect book values.

Even if it could be valuable to maintain historical cost basis of financial portfolios for taxation purposes, i.e. determination of gains and losses, or even in the event of bankruptcy,

when the relevant measure of debt holder's liability is the book value of debt rather that its market value (Banerjee, Heshmati and Wihlborg (2000), potential danger arises when relying exclusively on book value accounting. Other reason for using book values might be the relative facility and accuracy with which they can be measured. On the other hand, the market value reflects the real value of a firm.

During the decade prior to the 2008 financial crises, the seek for higher-investment yields led to the propagation of derivative instruments and securitization. Debt instruments where pooled for sale regardless of their creditworthiness. Even if there is a general trend towards the requirement to record derivatives at fair value on the balance sheet, the unsold debt instruments were held in books valued at book value, without reflecting their increasing deterioration of credit quality. Furthermore, the accounting treatment of securitizations can lead to leverage understatements. This highlights the importance of the accounting methodology adopted.

In some cases, both book and market measures of leverage are used in previous literature (Rajan and Zingales, 1995).
The utility of market value was specially discussed after the 2008 financial crisis. At the onset of the crises, financial institutions were highly leveraged. At high levels of leverage, even a small drop in the value of assets lead to considerable capital erosion. The value of assets and liabilities is subject to changing conditions of different markets (interest rates, equity, credit...). Exposure to market risk exists regardless of the accounting methodology.

The last financial crisis, which was driven largely by the collapse of the subprime mortgage bubble in US and problems in mortgage markets also in UK and Europe, has stressed the appropriateness of adopting market value accounting. The financial conditions had been precarious in the run-up to the last financial crisis. Market value accounting facilitates early identification of erosion of financial conditions, contributing to a more efficient management of financial risk and, therefore, to reduce the likelihood of insolvency.

Furthermore, financial leverage ratios incorporating marked-to-market measures of debt exhibits a greater association with market volatility than booked-value marked counterparts (Mulford, 1985). It follows that accounting measures may not provide good estimates for the market value of debt. Even if it is conventional to take the book value of debt as its market value, this approximation would be questionable in cases of high volatility of interest rates or credit quality (Mulford, 1985).

Additionally, the process of calculating market value should be fundamentally the same for both assets and liabilities in the interbank market context, as one bank's assets are other bank's liabilities.

In the following section, we show how the leverage dynamics become unstable when considering the marked-to-market leverage ratio, whereas this instability does not manifest itself when the book-value leverage ratio is considered. Hence, we can affirm that market value makes volatility more transparent.

1.4 Modelling leverage dynamics

This section studies the dynamics of leverage. As in the model of Banerjee, Heshmati, and Wihlborg (2000), we assume that the actual capital structure of a firm at a particular time does not necessarily equal the target capital structure of that firm. Firms -banks, in our case- adjust dynamically their capital structures -leverage- to a target.

1.4.1 The adjustment model

In the optimum, the leverage of a firm will equal its target leverage (de Haas and Peters, 2004). Ideally, the observed leverage of a firm i at time t should be not different from the optimal leverage, which implies that the change in actual leverage from the previous to the current period should be exactly equal to the change required for the firm to be at optimal at time t. The existence of adjustment costs may prevent firms from fully adjusting (Banerjee, Heshmati and Wihlborg, 2000).

The analysis of firms' adjustment toward a target leverage is essential in evaluating the credibility of capital structure theories[31]. Xu (2007) proofs that firms try to maintain the leverage ratio after reaching their optimum. The existing capital structure theories have different predictions about firms' adjustment mechanism toward a target leverage level. The static and dynamic trade-off theories predict that firms quickly revert to their optimal leverage ratio in response to deviations in their capital structures.

The static trade-off theory argues that firms select a value-maximizing leverage ratio by trading off the expected bankruptcy costs and tax benefits of debt. When shocks cause deviations from this optimum, firms will quickly rebalance toward the target. However, the adjustment will be incomplete in the presence of adjustment costs.

The dynamic trade-off theory suggests a firm's decision on adjustment is based on the trade-off between adjustment costs and deviation costs, i.e. the costs of operating with a suboptimal capital structure. The firm will readjust when the deviation costs outweigh

[31] Namely, the tradeoff, pecking order and market timing theories.

the adjustment costs. The speed of the adjustment will depend on firm's compliance in adjusting their leverage to meet the target leverage. When firms exhibit a strong compliance, deviations from the target leverage would be temporary and the speed of adjustment relatively fast. In contrast, the adjustment in response to a shock will be slow when firms are indifferent about leverage ratios.

This adjustment can be expressed using discrete dynamics as follows:

$$\lambda_{i,t} - \lambda_{i,t-1} = \epsilon_{i,t} \left(\lambda^T_{i,t-1} - \lambda_{i,t-1} \right) \tag{1.21}$$

or analogously

$$\lambda_{i,t} = (1 - \epsilon_{i,t})\lambda_{i,t-1} + \epsilon_{i,t}\lambda^T_{i,t-1} \tag{1.22}$$

where $\lambda^T_{i,t}$ is bank' i's target leverage at time t and $\epsilon_{i,t}$ is the adjustment speed during one period.
By specifying ${\lambda_{i,t-1}}^T$ rather than ${\lambda_{i,t}}^T$ it is assumed that the adjustment during the year t is made on the basis of beginning-of-year firm characteristics and that targets are not being revised during the year. Furthermore, from an econometric perspective, this specification reduces the potential for simultaneity bias since all determinants are included with a one-year lag in the estimations.

Equation 1.21 captures the standard partial adjustment model commonly used in literature for analysing the capital structure dynamics (de Haas and Peeters, 2006; Banerjee, Heshmati and Wihlborg, 2000) and estimating the speed of adjustment.

There are several other studies on the adjustment of leverage towards its target (Fischer, Henkel and Zechner, 1989; Leary and Roberts, 2005). In Fama and French (2002), for instance, the following partial adjustment model is estimated:

$$y_t = \alpha + \lambda(y_{t-1} - \mu_{t-1}) + \epsilon_t$$

where y is a measure of leverage and μ is the leverage target, itself often a function of other variables.
The speed (or degree) of adjustment at which the observed leverage ratio adjusts toward the target is commonly estimated using the standard partial adjustment model specified above[32].

[32] The speed of adjustment may itself be a function of some underlying variables affecting adjustment costs. We consider the dependency on the amplitude of the leverage gap.

Banerjee, Heshmati and Wihlborg (2000) discuss that if $\epsilon_{i,t} = 1$, then the entire adjustment is made within one period and the firm at time t is at its target leverage and, consequently, leverage at the end of the period will be equal to the beginning-of-period target: $\lambda_{i,t} = \lambda^T_{i,t-1}$, while if $\epsilon_{i,t} \leq 1$, then the adjustment from year t-1 to t is not enough to attain the target. On the other hand, $\epsilon_{i,t} \geq 1$ means that the firm adjusts more than is necessary, hence not achieving the optimal target. We disregard the latter scenario for the purposes of the analysis of leverage dynamics in this section, as we assume that it would take at least one period to the firm to achieve the leverage target. The value of the parameter ϵ capturing the bank's promptness in adjusting leverage deviations from target lies within the range [0,1].
Nevertheless, the leverage dynamics simulation in section 1.5 incorporates this scenario as short-sighted banks may over-adjust as they are not able to anticipate counterparties' adapting behaviour and, therefore, the effect of overlapping portfolios on their level of leverage.

In our case, the speed of adjustment depends on the distance between the target leverage and the actual leverage at the beginning of the period: $|\lambda^T_{i,t} - \lambda_{i,t-1}|$. In this way, we endogenize to some extent this parameter and also allow not only for adjustment across periods but for heterogeneity. The speed of adjustment will vary in time and differ among banks, given their target leverages and the actual levels of leverage affected by external-assets prices shocks.

Empirical evidence supports the existence of different rates of adjustment towards a target leverage among firms: over and under leveraged firms will adjust toward their targets at different rates, due bankruptcy costs, see e.g. Hovakimian et al. (2001), Faulkender, Flannery, Hankins, and Smith (2007). As shown in Elliott, Koeter-Kant and Warr (2008), firms above their optimal target hit a "hard" boundary and those that are below their target face a "soft" boundary, which translates to more rapid rates of adjustment for firms above their target, relative to those below their target.
Hence, fims's mean reverting behaviour toward a leverage target is evident and suggests not only that over leveraged firms revert more rapidly to the target leverage, but also that under leveraged firms adjust toward their target leverage at less than half the rate of over leveraged firms.

We are not interested in the absolute distance between the target and the actual leverage, but in the sign of this magnitude, in order to establish whether a bank is over or under leveraged. Since this distance from target is calculated as the target leverage minus the

observed, over-leveraged institutions will have a negative value for distance, whereas it will be positive for under-leveraged ones.

Regarding the assumption on the set of values the bank's promptness in adjusting leverage deviation from target can assume, the pecking order hypothesis predicts the adjustment speed to be zero, whereas according to the trade-off model the speed of adjustment should lie between zero and one (Fama and French, 2002).
Flannery and Ragan (2006) find that the speed of adjustment is 34,1% per year and argued that this fast adjustment is consistent with the dynamic trade-off theory. Additionally, they argue that targeting behaviour is evident in both market-valued and book-valued leverage measures and that their conclusions do not depend on the definition of the market debt ratio. Banerjee, Heshmati and Wihlborg (2000) estimate an average adjustment speed of 21% (UK data) and of almost 40% for US.

Asymmetries in the adjustment processes could be captured by setting different values for the adjustment parameter, depending on whether the bank is under or over leveraged. For the sake of simplicity, a value of 1 could be assigned to the adjustment parameter of over leveraged banks, and of 0,5 in the case of under leveraged ones. So banks with a negative distance would adjust at a faster rate (reducing 100 % of the distance from their target per period) than banks with positive distance (reducing 50 % of the distance per period). That's to say, for over leveraged banks full adjustment would be achieved within a period, while under leveraged banks would accomplish partial adjustments.

$$\epsilon_{i,t} = \begin{cases} 1 & \text{if } \lambda_{i,t}^T - \lambda_{i,t} < 0 \\ 0.5 & \text{if } \lambda_{i,t}^T - \lambda_{i,t} > 0 \end{cases}$$

The alternative adjustment model discussed in section 1.4.2 is one variant of the standard model that incorporate banks' expectations about their counterparties' adapting behaviour. This allows us to incorporate the effects of overlapping portfolios in the dynamics of the system.

1.4.1.1 Qualitative analysis

We now study the stability properties of the system given by the expression of the adjustment discrete dynamics in the case of two banks.
Initially, we don't account for interdependency of balance sheets. Later on, we analyse the dynamics considering overlap[33].

[33] We will simulate the system considering fully-leveraged banks affected by a shock in external asset prices

Whether a capital requirement policy implies a leverage ratio or the bank adopt a leverage target based on the VaR, a constraint of the form $\lambda_{i,t} \leq \lambda^T$ applies. We assume that banks always target this maximum allowed leverage.
Insofar as we are interested in analysing the behaviour of fully-leveraged or strategic banks for the purposes of this paper[34], banks initially perform at the level of committed leverage, i.e. they enter the dynamics system at the leverage requirement or target leverage. Hence, the analysis of leverage dynamics is performed around the equilibrium.
This is consistent with our purpose of delving into the spillover of sovereign risk to the banking system in the EU and of replicating the evidence from the last European sovereign debt crisis: the deterioration of banks' assets due to the Eurozone crisis and the negative response exhibited by sovereign exposures of highly leveraged financial institutions to increases in yield.

1.4.1.1.1 Book-value-based leverage ratio.

What we have is an autonomous first-order nonhomogeneous dynamic discrete system given by expression 1.22. We are assuming an exogenous target leverage, being $\lambda_{i,t}^T = \lambda_i^T$ a parameter set at time 0. In the case of two banks[35]:

$$\lambda_{1,t} = (1 - \epsilon_1)\lambda_{1,t-1} + \epsilon_1 \lambda_1^T \tag{1.23}$$

$$\lambda_{2,t} = (1 - \epsilon_2)\lambda_{2,t-1} + \epsilon_2 \lambda_2^T \tag{1.24}$$

or

$$\begin{bmatrix} \lambda_{1,t} \\ \lambda_{2,t} \end{bmatrix} = \begin{bmatrix} 1-\epsilon_1 & 0 \\ 0 & 1-\epsilon_2 \end{bmatrix} \begin{bmatrix} \lambda_{1,t-1} \\ \lambda_{2,t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_1 \lambda_1^T \\ \epsilon_2 \lambda_2^T \end{bmatrix} \tag{1.25}$$

[34]Present research could be extended by introducing an additional type of bank's financial behaviour. Banks could be considered as value investors, whose investing strategies would be ruled by a mispricing signal: banks take long positions in a specific asset whenever the asset price is below an agent-dependent fundamental value (Poledna, Thurner, Farmer and Geanakoplos, 2014). The strength of a bank's response to this mispricing signal would then depend on the so called "aggression parameter".
This approach would allow banks to perform far from (under) the efficient level of committed leverage. The analysis of the dynamic evolution of the system would be sensible in this case but it is out of the scope of this research, as we consider fully-leverage financial institutions.

[35]Given that $\epsilon_{i,t}$ is set whenever the leverage gap is not zero, it is considered as a constant during the periods affected by the adjustment process.

The equilibrium solution for the previous system is:

$$\lambda_i^* = \lambda_i^T \tag{1.26}$$

, being $i = 1, 2$. Once we have established that an equilibrium exists, we solve the system in order to study its stability. Considering the system in terms of deviations from equilibrium,

$$\lambda_{i,t} - \lambda_i^* = (1 - \epsilon_i)(\lambda_{i,t-1} - \lambda^*) \tag{1.27}$$

after solving for the eigenvalues $r = 1 - \epsilon_2$ and $s = 1 - \epsilon_1$[36] and finding the associated eigenvectors, we have that the solution would be:

$$\lambda_{i,t} = \lambda_i^* \tag{1.28}$$

It would be interesting to represent the previous system in its canonical form in order to call attention to the fact that, even if not having specified so far the expression for a bank's leverage as a function of their counterparties' leverage, the solution for $\lambda_{i,t}$ depends on parameters regarding the counterparty, such as the counterparty's adjustment rate and initial leverage.

$$z_{i,t} = (1 - \epsilon_j)^t \lambda_{j,0} \tag{1.29}$$

But we can gain insight into the dynamics of this system by looking at the phase plane. Considering the following system outlining discrete changes in λ_1 and λ_2:

$$\triangle\lambda_{1,t} = \epsilon_1(\lambda_1^T - \lambda_{1,t-1}) \tag{1.30}$$

$$\triangle\lambda_{2,t} = \epsilon_2(\lambda_2^T - \lambda_{2,t-1}) \tag{1.31}$$

We have already established that in equilibrium $\triangle\lambda_{1,t} = 0$; $\triangle\lambda_{2,t} = 0$, so $\lambda_1^* = \lambda_1^T$ and $\lambda_2^* = \lambda_2^T$, and the isoclines in the phase plane are given by $\lambda_{1,t-1} = \lambda_1^T$ and $\lambda_{2,t-1} = \lambda_2^T$, respectively.

Considering now points either side the isoclines: to the left (right) of the isocline $\triangle\lambda_{1,t} = 0$, variable $\lambda_{1,t-1}$ is increasing (decreasing); and below (above) the isocline $\triangle\lambda_{2,t} = 0$, variable $\lambda_{1,t-1}$ is increasing (decreasing).

$$\begin{cases} \triangle\lambda_{1,t} > 0 \Rightarrow \lambda_{1,t-1} < \lambda_1^T \\ \triangle\lambda_{1,t} < 0 \Rightarrow \lambda_{1,t-1} > \lambda_1^T \end{cases}$$

$$\begin{cases} \triangle\lambda_{2,t} > 0 \Rightarrow \lambda_{2,t-1} < \lambda_2^T \\ \triangle\lambda_{2,t} < 0 \Rightarrow \lambda_{2,t-1} > \lambda_2^T \end{cases}$$

[36] As $|r| < 1$ and $|s| < 1$ the system is expected to be dynamically stable.

By drawing these vector forces in our phase plane, it can be observed that regardless of the initial point, the system moves towards the equilibrium point where the two isoclines intersect: $(\lambda_1^*, \lambda_2^*) = (\lambda_1^T, \lambda_2^T)$ (See fig. 1.9).

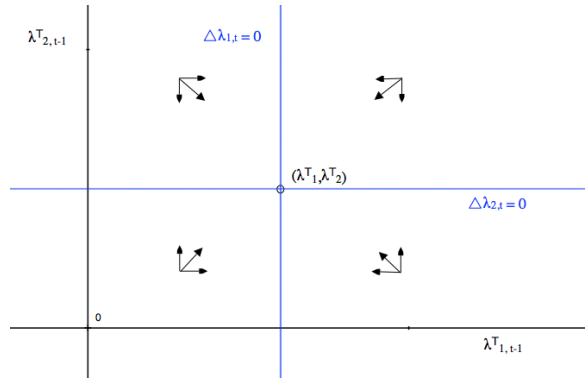


Figure 1.9: Adjustment model: Phase plane -book value-

1.4.1.1.2 Market-value-based leverage ratio.

When considering the marked-to-market leverage ratio, we have that $\lambda_{i,t-1}$ is a function $F(\lambda_{i,t-1}, \lambda_{-i,t-1})$.

It is possible to investigate the stability properties of this nonlinear system in the neighbourhood of the steady state as long as F is continuous and differentiable.Under such conditions the system can be linearised around one of the equilibrium points. The system we are studying has a fixed point at $(\lambda_1^*, \lambda_2^*)$ given by the condition $\lambda_i = \lambda_i^T$.
As we are considering marked-to-market leverage, λ_i can be substituted by the corresponding expression capturing the overlap among banks' portfolios. By doing so, we find new values for the fixed points, considering this time the effect of counterparties' leverage on the bank's own leverage. What can be derived from the resulting expression is that the value of one bank's leverage in the equilibrium depends on the counterparties' leverage target.

To compute the linear stability of the system we derive the Jacobian of the dynamical system and study the values of the resulting eigenvalues. As we have enough information about the functions $F(\lambda_{i,t-1}, \lambda_{-i,t-1})$, we can determine the characteristics of the equilibrium.

- When considering market value of assets, λ_i is replaced by the expression capturing the dependency on cunterparty's leverage, $F(\lambda_{-i})$. The Jacobian is of the form:

$$J = \begin{bmatrix} 0 & f_{\lambda_2}(\lambda_1^*, \lambda_2^*) \\ g_{\lambda_1}(\lambda_1^*, \lambda_2^*) & 0 \end{bmatrix} \tag{1.32}$$

  Given that both derivatives are positive, independently of the values of the equilibrium point, the determinant of this Jacobian is negative. This gives us the intuition that the equilibrium point is a saddle point in this case.

- When considering market value of assets an liabilities, λ_i is substituted by the corresponding linearised quadratic expression , $F(\lambda_{-i}, \lambda_i^*)$. The Jacobian is given by:

$$J = \begin{bmatrix} f_{\lambda_1}(\lambda_1^*, \lambda_2^*) & f_{\lambda_2}(\lambda_1^*, \lambda_2^*) \\ g_{\lambda_1}(\lambda_1^*, \lambda_2^*) & g_{\lambda_2}(\lambda_1^*, \lambda_2^*) \end{bmatrix} \tag{1.33}$$

  Given that all the derivatives are positive, independently of the values of the equilibrium point, the determinant of this Jacobian will be positive, so does its trace. This gives us the intuition that, independently of the sign of the discriminant, the equilibrium is unstable in this case.

So when introducing explicitly the dependancy of one bank's leverage on its counterparties' leverage, the dynamics of the model seem to become unstable.

1.4.1.1.2.1 An application to real systems, marked-to-market leverage.

Consider now the case of a regulator setting a target leverage for the banking system. According to literature on leverage cycles, banks' procyclical leverage policies destabilise the system leading to systemic effects. A regulator can potentially correct for this systemic risk by imposing a countercyclical policy consisting on allowing banks to vary their leverage in response to price shocks[37].

Regulator's choice of the level of target leverage would be equivalent to setting λ_i^T as a free variable to be determined given our adjustment model. The logic behind this approach is to set a target leverage according to the conditions of the market.

We have already seen how to expand a system of non-linear equations of the form $\lambda_{i,t} = \lambda(\lambda_{j,t}, p_{k,t})$ in a Taylor expansion around the point λ_i^T, which has turned out to be the equilibrium of the dynamic system λ_i^*.
In order to establish a connection with the analysis of the intersection point already performed, we now consider an unique common leverage target, $\lambda^T = \lambda^*$, and assume that the adjustment speed ϵ_i is equal to one. The non-linear dynamic system $\lambda_{i,t} - \lambda_{i,t-1} = \epsilon_i(\lambda_1^T - \lambda_{i,t-1})$ becomes $\lambda_{i,t} = \lambda_i^T (\equiv \lambda_i^*)$. We can impose this condition on the linearised equations of the form $\lambda_{i,t} = \lambda_i(p_{k,t}, \lambda^*)$, and solve for λ^*.
By doing so, a common equilibrium target leverage would then be determined, considering the real conditions of the banking sector.

Common knowledge of a target leverage and, therefore, capacity of perfect forecasting of counterparties' targets, allows for anticipation of others' balance sheet adjustments and the incorporation of these true expectations in own behaviour. Otherwise, as the leverage dynamics model simulations proves, the existence of equilibrium can not be ensured.

[37] In order to do so, in a scenario of decreasing external assets prices, for instance, the regulator could set a target leverage higher than the ones present in the market, so banks don't have to decrease their effective leverage when pursuing to close the leverage gap.

1.4.2 An alternative adjustment model

In order to capture more accurately the effects of overlapping portfolios in the dynamics of the system, it would be interesting to consider the following expression for the adjustment model:

$$\triangle \lambda_{i,t} = \lambda_{i,t} - \lambda_{i,t-1} = \epsilon_i(\lambda_i^T - \lambda_{i,t-1}) - \varsigma_{i,-i,t-1} E_{i,t-1}[\epsilon_{-i}(\lambda_{-i}^T - \lambda_{-i,t-1})] \tag{1.34}$$

where $\varsigma_{i,-i} = \frac{\delta\lambda_{1,t}}{\delta\lambda_{2,t}}$ is the sensibility of bank i to changes on counterparty's leverage, and the term $E_{i,t-1}[\epsilon_{-i}(\lambda_{-i}^T - \lambda_{-i,t-1})]$ captures bank i's expectations about the adapting behaviour of their counterparties. For the sake of simplicity, we assume rational expectations

$$E_{i,t-1}[\epsilon_{-i}(\lambda_{-i}^T - \lambda_{-i,t-1})] = [\epsilon_{-i}(\lambda_{-i}^T - \lambda_{-i,t-1})]$$

and the parameters ϵ_{-i} and $\varsigma_{i,-i,t-1}$ to be constant.

We do not explicitly specify the overlap issue in the expression for bank i's leverage λ_i, but it's captured by the additional term. When closing the leverage gap, banks anticipate the effects of the simultaneous adjustment on own leverage made by their counterparties and incorporate them to their behaviour.

We now study the stability properties of the dynamic system given by 1.34 for the case of two banks. Given

$$\triangle \lambda_{i,t} = \epsilon_i(\lambda_i^T - \lambda_{i,t-1}) - \varsigma_{i,j}\epsilon_j(\lambda_j^T - \lambda_{j,t-1}) \tag{1.35}$$

we obtain the equilibrium values $\lambda_i^* = \lambda_i^T$, $i, j = 1, 2$.

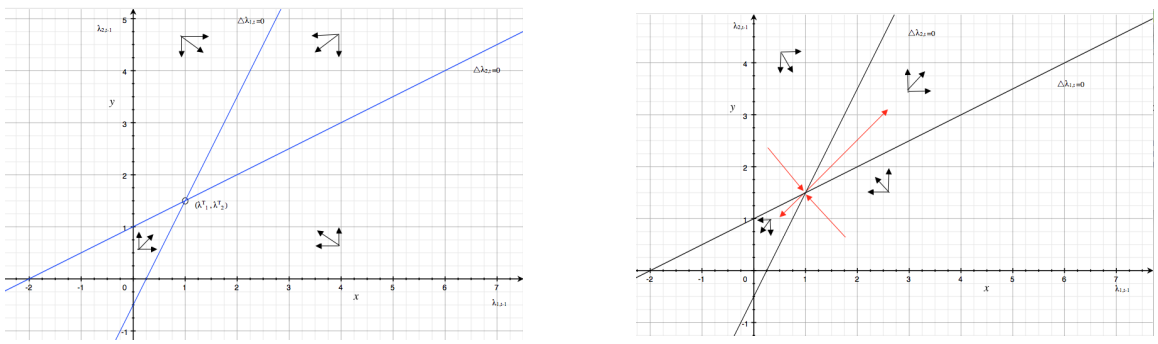
The isoclines are given by $\lambda_{1,t-1} = \frac{\epsilon_1}{\epsilon_2\varsigma_{1,2}}[\lambda_{1,t-1} - \lambda_1^T] + \lambda_2^T$ and $\lambda_{2,t-1} = \frac{\epsilon_1\varsigma_{2,1}}{\epsilon_2}[\lambda_{1,t-1} - \lambda_1^T] + \lambda_2^T$, respectively.
Considering now points either side the isoclines: above the isocline $\triangle\lambda_{1,t} = 0$, variable $\lambda_{1,t-1}$ increases, whereas it decreases below. On the other hand, below the isocline $\triangle\lambda_{2,t} = 0$, variable $\lambda_{2,t-1}$ increases, while it decreases above it.

When looking at the phase plane, the system presents two different stability behaviours. Stability also depends on the relation between the slope of the system isoclines, determined ultimately by the effect of changes in counterparty's leverage, i.e., by the balance sheet structure:

- If the slope of the isocline given by $\triangle\lambda_{1,t} = 0$, $\frac{\epsilon_1}{\epsilon_2\varsigma_{1,2}}$, is greater than the slope of isocline given by $\triangle\lambda_{2,t} = 0$, $\frac{\epsilon_1\varsigma_{2,1}}{\epsilon_2}$, the equilibrium would be an stable proper node, as

in the previous adjustment model when considering book-value-based leverage (See fig.1.10a).

- On the contrary, if the slope of the isocline given by $\triangle\lambda_{2,t} = 0$ is greater than the slope of isocline given by $\triangle\lambda_{1,t} = 0$ we realise that the equilibrium point is an unstable saddle point, so we would expect the system to move away from the equilibrium point, except for initial values lying on the stable manifold (See fig.1.10b)[38].



(a) Stable case

(b) Unstable case

Figure 1.10: Adjustment model with overlapping portfolios: Phase plane -market value-.

In our case, the initial conditions are given by $\lambda_{1,0} = \lambda_1^T$ and $\lambda_{2,0} = \lambda_2^T$, so the initial values are those one in the equilibrium. Therefore, we expect the system to be stable in the presence of rational expectations.

1.4.2.1 Application to real systems

This last approach to the adjustment model would be particularly suitable when allowing for changes in target leverage. Even if in our model the target leverage is treated as an exogenous variable, it would be useful to study the case in which a shock in the target leverage takes place. This could be the real case of a regulator -macroprudential supervisory authorities- updating the target leverage for the banking system as part of the implementation of countercyclical policies.

It's relevant to study the situation where we have as equilibrium an unstable saddle point. Therefore, in terms of figure 1.10b, an increase in λ_2^T shifts both the $\triangle\lambda_{1,t} = 0$ and $\triangle\lambda_{2,t} = 0$ lines to the left, being the magnitude of the shift greater for the isocline $\triangle\lambda_{1,t} = 0$-. On the contrary, an increase in λ_1^T shifts both the $\triangle\lambda_{1,t} = 0$ and $\triangle\lambda_{2,t} = 0$ lines to the right by the same magnitude. The system will reach a new equilibrium, and an

[38] Ultimately, these conditions depend on the structure of the balance sheet.

adjustment from the first equilibrium to the new one is expected (See fig. 1.11).

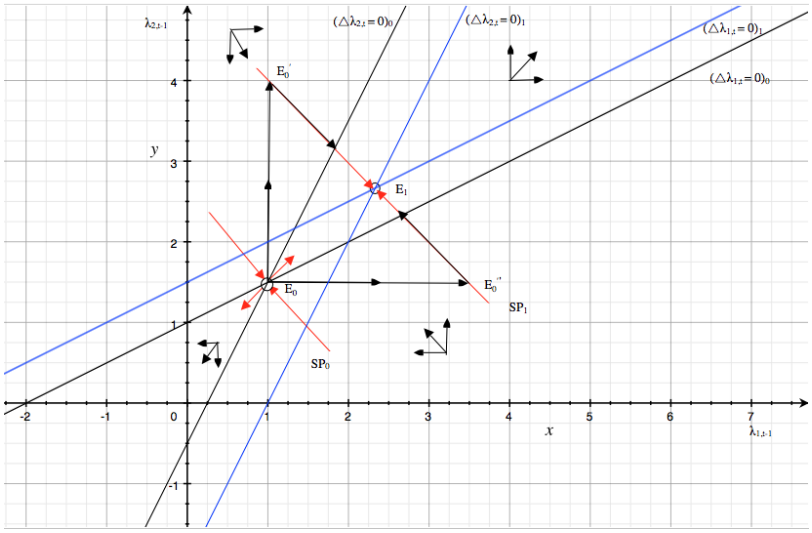


Figure 1.11: Countercyclical policy in the adjustment model

A shock away from the equilibrium won't be corrected in the model unless the economy is in the saddle path. Given the perfect foresight hypothesis, agents know that the economy will fall apart in other case and they expect the economy to be in the saddle path. By this behaviour this expectation is correct (Heijdra and Van der Ploeg, 2002).

A countercyclical policy produces an overshooting, allowing the system to converge to a new equilibrium. An increase in the target leverage causes a leverage gap, so a balance-sheet management mechanism affecting external assets holdings is triggered. This initially results in an increase in effective leverage, i.e., leverage overshoots as banks manage to close the leverage gap. Each agent will increase his effective leverage in response to the increase in target leverage, approaching points E_0' and E_0'' in fig. 1.11, respectively. But banks perfectly predict the behaviour of other banks, so they expect their counterparties to adjust simultaneously their effective leverages, which will affect the former bank's own level. Because of that, a gradual adjustment along the saddle path SP_1 leads to the economy back to the equilibrium E_1. Hence, E_1 must be approached from a north-westerly direction in the case of bank 2, and from a south-easterly one for bank 1.

The policy implication of our analysis of leverage dynamics is that the regulatory authorities should manage the system leverage, by reducing allowed leverage during economic upturns and supporting it during economic downturns. According to Feinstein and El-Masri (2015), in a system of fully-leveraged banks, a stricter leverage requirement (during economic expansion) leads to fewer assets to be impacted by price fluctuations through mark-to-market accounting. Thus stricter leverage requirements produce a system more robust to systematic shocks.
As argued, the application of a regulatory leverage ratio would enable perfect forecasting of counterparties' targets, allowing for the incorporation of expectations of others' adjustment in own balance sheet management and, therefore, avoiding the scenario that lead to instability. Instability in this case is understood as the incapacity of the system to return to the equilibrium leverage level.

Procyclical leverage is a consequence of the active balance sheet management undertaken by financial intermediaries in response to changes in the prices and measured risk. In this scenario of adjusted asset and liability holdings, leverage is considered as procyclical in the sense that it tends to increase when the economy is growing and to decrease during recessions.
Previous work has shown that procyclical leverage can have aggregate consequences and has documented also that changes in balance sheets have asset pricing consequences through shift in risk appetite (Adrian and Shin, 2010). To this regard, leverage is considered as procyclical in the sense that leverage dynamics can magnify financial fluctuations.

The key behavioural hypothesis in our analysis is that banks maintain a leverage target and manage actively their balance sheets when the target becomes binding due to a change in actual leverage induced by a negative shock in asset prices. The consequent downward adjustment of leverage entails a sell of assets. A potential feedback effect may then arise: weak balance sheets lead to greater sales of the asset, which in turn depresses the asset price and lead to even weaker balance sheets. Under this scenario, leverage adjustments and price changes will reinforce each other, amplifying the financial cycle (Adrian And Shin, 2010).
The initial change in leverage is transmitted indirectly to counterparties through their exposure to the shocked institution. Further amplification occurs as the deleveraging causes additional changes in asset value, as explained above, and as counterparties manage their balance sheets in turn. The financial crisis of 2007/2008 illustrated how an initial shock can be amplified in absence of policy responses that would mitigate the shock amplification

resulting from the initial deleveraging.

When balance sheets are marked to market, i.e. the dependency of one's leverage on its counterparties' leverage is explicitly introduced in the leverage formula, the reaction to price changes may entails disproportionate responses.
The size of the sub-prime mortgages exposures was small relative to the liabilities of the financial system but the last crisis demonstrated that the impact of a shock in asset prices can be large (Adrian and Shin, 2010).

Amplification of financial shock through balance sheet channels is considered as the other source of unstable dynamics in the financial system. The dependency of a financial intermediary' assets-to-capital ratio on counterparties' financial robustness, i.e. on leverage, can also lead to instability in the sense that the amplification effects on prices of banks' active management of balance sheets can be reinforced by the larger deviation from the target leverage caused by the changes in counterparties' leverage through the adjustments of asset and liability holdings.

1.5 Leverage dynamics model simulation.

Classical literature has failed to predict some stylised dynamics of financial markets.
Stylized dynamics refer to the occurrence of certain kinds of instabilities, patterns, phase transitions/regime shifts within financial markets. Stylized features may relate to recurrence of bubbles and crushes, volatility excess and volatility clustering [39], amplifications mechanisms of financial distress, assets market cycles or procyclical leverage dynamic, for instance.
Some stylized dynamics may also refer to observed properties of financial cycles.
Empirical work suggests that the financial cycle has a much lower frequency than the traditional business cycle and that peaks in the financial cycle tend to coincide with episodes of financial distress. These crises seem to be exposed to financial cycles abroad and are typically preceded by credit booms, as was the case of the recent crisis.

The stylized dynamics of financial markets relevant for the scope of this paper are the ones related to leverage fluctuations. There is empirical evidence (Adrian and Shin, 2010; Geanakoplos, 2010; Nuo and Thomas, 2012; Adrian and Shin 2013) of a procyclical assets-to-equity ratio (e.g. with respect to GDP and assets), of a strongly positive relationship

[39] the probability of observing a large change in the market tomorrow is higher than on average if today the market has been very volatile

between changes in leverage and changes in financial intermediaries' balance sheets, of a negative co-movement between leverage and capital and of procyclical leverage as a source of amplification.

The evidence indicates that financial intermediaries adjust their balance sheet actively, and do so because they target a fixed leverage ratio, so that leverage is high during booms and low during busts, i.e. leverage is procyclical. Yet variation in leverage has a significant impact on the price of assets, contributing to economic bubbles and busts. In the absence of intervention, this lead to sensitively high prices in boom times and too low prices in crisis times. This is what Geanakoplos (2010) defines as the leverage cycle and describes as a recurring phenomenon, as evidenced by the crash of 1987, the financial derivatives crisis in 1994 and the emerging markets crisis of 1998. The author argued that these episodes were or seemed to be at the tail end of a leverage cycle.

The last financial crises has posed the urgency of incorporating some crucial facts in order to delve into the functioning of the financial system. It's important to account for heterogeneity of financial agents and products, for potential amplification and contagion mechanisms, as well as for different conditions of the financial markets, regarding liquidity, the level of leverage within the system, regulation...
The incorporation of these elements in modelling becomes complicated within an analytical framework or when employing the classical model approach based on a representative agent.

We develop a model representing leveraged interconnected banks taking positions in several external securities.
In a financial system with interlinked claims and obligations, lenders and borrowers are interconnected through credit contracts, i.e., one party's internal liabilities are other parties' assets. Therefore, not only direct effects of external asset price shocks are considered but also the propagation effects through others' exposures to banks affected directly by those shocks[40].

Heterogeneity among agents and products is introduced through the holdings of external assets, while we rely, on one hand, on the network diversification -d- and integration degree -w- and, on the other, on system leverage levels, i.e. initial capitalization, to capture different market conditions.

[40]The external network could be also considered as an additional source of connectivity among banks in the sense that banks can be exposed to the same external assets and liabilities and, therefore, there will be overlaps among portfolios of external holdings. Nevertheless, that is beyond the scope of this paper.

In our model, banks are performing at the level of committed leverage. Financial institutions are *fully-leveraged or strategic banks* that actively manage their balance sheets in order to match a target leverage. This target can be either individually determined by risk management models (VaR), as seen before, or imposed by a policy regulator. By doing so, these agents pursue a procyclical leverage policy. The initial condition for leverage for these agents is: $\lambda_{i,0} = \lambda_i^T$.

We disregard the role of financial institutions as active traders, so that the adjustment to a target leverage through variations in external assets won't induce prices movements in the same direction of the trading. Therefore, banks manage their balance sheets in response to asset price movements, but no price tâtonnement process is activated.

1.5.1 Some considerations on Balance Sheet Management.

Different circumstances can be identified as trigger for shocks in government debt prices. For instance, an increase in interest rates causes the bond prices to fall, and, consequently, older bond yields to increase, bringing them into line with newer bonds being issued with higher coupons. So we can understand the shock in prices in our model as a consequence of new issues in the case of sovereign' robustness deterioration, which translates into higher interest rates.
There could be also market facts triggering debt price shocks, such as macroeconomic indicators data releases indicating an economic deterioration (or recovery) in the issuer country (GDP...) or rating agencies' announcements.

In the case of fully-leveraged banks, leverage λ_i is initially set equal to λi^T ($\lambda_{i,0} = \lambda i^T$) at a given equilibrium price. Deviations of prices from this reference price on the asset side of banks' balance sheets lead to changes in leverage, given that changes in prices induce greater changes in equity. Defining:

$$\bar{\lambda}_{i,t} = \frac{\sum_k n_{ik,t-1} p_{k,t} + \sum\limits_j \omega_{ij} \frac{b_j}{R+\xi_j \bar{\lambda}_{j,t}}}{\bar{e}_{i,t}}$$

in the presence of a shock to external assets prices, a $\delta\%$ of change in prices from $p_{k,t-1}$ to $p_{k,t}$ causes a $\bar{\lambda}_{i,t}\delta\%$ change in equity (Thurner et al., 2010). These changes in equity result in surplus ($e_{i,t} > e_{i,t-1}$) or deficit ($e_{i,t} < e_{i,t-1}$) of capital that will affect external holdings. Considering, for instance, a downward shock to the price of an specific external asset. At a lower price fully-leveraged banks would start selling the external asset in order to adjust down its leverage and close their leverage gap[41].

[41] Conversely, an upward price shock would induce asset purchases so banks adjust up the leverage and

If we were considering any price tâtonnement process, in the absence of other players in the market this would increase the supply of assets above demand, and the market price would adjust to clear the market, pushing the prices further down. This is not captured in our model, where the positive feedback loop affecting leverage is triggered by the variation in the value of external securities holdings in interconnected balancesheets, as we will see. The extend of the procyclical effects triggered by the fully-leverage banks' behaviour will depend on the capital structure of agents within the banking system and on the phase of the financial cycle. In a high-leveraged financial system, both stronger shock amplification, and, therefore, greater systemic risk are expected -during downturns and upturns-.

The expansion -or shrinking- of the balance sheets is not conducted through banks' active management of equity but through assets and liabilities, given our assumption of "sticky" equity[42].

In our model, own equity initially changes only via changes in external asset price in which banks hold positions:

$$\bar{e}_{i,t} = e_{i,t-1} + \sum_k n_{ik,t-1} \left(p_{k,t} - p_{k,t-1}\right) \tag{1.36}$$

and adjustments on the balance sheets are made via external securities, both on the assets and liabilities sides:

$$d_{i,t} - d_{i,t-1} \equiv \sum_k n_{ik,t} - \sum_k n_{ik,t-1} \tag{1.37}$$

But given the overlapping nature of banks' financial structures, own equity is also affected by counterparties' balancesheet variations, i.e., net wealth evolves according to the performance of other banks' trading.

The dynamics of the balance sheet management of fully-leveraged financial intermediaries work as follows:

- In a scenario of increasing external asset prices, leverage falls. The resulting surplus capacity will lead to an expansion of the balance sheet, by taking on more external funding -on the liabilities side- and searching for potential external investment opportunities -on the assets side-. So both debt and credit will increase.

close the gap.

[42]This assumption is in line with the results of Adrian and Shin (2011) showing that banks' balance sheet management reveals a relative "stickiness" of equity, which behaves as the pre-determined variable, even during upturns. This is captures also by the fact that leverage and asset growth are positively related.

- Under falling external asset prices, banks face equity deficit and will adjust down their increased leverage by shrinking their balance sheet: they will sell part of their external assets and pay down a portion of their external debt.

In order to reduce exogenous constraints, we assume that banks do not face funding restrictions in real economy, i.e. there are always willing depositors; and that sovereigns face funding necessity so the offer covers the demand of sovereign bonds.

1.5.1.1 BSM Accounting Rules.

The identification of shocks in sovereign debt prices as the triggering event for the amplification mechanisms motivates the analysis of the decisions financial institutions make about their holdings on sovereign debt.

As said previously, the connections among financial institutions and governments are based on the decision banks make about their holdings of sovereign debt. This decision process could be expressed through an *accounting rule* that would govern the balance sheet management process, capturing the rate of change for asset k for bank i (the derivation is provided in Appendix D):

$$\frac{n_{ik,t} - n_{ik,t-1}}{n_{ik,t-1}} = -\frac{\epsilon_{i,t}}{\alpha_{i,t-1}} \frac{(\lambda_{i,t-1} - \lambda_i^T)}{\lambda_{i,t-1}} = \frac{\epsilon_{i,t}}{\alpha_{i,t-1}\lambda_{i,t-1}}(\lambda_i^T - \lambda_{i,t-1}) \tag{1.38}$$

where $\alpha_{i,t}$ is the ratio of external assets value to total assets value at time t-1.

Therefore, a fully-leveraged bank's demand on external assets at time t will be given by:

$$n_{ik,t} = n_{ik,t-1}\left[1 + \frac{\epsilon_{i,t}}{\alpha_{i,t-1}\lambda_{i,t-1}}(\lambda_i^T - \lambda_{i,t-1})\right] \tag{1.39}$$

Then, from the assumption of "sticky" equity:

$$\frac{d_{i,t} - d_{i,t-1}}{d_{i,t-1}} = \frac{\epsilon_{i,t}}{\beta_{i,t-1}(\lambda_{i,t-1} - 1)}(\lambda_i^T - \lambda_{i,t-1}) \tag{1.40}$$

where $\beta_{i,t}$ is the ratio of the value of external debt to the value of total debt for bank i at time t.

Given the previous expression, we have:

$$d_{i,t} = d_{i,t-1}\left[1 + \frac{\epsilon_{i,t}}{\beta_{i,t-1}(\lambda_{i,t-1} - 1)}(\lambda_i^T - \lambda_{i,t-1})\right] \tag{1.41}$$

It's worth mentioning that the demand for external assets is a function directed related to the deviation from the leverage target and inversely to the ratio of external assets to

total assets, while the demand for funds is inversely related to to the ratio of external debt to total debt[43].

1.5.2 The model.

In this section we develop a model that gives some insights into leverage dynamics within an overlap financial network.
Our goal is to simulate the effects of marked-to-market overlapping portfolios in the dynamics of leverage and assess how the stability of the financial system, that we have previously qualitatively analysed, depends on the degree of diversification, integration and indebtedness within the network.

The simulation model is based on the idea that linkages among financial institutions can be modelled through dependency matrices, following Elliot et al. (2014). The main concept behind our leverage-dynamics analysis is that the value of a financial organization ultimately depends on own and counterparties' external assets.

The modelling aggregates the effects of linkages among banks on their financial situation, leverage, into a linear dependence of each bank on others. In the literature, we find different linkages patterns. Elliot, Golub and Jackson (2014), in line with Brioschi, Buzzacchi and Colombo (1989) and Fedenia, Hodder and Triantis (1994), consider linkages among the assets of different firms, for instance. We improve this framework accounting for cross-holdings by looking at the presence of linkages among both assets and liabilities, arising from mutual lending and borrowing relationships.

In order to capture these linkages, we assume that the value of bank i's assets is ultimately related to the value of the assets of bank j, for $j \neq i$.
The actual value of bank i's assets depends on its counterparties' capacity of meeting their obligations, that is, on the value of assets of its obligors, which depends in turn on the strength of obligors' counterparties' balance sheets. This creates an interdependency among the value of assets of all banks in the network, namely, among banks' financial robustness, which is consistent with the reasoning followed so far: bank i's leverage depends ultimately on counterparties' leverage.
Financial linkages are modelled as linear dependences, as we will see.

[43] This could be indicative of a kind of "saturation".

1.5.2.1 Cross-holdings definition and bank's value.

Previous specification of leverage is slightly modified in order to work with matrix notation.

There are N banks connected through a network of cross-holdings, via debt contracts. According to what expounded above, banks' values depend ultimately on the market value of their holdings of external assets: shocks hitting counterparties' balance sheets affect own leverage through interbank marked-to-marked exposures. There are K external assets, and the market price (present value) of each external asset, $p_{k,t}$, is subject to shocks. **S** denote a nonnegative, column-stochastic matrix with generic entry $s_{i,k} \geq 0$, that represents the share of the value of external asset k held by bank i. It holds that $\sum_i s_{i,k} = 1$.

Recalling previous notation, we have that $n_{ik,t} = s_{ik,t} n_k$[44], being n_k the total supply of external asset k. Therefore, $n_{ik,t} p_{k,t}$ denotes the value of external asset k held by i at time t. A bank is shocked whenever

$$n_{ik,t} p_{k,t} - n_{ik,t} p_{k,t-1} \neq 0$$

being the size of the shock given by $\Delta p_t = p_{k,t} - p_{k,t-1}$.

Let **q** denote the column matrix whose k entry is equal to the Hadamard product of price and supply vectors **p** and **n**, respectively: $p \circ n = (n_k p_k)$.

Interbank cross-holdings are captured by matrix **W. W** is still a column-stochastic weight matrix, where $w_{i,j} \geq 0$ is defined now as the fraction of bank j's value held by bank i, being $w_{i,i} = 0$ for all i.

There is also a fraction of a bank's value held by agents in the external network. Diverse market conditions affecting leverage dynamics are incorporated to our linear model by considering a range of leverage and capitalization levels. These are captured through different definitions of the external network structure regarding liabilities.

Matrix $\hat{\mathbf{W}}$ captures the linkages between banks and outside investors. The off-diagonal entries of this matrix are defined to be 0, representing $\hat{w}_{i,i} = 1 - \sum_j w_{ji}$ the fraction of bank i's value owned by an external investor. When differentiating between external creditors and shareholders, the definition of matrix $\hat{\mathbf{W}}$ changes: $\hat{w}_{i,i} \neq 1 - \sum_j w_{ji}$, but $\hat{w}_{i,i} \geq 0$, while there remains a fraction of bank i's value held by shareholders. These latter weights

[44] We consider a fixed total supply of external asset k, n_k, normalized to one.

are given by the matrix $\tilde{W}$, where $\tilde{w}_{i,i} = 1 - \sum_j w_{j,i} - \hat{w}_{i,i}$.

We define the book value V_i of a bank i as the total value of the liability side of its balance sheet -liabilities and capital-. This is equal to the value of bank i's external assets plus the value of its claims on other banks.

$$V_{i,t} = \sum_j \omega_{ij} V_{j,t} + \sum_k n_{ik,t} p_{k,t} \tag{1.42}$$

For our purposes, we will consider that the ultimate dependency of each bank on others is instrumented through external assets. Therefore, equation 1.42 can be written in matrix notation as $\mathbf{V} = \mathbf{WV} + \mathbf{Sq}$. Solving for V we have:

$$\mathbf{V} = (\mathbf{I} - \mathbf{W})^{-1}\mathbf{Sq} \tag{1.43}$$

Equation 1.43 shows that a bank's value, and consequently, given the accounting identity, the value of its assets, can be expressed in terms of the value of external assets, not only own ones but also counterparties'. The value of external assets held directly by bank i affects bank i's value and also the books of the banks that hold a fraction of bank i's value. This interdependence among organizations is captured through the *dependency matrix*

$$\mathbf{D} = (\mathbf{I} - \mathbf{W})^{-1} \tag{1.44}$$

, which accounts for cross-holdings[45].

We define the market value of a bank in two ways, according to previous differentiation regarding external liabilities:

1. The following equation is derived to represent bank's equity value, $v_{i,t} = \hat{w}_{ii} V_{i,t}$, understood as the value held by outside investors:

$$v_{i,t} = \sum_j \omega_{ij} V_{j,t} + \sum_k n_{ik,t} p_{k,t} - \sum_j \omega_{ji} V_{i,t} \tag{1.45}$$

   or in matrix notation, after substituting for the book value from 1.43:

$$\mathbf{v} = \hat{\mathbf{W}}\mathbf{V} = \hat{\mathbf{W}}(\mathbf{I} - \mathbf{W})^{-1}\mathbf{Sq} \tag{1.46}$$

[45]The value of external assets held by bank i contributes directly to i's equity value, but also counted partially on the books of creditors within the interbank network holding a share of bank i's value (Elliot, Golub and Jackson, 2014). These cross-holdings lead to a double-counting of external assets and, therefore, to an overstatement of a bank's value (French and Poterba, 1991), as is reflected by the fact that **D** is not column-stochastic.

In this case, the matrix capturing the dependency of equity value on own and others' external asset holdings is defined

$$\hat{\mathbf{D}} = \hat{\mathbf{W}}(\mathbf{I} - \mathbf{W})^{-1} \tag{1.47}$$

2. If equity value is defined as the value of shareholders' holdings, the market value of equity, $v_{i,t} = \tilde{w}_{ii}V_{i,t}$ can be expressed as the current value of assets net of overall liabilities:

$$v_{i,t} = e_{i,t} = \sum_j \omega_{ij}V_{j,t} + \sum_k n_{ik,t}p_{k,t} - \sum_j \omega_{ji}V_{i,t} - \hat{w}_{ii}V_{i,t} \tag{1.48}$$

Equation 1.48 can be written in matrix notation. After substituting for V from 1.43, it becomes

$$\mathbf{v} = \tilde{\mathbf{W}}\mathbf{V} = \tilde{\mathbf{W}}(\mathbf{I} - \mathbf{W})^{-1}\mathbf{Sq} \tag{1.49}$$

Any shock in external asset value is transmitted to a bank's equity value through the matrix of interconnections, given in this case by matrix

$$\tilde{\mathbf{D}} = \tilde{\mathbf{W}}(\mathbf{I} - \mathbf{W})^{-1} \tag{1.50}$$

Equations 1.46 and 1.49 show that a bank's market value of equity can be represented as a weighted sum of the values of own and others' external asset holdings[46].

The *dependency matrix* captures indirect and direct holdings. Let assume that each bank is endowed with the 100% of one external asset k, so that $N = K$ and $\mathbf{S} = \mathbf{I}$. Hence, the (ij)th entry of matrix $\mathbf{D}$ -analogously, $\hat{\mathbf{D}}$ or $\tilde{\mathbf{D}}$, depending on the structure of the external network considered- describes the dependency of bank i's value on the value of bank j's external asset holdings.

The balance sheet of a bank at time t is now represented as follows:

Case 1		Case 2	
Assets	**Liabilities**	**Assets**	**Liabilities**
$\sum_j \omega_{ij}V_{j,t}$	$\sum_j \omega_{ji}V_{i,t}$	$\sum_j \omega_{ij}V_{j,t}$	$\sum_j \omega_{ji}V_{i,t}$
		$\sum_k n_{ik,t}p_{k,t}$	$\hat{w}_{ii}V_{i,t}$
$\sum_k n_{ik,t}p_{k,t}$	$v_{it} = \hat{w}_{ii}V_{i,t}$		$e_{it} = \tilde{w}_{ii}V_{i,t}$

[46]Furthermore, when calculating the aggregate equity value of the financial system, interbank claims and obligations cancel out. In the aggregation, the marked-to-market value of the net (external) assets -external assets net of external liabilities- remains as the equity value of the financial system as a whole (Shin, 2008).

1.5.2.2 Shock transmission: contagion.

The overlap structure of the financial system enables contagion, as the shock affecting one bank constitutes a shock itself for the banks with whom the bank is linked through the interbank exposures captured by the dependency matrix. Furthermore, a domino effect materialises given that counterparties' adjustments in response to the shock affect subsequently assets values, which constitutes an additional negative shock.

The presence of financial interconnections, as described previously, implies that all banks are affected by any shock hitting the value of external asset holdings of any one within the interbank network. We track how negative shocks impacting on external asset value propagate through the network of financial cross-holdings and how the balance sheet-management mechanism induces further value disruptions affecting financial conditions -leverage-.

Even small and bank-specific shocks can be noticeably amplified. As the simulation results show, not only the magnitude of the shock affects the occurrence of events such as selloffs or defaults and the capacity of the system to converge to an equilibrium -the target leverage-, but also market conditions.

In our model, selloffs occurred when the disposal of external assets after a balance sheet adjustment leads to a decline in the value of the security so that $n_{ik,t}p_{k,t} \leq 0$.
The solvency of a bank is established with reference to a bank's market value: if the value of bank i's assets is equal or lower than the value of i's liabilities, the bank defaults, i.e., a default occurs whenever $v_{i,t} \leq 0$.

The market conditions are captured by some network properties and node-specific characteristics. The *connectivity* and *integration* of the network, bank balance sheet and, therefore, *financial robustness* defined through the leverage ratio, are critical for financial stability.

Integration measure is given by the fraction of a financial institution's value -liabilities- held by other financials institutions and it captures the depth of borrowing-lending linkages. A more integrated banking system is characterised by lower holdings of outside investors in each bank within the network and, consequently, higher total cross-holdings of counterparties in each organization. Accordingly, $w = \sum_j w_{ij} \forall j \neq i$ is higher in more integrated systems, which is equivalent to say that $\hat{w}_{ii}$ is lower.

As for financial interconnectedness, it captures the spread of interbank cross-holdings. The level of diversification of the financial system is given by $d = d_i = \sum_j g_{ij}$, the average degree, i.e., the expected number of interbank counterparties bank i lends to and the ex-

pected number of other financial institutions bank i borrows from[47].

Regarding financial robustness, various scenarios are recreated by defining different structures of the balance sheet's liability side, as discussed previously.

Spillover effects and steady systemic instability are expected to be more significant in highly leveraged and more integrated financials systems.

1.5.2.3 Simulation on random networks.

In this section, we show that previous analytic results regarding leverage dynamics hold.

As this paper focuses on convergence and divergence leverage dynamics and explores how financial institutions' balance sheet management and interlinkages influence these dynamics, the tracked outcome variable is the actual level of leverage resulting from variations of external-asset value on account of shocks or balance-sheet adjustment processes.

We design the simulation framework with respect to:

1. The capital structure, regarding:

   - Node characteristics: level of target leverage. We allow for different levels of capitalization. We vary the level of initial leverage, $\lambda_{i,0} = {\lambda_i}^T$, by changing the definition of a bank's equity value. We consider the following cases:

     (a) $v_{i,t} = \hat{w}_{ii} V_{i,t} \equiv \mathbf{v{=}\hat{D}Sq}$. Equity is defined as bank's assets net of interbank liabilities - *outside-investors equity* -.

     (b) $v_{i,t} = e_{i,t} = \tilde{w}_{ii} \equiv \mathbf{v{=}\tilde{D}Sq}$. Equity is defined as bank's assets net of overall liabilities - *shareholders' equity* -.

     (c) $v_{i,t} = V_{i,t}/\lambda_i^T \equiv \mathbf{v} = \mathbf{V}/\lambda^{\mathbf{T}}$. Equity is derived from the leverage ratio - *assets-to-target leverage ratio* -.

     In approaches (a) and (b), the initial level of leverage -target leverage- is endogenously derived, while in (c) is defined exogenously. When defining a bank's equity value as in (a) and (b), the target leverage is expressed as the asset-to-equity ratio, having been both assets and equity previously defined in terms of dependency on external assets values, according to expressions 1.43, and 1.46 or 1.49, respectively. In the remaining case, the target leverage is firstly set and the value of assets is obtained from expression 1.43, being equity determined afterwards.

[47] When the graph representing the banking network is regular, indegree and outdegree of each node is equal to each other, so that $\bar{d}_i = \underline{d}_i = d_i$.

- Network properties: the range of w -$\hat{w}$ and $\tilde{w}$- and d. We consider different funding policies based on the weight of external funds versus funds raised form the interbank market on the liabilities side. Each bank is equally exposed to other banks in the banking system, holding that $\sum_j w_{ji} = \sum_i w_{ij} \forall i$. The level of integration varies in fixed increments with values $w \in [0, 1]$.
  Regarding the level of diversification, it holds that $0 \leq d \leq n - 1$, being d randomly chosen.

As a result, our simulation consists of multiples capital structures where we consider low, medium and high levels of capitalization and interconnectedness.

2. The magnitude of the price shock. The exogenous shock is assumed to be asset-specific. It is also a single shock, in the sense of hitting external assets only once at the beginning of the simulation. We pick bank i and drop the value of the external asset held by it. In order to assess the effect of the shock size on leverage dynamics, we consider different shock intensities.

1.5.2.3.1 Simulation framework.

In our model simulation, we consider the case of two banks[48] presenting accounting symmetry with the exception of external asset holdings. Each bank fully owns one external asset, so that $K = 2$, being the dependency matrix equal to the identity matrix.
Consequently, we restrict our analysis to a regular and complete network, where $w_{ij} = w_{ji} = w$. Accordingly, the dependency matrix is symmetric and without loss of generality the bank hit by the shock is chosen randomly.

In a banking system consisting of two banks, $i = 1, 2$, and, therefore, two external assets, k=1,2, we have that:

$$\mathbf{Sq} = \begin{pmatrix} s_{ii,t} & 0 \\ 0 & s_{jj,t} \end{pmatrix} \begin{pmatrix} n_{k_i} p_{k_i,t} \\ n_{k_j} p_{k_j,t} \end{pmatrix} \begin{pmatrix} s_{11,t} & 0 \\ 0 & s_{22,t} \end{pmatrix} \begin{pmatrix} q_{1,t} \\ q_{2,t} \end{pmatrix} = \begin{pmatrix} s_{11,t} q_{1,t} \\ s_{22,t} q_{2,t} \end{pmatrix} \tag{1.51}$$

For the sake of simplicity, we additionally assume that external assets -government bonds- are initially priced at par: $p_{k,0} = 1 \equiv 100\%$. Therefore, each bank holds a single external asset with value 1 and the expression in matrix notation $\mathbf{Sq}$ is equivalent to a

[48] Obviously, a banking network of two banks imposed automatically a complete network design. We diminish the relevance of this restriction for our simulation purposes for two reasons: firstly, considering a banking system consisting of just two banks already allows for establishing the existence of unstable leverage dynamics, and secondly, some part of the literature shows that the role of diversification can be ambiguous.

kx1 column vector with all k elements equal to one, $\mathbf{z}$. Hence, at t=0, previous expression becomes:

$$\mathbf{Sq} = \begin{pmatrix} s_{11,0} & 0 \\ 0 & s_{22,0} \end{pmatrix} \begin{pmatrix} q_{1,0} \\ q_{2,0} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \mathbf{z} \tag{1.52}$$

Two main scenarios are considered regarding the intensity of shocks affecting the diverse capital structures, as we select the external asset held by bank 1 and make its value drops either 1% or 10%.

The conclusions change depending on whether we consider short or long-term dynamics. We allow for this time-horizon effect by setting the number of periods of each realization $T = 10$ or $T = 50$.

In order to analyse how integration, connectivity and initial levels of capitalization affect the leverage dynamics triggered by external asset price shocks, we perform simulations on random networks following an approach similar to the one used in Elliot et al. (2014). We combine those network measures to determine the magnitude of impact of network effects on contagion and shock amplification. Working with a random graph allows for comparative statics by imposing some structure on cross-holdings distribution.
In a ER random graph model, the expected degree is computed as $d = p(n-1)$, where p is the probability of two nodes being connected.

The interbank cross-holdings matrix $\mathbf{W}$ is derived from the graph (N, g), whose off-diagonal entries are w, the fraction of each bank held by the other bank in the financial system.
Different funding scenarios are captured through w. In the simulation, we loop over the values of integration parameter w within the range [0.1,0.7], varying in increments of 0.1.

Regarding external liabilities cross-holdings, if $v_{i,t} = \hat{w}_{ii} V_{i,t}$, the remaining fraction of each bank is held by outside investors, so that the diagonal elements of matrix $\hat{W}$ are defined $\hat{w} = 1 - w$.

However, if $v_{i,t} = e_{i,t} = \tilde{w}_{ii} V_{i,t}$, the outstanding bank's value is distributed among shareholders, $\tilde{w}$, and external creditors, $\hat{w}$. In this context, $\hat{w}$ is defined as a vector of the same length as w, with the order of its elements reversed. As for shareholders' equity, it is defined as $\tilde{w} = 1 - w - \hat{w}$.

The simulation is structured as follows[49]:

1. After defining some of the variables and parameters as explained above, a random graph $\mathbf{G}$ with N=2 nodes and randomly-set degree $d \leq N$ is generated. We construct a 2-by-2 column-stochastic matrix of iid uniform random variables, with directed link-formation probability of $p = d/(n-1) = d$. The diagonal entries are set to zero to avoid self-links.

2. The matrix of cross-holdings is calculated from the directed random network: $\mathbf{W}$=$w\mathbf{G}$. The remaining matrices are defined as $\hat{\mathbf{W}} = \hat{w}\mathbf{I}$ and $\tilde{\mathbf{W}} = \tilde{w}\mathbf{I}$, when applicable.

3. Dependency matrices are generated according to expressions 1.44 and either 1.47 or 1.50, depending on the selected funding scenario.
   In this case of two banks, $i = 1, 2$, and two external assets, k=1,2, the generic form of dependency matrices is follows:

$$\mathbf{D} = \begin{pmatrix} d_{ii} & d_{ij} \\ d_{ji} & d_{jj} \end{pmatrix} = \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} \tag{1.53}$$

4. Value initialization ($t = 0$). Initial values for assets, $V_{i,0}$, are calculated using expression 1.43, while initial equity values, $v_{i,0}$, are determined as stated in equations 1.46 and 1.49. Once these values are obtained, we proceed to assess the initial value of leverage, defined as the assets-to-equity ratio, and, therefore, of target leverage, given our assumption $\lambda_{i,0} = \lambda_i^T$.
   Only in scenario (c), when the target leverage is exogenously set, the sequence followed in the last step is reversed. In this case, we firstly set the level of target leverage and determine the value of assets following 1.43, to subsequently derive the value of equity.

5. Initial price shock ($t = 1$). We drop the value p_k of the external asset held by randomly-chosen bank i and update matrix $\mathbf{Sq}$. After hitting the system with this shock in prices, the new values of assets, $\bar{V}_{i,1}$, equity, $\bar{v}_{i,1}$, and, therefore, the actual leverage, $\bar{\lambda}_{i,1}$, are recalculated.

6. Adjustment process: ($t = 1 + \tau/\tau := [1, T]$). The price shock implies a deviation from equilibrium by deviating $\lambda_{i,t}$ from λ_i^T. Considering the dependency analysis, and according to expression 1.51 and 1.53, the new lambda after the shock hitting

[49]The simulation design is implemented so it can be easily extrapolated to a N-banks case.

bank 1 is defined[50]:

$$\bar{\lambda}_{1,t} = \frac{(d_{11}s_{11,t-1}q_{1,t}) + (d_{12}s_{22,t-1}q_{2,t-1})}{\bar{e}_{1,t}} \tag{1.54}$$

Whenever the leverage gap materializes, i.e., $\lambda_{i,t} \neq \lambda_i^T$, an adjustment process begins. When confronting negative price shocks, banks adjust down leverage by selling assets worth x, and paying down x worth of debt. Variable $x_{i,t}$ quantifies the adjustment undertaken by shocked bank i at time t and is defined as the variation in the value of external asset holdings. In our specific case of N=2:

$$x_{1,t} = s_{11,t}q_{1,t} - s_{11,t-1}q_{1,t} = \frac{\lambda_1^T \bar{e}_{1,t} - d_{11}(s_{11,t-1}q_{1,t}) - d_{12}(s_{22,t-1}q_{2,t-1})}{d_{11}} \tag{1.55}$$

such that, theoretically:

$$\lambda_1^T = \frac{[d_{11}(s_{11,t-1}q_{1,t} + x_1)] + (d_{12}s_{22,t-1}q_{2,t-1})}{\bar{e}_{1,t}} \tag{1.56}$$

As shown in expression 1.42, bank i's counterparties are indirectly affected by shocks hitting i, being the new level of bank 2's leverage given by:

$$\bar{\lambda}_{2,t} = \frac{(d_{22}s_{22,t-1}q_{2,t-1}) + (d_{21}s_{11,t-1}q_{1,t})}{\bar{e}_{2,t}} \tag{1.57}$$

Consequently, there is a simultaneous adjustment performed by financial organizations exposed to i. In the network under consideration, this adjustment is equivalent to:

$$x_{2,t} = s_{22,t}q_{2,t-1} - s_{22,t-1}q_{2,t-1} = \frac{\lambda_2^T \bar{e}_{2,t} - d_{22}(s_{22,t-1}q_{2,t-1}) - d_{21}(s_{11,t-1}q_{1,t})}{d_{22}} \tag{1.58}$$

so the following expression hypothetically holds

$$\lambda_2^T = \frac{[d_{22}(s_{22,t-1}q_{2,t-1} + x_2)] + (d_{21}s_{11,t-1}q_{1,t})}{\bar{e}_{2,t}} \tag{1.59}$$

We assume absence of rational expectations in our model, i.e., short-sighted banks are not able to anticipate counterparties' adapting behaviour, and so expressions 1.56 and 1.59 do not actually hold. For these expressions to hold, the categorical value of external assets held by the counterparty to be incorporated in adjustments 1.55 and 1.58 and, therefore, in previous expressions should be given by $s_{22,t}q_{2,t-1}$ instead of by $s_{22,t-1}q_{2,t-1}$, and by $s_{11,t}q_{1,t}$ instead of by $s_{11,t-1}q_{1,t}$, respectively.

[50] As anticipated, equity initially changes only via changes in external asset price in which banks hold positions:

By not being capable to foresee the effect of other banks' adjustments on own assets and equity, banks do not incorporate true expectations in own behaviour.
With regard to equity, an analog process emerges. As balance-sheet adjustments are made via external securities, both on the assets and liabilities sides, own equity should remain unchanged during the adjustment process, $e_{i,t} = \bar{e}_{i,t}$. On the contrary, changes in the value of assets due to a bank's own balance sheet management induce further variations in the value of equity of connected banks. Consequently, equity indeed varies in counterparties' balance sheets without banks anticipating this effect. This deviation in equity can be expressed as follows:

$$e_{i,t} - \bar{e}_{i,t} = d_{ii}[s_{jj,t}q_{j,t-1} - s_{jj,t-1}q_{j,t-1}] = d_{ij}x_{j,t} \tag{1.60}$$

Therefore, the intensity of the adjustment will be erroneous and the leverage gap will not be closed by the end of the period. Iterated processes of adjustments will then trigger and perpetuate until either both banks achieve their objective of reaching the target leverage level, at least one default or one asset selloff occurs.
The adjustment in this context would be expressed as follows:

$$x_{i,t+1} = \frac{\lambda_i^T \bar{e}_{i,t+1} - d_{ii}(s_{ii,t}q_{i,t}) - d_{ij}(s_{jj,t}q_{j,t})}{d_{ii}} \tag{1.61}$$

Figure 1.12 represents the market dynamics triggered by fully-leveraged banks' balance sheet management, given an exogenous shock in sovereign debt prices, and the amplification mechanism:

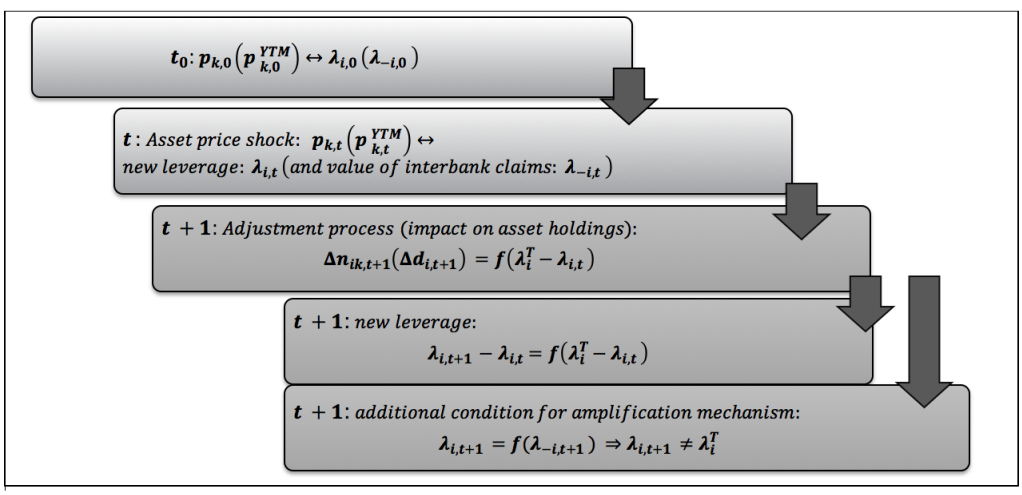


Figure 1.12: Flow-chart of the leverage-asset holdings cycle.

1.5.2.3.2 Simulation results.

Three main scenarios are simulated, based on different levels of target leverage. For each of these frameworks, we vary the level of integration while holding the level of diversification fixed, in order to assess how the stability of the system behaves according to the degree of interbank connectedness.
Holding the network degree fixed, increasing w increases integration but not connectivity. As w increases, so does the fraction of bank i's cross-holdings in other banks it has already established financial linkages with. Therefore, the (ij)th entry of the dependency matrices described previously in 1.44, 1.47 and 1.50 is increasing in w.

The same leverage-asset holding cycle is evidenced in each scenario and for every loop over ω. It can be summarized as follows:

- At $t = 0$, leverage $\lambda_{i,0}$ is set equal to the target leverage, λ_i^T. This level is the same for all banks, but not common knowledge -in order to enable forecasting deficiencies-.
- At $t = 1$, a price shock materializes, involving direct and indirect effects on leverage. It induces an immediate variation on leverage of the bank directly exposed to the shocked asset, as well as a further deviation as the value of its internal assets -counterparties' obligations - is affected by the indirect exposure of the obligor to external asset shocks. This exposure is captured in our model through the dependency matrix in 1.44. Thus, the shock affects all organizations within the interbank network, through either direct or indirect channels, and initially deviates leverage out of the equilibrium, from λ_i^T to $\lambda_{i,1}$.
- Banks manage their balance sheets actively to as to maintain their leverage ratio at the target level. At $t = 2$, this re-sizing of banks' balance sheet is implemented through an adjustment in external asset holdings and external funds according to the accounting rule in 1.37 and expressions 1.55 and 1.58, respectively. This latter adjustment has a subsequent impact on the value of external assets, and consequently a further effect on leverage. As banks are not capable to anticipate and incorporate into their adjustment routine the effect of counterparties' behaviour, by the end of the period leverage is expected to not return to "equilibrium" levels. This latter condition concludes a whole the leverage-asset holdings cycle, and cause the beginning of a new one.
- At $t = 3$ a new cycle is indeed set in motion. Henceforth, banks perform iterative adjustments as stated in 1.37 and 1.61, ceasing the process only once leverage returns to the equilibrium level or either a default or selloff occurs.

We find that convergence dynamics of leverage[51] depend on the interaction between some characteristics of a bank's capital structure -integration and capitalization- and between these properties and the level of the price shock. Depending on different contexts arising from diverse interplays, leverage dynamics evolves as follows:

Result 1: **High level of capitalization**. Equity is defined as bank's assets net of interbank liabilities. Making no distinction between external liabilities and shareholders equity implies an overestimated capitalization and, therefore, an underestimated leverage.

Figure 1.13 illustrates leverage dynamics as the level of integration varies, remaining the level of diversification fixed.

The extent of the effects of a price shock on leverage dynamics increases in network integration. The depth of interbank cross-holdings affects the leverage-gap amplitude and, consequently, the intensity of the required adjustment. The higher w, the deeper the financial interdependencies and the stronger the scope of the shock within the interbank market. As we continue to increase integration, the collateral effects through the exposure to the shocked bank intensify and the proportional relation between adjustment decreases, as the affected counterparty becomes more vulnerable to shocks and is forced to strengthen the adjustment.

We can distinguish different dynamics patterns regarding the convergence behaviour.

- When the system converges simultaneously, asymptotically or not, to the leverage target, a bank's leverage gap resulting from the non-anticipated effects of the counterparty's balance sheet management decreases over time, and so do the required adjustment. This holds for levels of $w \in \left[0.1, 0.4\right]$ as shown in figure 1.13.
- The system can also manifest an overall but non-simultaneous convergent behaviour, i.e., the system globally converges to the leverage target but individual leverage levels do not converge simultaneously to the target. Periods of individual convergence may alternate with periods of divergence. Convergence occurs when the magnitude of the effect of the own adjustment is greater than the

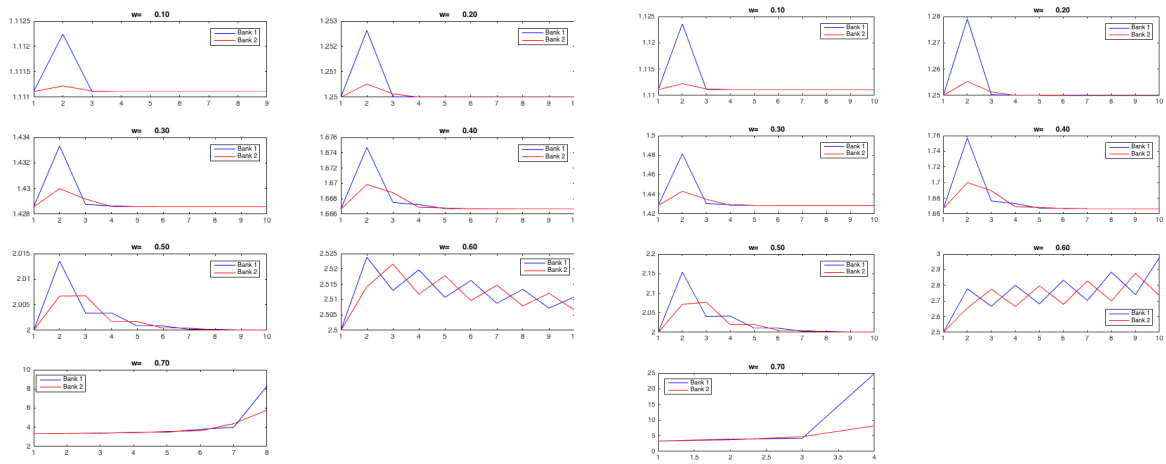
[51] Systemic convergency is achieved when all agents within the interbank network manage to close their leverage gaps. We assume this leverage gap to be significant whenever it holds $\lambda_{i,t} - \lambda^T < 1 * 10^{-3}$. By changing the decimal-digits precision, the results of our simulation can vary.

magnitude of the effects of the counterparty's adjustment: $d_{ii}x_{i,t} > d_{ij}x_{j,t}$. Alternatively, there is divergence whenever the effect of own adjustment is equal to the effect of others', $d_{ii}x_{i,t} = d_{ij}x_{j,t}$, so own leverage is boosted to levels even higher than in previous period: $\lambda_{it} > \lambda_{i,t-1} > \lambda^T$, but not higher than those ones in $t-2$, so the convergence to be possible. This happens for $w \in [0.5, 0.6]$ in panel (a) of figure 1.13 and for $w = 0.5$ in panel (b).

- On the contrary, the system can also show an overall non-simultaneous divergent pattern. Periods of individual convergence alternate with periods of divergence, but this time the strength of own adjustment is not great enough to push leverage down to a level lower than the one reached in $t-2$ during periods of convergence, and the effects of the counterparty adjustments boost leverage to the highest level so far during periods of divergence. This is the case for $w = 0.6$ in panel (b) of figure 1.13.
- Lastly, when the system diverges simultaneously, we observe that a bank's leverage gap increases over time and so do the intensity of its balance sheet management. Agents within the interbank network become more susceptible to the behaviour of the counterparty due to higher financial integration, $w = 0.7$.

Additionally, the consequences of integration are influenced by the level of the price shock:

- **Shock: 1%**. For low levels of interbank exposure ($0.1 \leq w \leq 0.3$), the system absorbs immediately the price shock after a one-period adjustment. Then, the number of periods required to reach the target value increases as the interbank market becomes more integrated. For higher values of w, the dynamics of the system change. Specifically, for $w = 0.6$, the system asymptotically converges to the target value, whilst it diverges for higher levels of integration and both banks defaults in the short term.
- **Shock: 10%**. Similarly to what happens in the presence of a small shock, for low levels of integration, $w \in [0.1, 0.5]$, the system manage to absorb the shock and turn to the equilibrium. Nevertheless, the convergence to equilibrium will be more gradual and extended over time. As interbank cross-holdings become deeper, the capacity of the system to return to pre-shock leverage levels severely decreases. This inability leads to divergent dynamics that result in defaults and selloffs for the highest levels of interbank exposure.



(a) Low-level price shock

(b) High-level price shock

Figure 1.13: Integration effects on leverage dynamics -high level of capitalization-

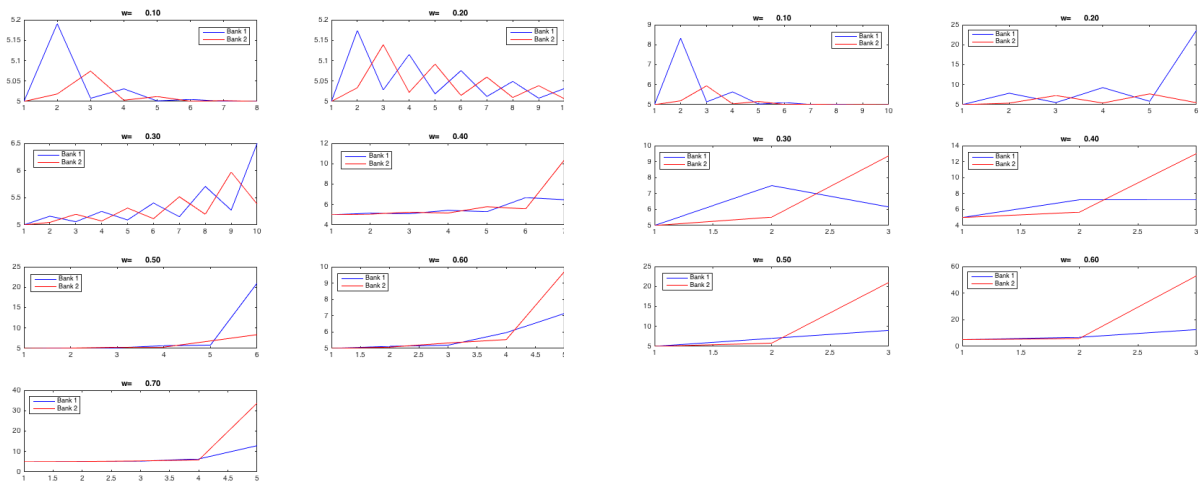
Result 2: **Medium level of capitalization**. Equity is now defined as bank's assets net of overall liabilities. Consequently, leverage is not underestimated; still we work with levels of leverage lower than the maximum suggested or imposed by regulators.

Figure 1.14 illustrates how integration affects leverage dynamics in this new context of lower capitalization. It follows that the vulnerability of the system not only depends on the degree of financial integration but also on the initial financial robustness of banks.

As previously, we can observe diverse patterns of convergence dynamics influenced by the size of the shock and the depth of interconnectivity.

- **Shock: 1%**. In the case of a more leveraged system, only when w is sufficiently low, $w = 0.1$, the system absorbs the price shock and converges to the target leverage, but not simultaneously. The number of periods required to reach the target value is higher than in previous scenario, though. When the level of integration is still low, $w = 2$, the system converges asymptotically to the equilibrium level. For values of integration within the range $w := \{0.3, ..., 0.7\}$, leverage dynamics diverge from the target and both defaults and selloffs occur even in the short term. As w increases, the number of periods to any of these events decreases.

– **Shock: 10%**. Similarly to what happens in the presence of a small shock, for the lowest level of integration the system manage to absorb the shock and turn to the equilibrium, but this process extends over a longer period of time due to the magnitude of the shock. When w is still low, $w = 0.2$, the system manifests a divergent non-simultaneous behaviour that results in rapid selloffs. For any other value of $w \in \left[0.2, 0.7\right]$, the system is not able to assimilate the shock and default events and selloffs materialize in the very short term. Furthermore, the initial shock triggers an immediate default when integration has reached its maximum.



(a) Low-level price shock

(b) High-level price shock

Figure 1.14: Integration effects on leverage dynamics -medium level of capitalization-

Result 3: **Low level of capitalization**. The level of leverage is imposed exogenously and equity is then derived from the leverage ratio - *assets-to- target leverage ratio* -. By doing so, we set a more realistic leverage value that results in a lessened financial robustness.

Figure 1.15 shows how the effects of a price shock vary with w in a context of low initial capitalization. The resistance of the system to shocks decreases with integration, so higher levels of w correspond to earlier defaults.

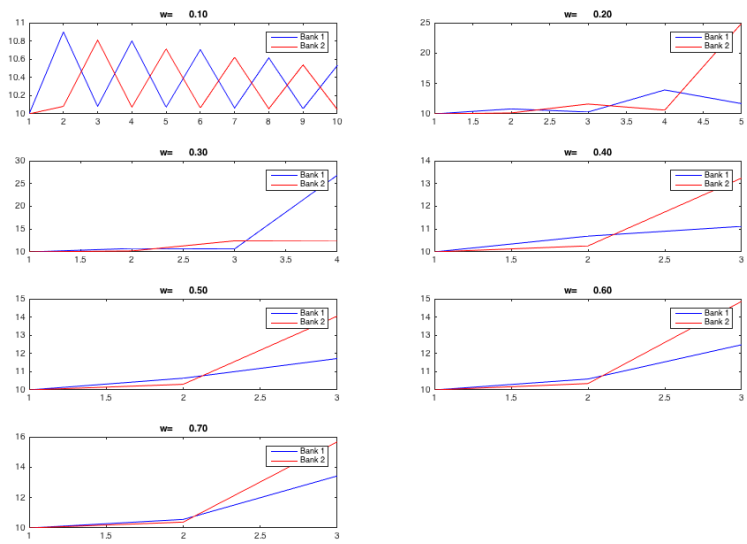


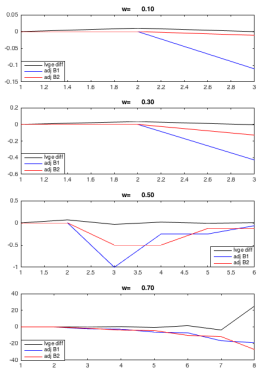
Figure 1.15: Integration effects on leverage dynamics -low-level capitalization and price shock-

In the presence of an small shock, leverage can be returned to its equilibrium value only when integration is at its minimum level, but this takes considerably longer than in previous simulation scenarios. For $w \in [0.2, 0.7]$, there is no convergence to the target and the system defaults in the short term. The promptness with which the equity of an organisation falls below zero increases with integration.

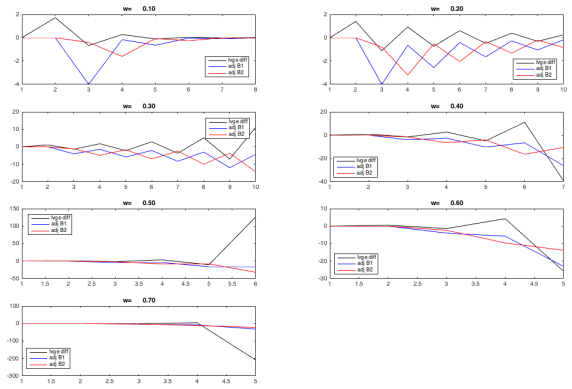
When a high-level price shock occurs, the system does not manage to absorb it. Either the value of the external asset held by the shocked bank falls below zero as a consequence of asset selloffs or both institutions default during the first period of adjustment.

In any case, whatever the convergence pattern, the system exhibits synchronized dynamics given by banks' active balance sheet management.

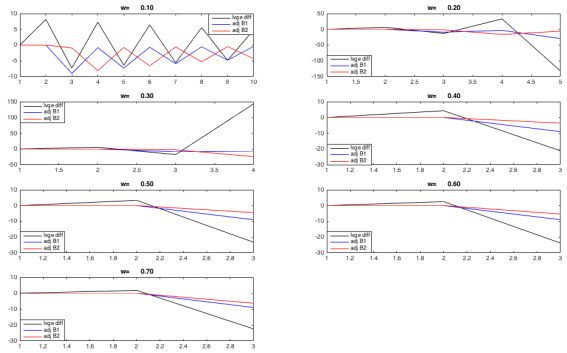
The absence of expectations about others' performance lead to overadjustments that result in persistent deviations from the leverage target. Only if this gap decreases over time, there is convergence to the equilibrium.
Furthermore, the magnitude of the others' adjustments on assets affects the size of own's leverage gap at the end of each period. The greater the counterparty's adjustment, the greater own leverage deviation from the target due to interdependence effects. Hence, the bank will be forced to adjust more the next period.



(a) High capitalization



(b) Medium capitalization



(c) Low capitalization

Figure 1.16: Synchronization of leverage dynamics

Panels of figure 1.16 shows how banks adjust more than the other periodically. Even if the proportional relation between adjustment decreases in w, it remains constant over alternate iteration periods for a fixed integration degree, i.e., $x_{i,t+1}/x_{j,t+1} = x_{j,t}/x_{i,t} = x$.

As initial capitalization deteriorates, the difference between banks' adjustments increases, and so does the difference between the respective leverage levels over time. The

magnitude of the adjustments is also increasing in capitalisation deterioration.

It can be observed that whenever the system exhibits divergence, we observe that a banks' adjustments in t is greater than the one in $t-2$. The opposite holds for convergence.

To summarise, our simulation results proof that unstable leverages dynamics arise when considering marked-to-market leverage and under the assumption of banks' incapacity to form correct expectations on the performance of their counterparties within the interbank network. Instability manifests through the inability of systemic leverage to converge to the equilibrium level: the target leverage. We have shown that convergence is affected by some network properties and node-specific characteristics.
More specifically, the conclusion derived from the analysis of leverage dynamics is that increasing integration reduces the resilience of the system to shocks affecting external asset price in the specific case of two banks. The vulnerability of the system depends subsequently on the financial robustness of the banks it comprises and on the size of the shock that distresses it. Therefore, the effects of integration are magnified as financial robustness deteriorates and the magnitude of the shock increases.

1.6 Conclusions.

In analyzing the interactions and feedback relationships within the financial network, we have followed an analytical and quantitative approach to overlap and contagion.
One of the novelties of our work if that we have provided explicit analytical foundation to leverage dynamics and the functioning of the overlap financial structure. In doing so, we have considered diverse leverage valuation methodologies, namely the ones adopted in finance theory and empirical finance. The conclusions derived have then been tested by simulating leverage dynamics within a heterogeneous financial network.

We have firstly shown how the chosen analytical method affects significantly the results of the equilibrium analysis, i.e., different valuations of leverage lead to different potential equilibriums.
Additionally, we have evidenced that the existence of an equilibrium is ultimately strongly affected by the structure of balance sheets and, therefore, by the level of leverage of the system.

Within the banking network, banks reveal an individualistic behaviour as leverage-targeting institutions. But homogenised dynamics may arise as banks adopt the same

strategies. This would be the case of fully-leveraged financial institutions reacting to an external event impacting on their net worth. Affected banks actively manage their balance sheets -by adjusting the level of external securities- to maintain leverage at the target level. This synchronised behaviour may amplify, under certain circumstances, the effects of external shocks, even if they are small.

Our simulation results confirm the importance of leverage in the origination of endogenous financial processes, ultimate leading to crises. Excessive leverage increases the vulnerability of system to small fluctuations and may lead to systemic events.
The level of leverage determines the intensity of the balance sheet management process. This intensity is crucial in determining further feedbacks effects affecting the value of of the holdings of external assets.

We have analysed how the stability of the financial system is determined by the interaction among bank, market and network attributes. Particularly, convergence dynamics of leverage depends on capitalisation, diversification integration, the level of the price shock and the rate of adjustment to the target leverage.

The dynamics of the model become unstable when introducing explicitly the dependency of one bank's leverage on their counterparties' leverage.
The main concept behind our leverage-dynamics analysis is that the value of a financial organisation ultimately depends on own and counterparties' external assets. Consequently, we have modelled the linkages among financial institutions through dependency matrices.
Different scenarios have been simulated, based on different levels of target leverage. For each of these frameworks, we varied the level of integration while holding the level of diversification fixed, in order to assess how the stability of the system behaves according to the degree of interbank connectedness.

The developed agent-based model has made it possible to improve the analysis concerning some ambiguous relationships among variables, even if the role of diversification may still be ambiguous in the presence of amplification mechanisms. To this regard, some additional simulations have been made for a financial system consisting of N banks, with $N > 2$.
When considering different levels of diversification, accounting also for incomplete networks, patterns similar to the ones captured in the 2-bank case are observed. Nevertheless, this analysis will be thoroughly covered in future research.

Our simulation results proof that unstable leverages dynamics arise when considering marked-to-market leverage and under the assumption of banks' incapacity to form correct expectations on the performance of their counterparties within the interbank network.

Additionally, some conclusions are drawn from our analysis regarding to the suitability of imposing countercyclical macroprudential policies.
We have analytically illustrated that the amplification mechanism may drive leverage to an "overshoot" equilibrium in the presence of an external asset bubble burst only if these kind of policies are applied.
Common knowledge of a target leverage and, therefore, perfect-forecasting capacity of counterparties' targets, allows for anticipation of others' balance sheet adjustments and the incorporation of these true expectations in own behaviour. Otherwise, as the leverage dynamics model simulations proves, the existence of equilibrium can not be ensured.
In this sense, the imposition of counter-cyclical leverage requirements may be effective. Nevertheless, the choice of leverage ratios by macroprudential authorities should be based on the actual structure in order to ensure a systemic equilibrium.

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Appendices

Appendix A

A Static Analysis of Interaction in Overlapping Portfolios

A.1 Mark-to-market asset values, book value of liabilities: Asymmetric case of two banks

A.1.1 Curvature of the leverage function

In the following we study the curvature of the leverage function. Being the expression for the second derivative of bank i's leverage with respect to the counterpart's leverage:

$$\frac{\partial^2 \lambda_i}{\partial \lambda_j^2} = \frac{\left[\frac{2\xi^2 b_j (d_i+b_i)}{(R+\xi\lambda_j)^3}\right]\left[n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j} - (d_i+b_i)\right]\left[\frac{b_j}{R+\xi\lambda_j} - \left(n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j} - (d_i+b_i)\right)\right]}{\left[n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j} - (d_i+b_i)\right]^4} \tag{A.1}$$

Some considerations have to be made about the sign of this derivative:

The **denominator** will always be positive, as the expression in brackets -which is the definition of net worth- is to a even number power. Furthermore, by assumption, the equity/ net worth has to be greater than 0 (an equity equal or lower than 0 means bankruptcy), so the derivative can not became infinite.

Regarding the **numerator**, we have to consider that, by assumption:

- The rate of return can not be negative, so $R + \xi\lambda_{-i} > 0$. So this expression to any power will be positive.
- Always by assumption, the liabilities can not be negative, so $(d_i + b_i) > 0$. Technically a negative liability is an asset, and should be classified so.
- The equity/ net worth has to be greater than 0, as said previously.

That said, for the numerator -and, consequently, the previous derivative- to be positive, it must hold:

$$\frac{b_j}{R+\xi\lambda_j} - \left(n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j} - (d_i+b_i)\right) > 0 \tag{A.2}$$

which could be, for instance, the case in which a bank does not have positions in external assets/ liabilities, with interbank liabilities being greater than zero. Or it may happen that the holdings of external assets are lower than total liabilities, so $n_{i,k}p_k < d_i + b_i$.
On the other hand, for the numerator -and, consequently, the previous derivative- to be negative, the opposite must hold.

When analysing the effects of the exposure to a counterparty on the slope of the leverage curve:

$$\frac{\partial}{\partial b_j}\left(\frac{\partial\lambda_i}{\partial\lambda_j}\right) = \frac{\frac{\xi(d_i+b_i)}{(R+\xi\lambda_j)^2}\left(n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j} - (d_i+b_i)\right)\left[\left(n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j} - (d_i+b_i)\right) - \frac{2b_j}{R+\xi\lambda_j}\right]}{\left(n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j} - (d_i+b_i)\right)^4} \tag{A.3}$$

we have that the sign of this expression depends on whether

$$n_{i,k}p_k - \frac{b_{-i}}{R+\xi\lambda_{-i}} > (d_i+b_i) \tag{A.4}$$

So the difference between the value of external and internal assets has to be grater than the value of total liabilities in order to have $\frac{\partial}{\partial b_{-i}}\left(\frac{\partial\lambda_i}{\partial\lambda_{-i}}\right) > 0$.

Additionally, the sign of the following derivative:

$$\frac{\partial}{\partial n_k}\left(\frac{\partial\lambda_i}{\partial p_k}\right) = \frac{-(d_i+b_i)\left(n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j} - (d_i+b_i)\right)^2 + \left[2p_k n_{i,k}(b_i+d_i)\left(n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j} - (d_i+b_i)\right)\right]}{\left(n_{i,k}p_k + \frac{b_j}{R+\xi\lambda_j} - (d_i+b_i)\right)^4} \tag{A.5}$$

depends on whether $n_{i,k}p_k - \frac{b_j}{R+\xi\lambda_j} > d_i + b_i$ (to be positive) or not.

A.1.2 Equilibrium of the system: the intersection point in the plane.

We are interested in finding a point of equilibrium. So we will consider the two graphs consisting on the expressions for λ_1 and λ_2 and calculate the intersection point(s) of both.

$$\lambda_1 = \frac{n_{1,k}p_k + \frac{b_2}{R+\xi\lambda_2}}{n_{1,k}p_k + \frac{b_2}{R+\xi\lambda_2} - (d_1+b_1)} \tag{A.6}$$

$$\lambda_2 = \frac{n_{2,k}p_k + \frac{b_1}{R+\xi\lambda_1}}{n_{2,k}p_k + \frac{b_1}{R+\xi\lambda_1} - (d_2+b_2)} \tag{A.7}$$

Solving expression A.7 for λ_1:

$$\lambda_1 = -\frac{R}{\xi} + \frac{b_1(1-\lambda_2)}{\xi[(\lambda_2 - 1)n_{2,k}p_k + \lambda_2(d_2 + b_2)]} \tag{A.8}$$

and setting the previous equation for λ_1 equal to expression A.6 and solving for λ_2, we have that the λ_2-coordinate of the intersection point will be the solution for the quadratic equation:

$$\begin{aligned} \lambda_2^2\xi[(P_2 - N_2)[\xi P_1 + R(P_1 - N_1)] - (P_1 - N_1)b_1] \\ +\lambda_2[(P_2 - N_2)(R[\xi P_1 + R(P_1 - N_1) - b_2] + \xi b_2) - (P_1 - N_1)[Rb_1 + \xi P_2] + b_1 b_2 - \xi^2 P_1 P_2] \\ +R(P_1 - N_1)(P_2 + B_1) + R(b_2 P_2 + \xi P_1 P_2) - b_1 b_2 - P_1 P_2 = 0 \end{aligned} \tag{A.9}$$

where $P_i = n_{i,k}p_k$ and $N_i = d_i + b_i$

By plugging the solution to the previous expression into A.7, we will get λ_1-coordinate of the intersection point.

A.2 Mark-to-market asset values and liabilities: Asymmetric case of two banks: General resolution.

A.2.1 Linearisation of the system of non-linear equations: $\lambda_1^* \neq \lambda_2^*$

The system of non-linear equations is given by:

$$f(\lambda_1, \lambda_2) = [\xi (N_1 - d_1) - 1] \lambda_1^2 + [N_1 (R - \xi) - d_1 R] \lambda_1 - N_1 R = 0 \tag{A.10}$$

$$g(\lambda_1, \lambda_2) = [\xi (N_2 - d_2) - 1] \lambda_2^2 + [N_2 (R - \xi) - d_2 R] \lambda_2 - N_2 R = 0 \tag{A.11}$$

When linearising the expressions A.10 and A.11 using the Taylor expansion and solving for λ_1 and λ_2, we get expressions of the form $\lambda_1 = F(\lambda^*, p_k)$ and $\lambda_2 = G(\lambda^*, p_k)$:

$$\begin{aligned}
\lambda_1 = &\left(\left[n_{1,k}p_k + \frac{b_2}{R+\xi\lambda_2^*}\right][\xi(2\lambda_1^*-1)+R] - 2\lambda_1^*(\xi d_1+1) - d_1 R\right)^{-1} \\
&\left[\lambda_2^* \frac{\xi b_2}{(R+\xi\lambda_2^*)^2}(R-\lambda_1^*[\xi\lambda_1^*+(R-\xi)]) - (\lambda_1^*)^2\left[1+\xi d_1 - \xi\left(n_{1,k}p_k + \frac{b_2}{R+\xi\lambda_2^*}\right)\right] + R\left(n_{1,k}p_k + \frac{b_2}{(R+\xi\lambda_2^*)}\right)\right. \\
&\left. + \frac{\xi b_2}{(R+\xi\lambda_2^*)^2}(R-\lambda_1^*[\xi\lambda_1^*+(R-\xi)])\right] \\
&\left(1 - \left[\left[n_{2,k}p_k + \frac{b_1}{R+\xi\lambda_1^*}\right][\xi(2\lambda_2^*-1)+R] - 2\lambda_2^*(\xi d_2+1) - d_2 R\right]^{-1}\right. \\
&\left.\left[\frac{\xi b_1}{(R+\xi\lambda_1^*)^2}(R-\lambda_2^*[\xi\lambda_2^*+(R-\xi)])\right]\left[\frac{\xi b_2}{(R+\xi\lambda_2^*)^2}(R-\lambda_1^*[\xi\lambda_1^*+(R-\xi)])\right]\right)^{-1} \\
&\left(\left(\left[n_{2,k}p_k + \frac{b_1}{R+\xi\lambda_1^*}\right][\xi(2\lambda_2^*-1)+R] - 2\lambda_2^*(\xi d_2+1) - d_2 R\right)^{-1}\right. \\
&\left[\lambda_1^* \frac{\xi b_1}{(R+\xi\lambda_1^*)^2}(R-\lambda_2^*[\xi\lambda_2^*+(R-\xi)]) - (\lambda_2^*)^2\left[1+\xi d_2 - \xi\left(n_{2,k}p_k + \frac{b_1}{R+\xi\lambda_1^*}\right)\right] + R\left(n_{2,k}p_k + \frac{b_1}{(R+\xi\lambda_1^*)}\right)\right. \\
&\left. + \left[\frac{\xi b_1}{(R+\xi\lambda_1^*)^2}(R-\lambda_2^*[\xi\lambda_2^*+(R-\xi)])\right]\left(\left(\left[n_{1,k}p_k + \frac{b_2}{R+\xi\lambda_2^*}\right][\xi(2\lambda_1^*-1)+R] - 2\lambda_1^*(\xi d_1+1) - d_1 R\right)^{-1}\right. \\
&\left.\left.\left[\lambda_2^* \frac{\xi b_2}{(R+\xi\lambda_2^*)^2}(R-\lambda_1^*[\xi\lambda_1^*+(R-\xi)]) - (\lambda_1^*)^2\left[1+\xi d_1 - \xi\left(n_{1,k}p_k + \frac{b_2}{R+\xi\lambda_2^*}\right)\right] + R\left(n_{1,k}p_k + \frac{b_2}{(R+\xi\lambda_2^*)}\right)\right]\right)\right]\right)\right]
\end{aligned} \tag{A.12}$$

$$
\begin{aligned}
\lambda_2 = \Bigg(1 - \Bigg[\left[n_{2,k}p_k + \frac{b_1}{R+\xi\lambda_1^*}\right]\left[\xi\left(2\lambda_2^* - 1\right) + R\right] - 2\lambda_2^*\left(\xi d_2 + 1\right) - d_2 R\Bigg]^{-1} \\
\left[\frac{\xi b_1}{\left(R+\xi\lambda_1^*\right)^2}\left(R - \lambda_2^*\left[\xi\lambda_2^* + (R-\xi)\right]\right)\right]\left[\frac{\xi b_2}{\left(R+\xi\lambda_2^*\right)^2}\left(R - \lambda_1^*\left[\xi\lambda_1^* + (R-\xi)\right]\right)\right]\Bigg)^{-1} \\
\Bigg(\Bigg(\left[n_{2,k}p_k + \frac{b_1}{R+\xi\lambda_1^*}\right]\left[\xi\left(2\lambda_2^* - 1\right) + R\right] - 2\lambda_2^*\left(\xi d_2 + 1\right) - d_2 R\Bigg)^{-1} \\
\left[\lambda_1^*\frac{\xi b_1}{\left(R+\xi\lambda_1^*\right)^2}\left(R - \lambda_2^*\left[\xi\lambda_2^* + (R-\xi)\right]\right) - \left(\lambda_2^*\right)^2\left[1 + \xi d_2 - \xi\left(n_{2,k}p_k + \frac{b_1}{R+\xi\lambda_1^*}\right)\right] + R\left(n_{2,k}p_k + \frac{b_1}{\left(R+\xi\lambda_1^*\right)}\right)\right. \\
+\left[\frac{\xi b_1}{\left(R+\xi\lambda_1^*\right)^2}\left(R - \lambda_2^*\left[\xi\lambda_2^* + (R-\xi)\right]\right)\right]\Bigg(\Bigg(\left[n_{1,k}p_k + \frac{b_2}{R+\xi\lambda_2^*}\right]\left[\xi\left(2\lambda_1^* - 1\right) + R\right] - 2\lambda_1^*\left(\xi d_1 + 1\right) - d_1 R\Bigg)^{-1} \\
\left.\left[\lambda_2^*\frac{\xi b_2}{\left(R+\xi\lambda_2^*\right)^2}\left(R - \lambda_1^*\left[\xi\lambda_1^* + (R-\xi)\right]\right) - \left(\lambda_1^*\right)^2\left[1 + \xi d_1 - \xi\left(n_{1,k}p_k + \frac{b_2}{R+\xi\lambda_2^*}\right)\right] + R\left(n_{1,k}p_k + \frac{b_2}{\left(R+\xi\lambda_2^*\right)}\right)\right]\Bigg)\Bigg]\Bigg)
\end{aligned}
\tag{A.13}
$$

A.2.2 Linearisation of the system of non-linear equations: $\lambda_1^* = \lambda_2^*$.

When linearising around the point $\lambda^* = \lambda_1^* = \lambda_2^*$ we get:

$$f(\lambda_1, \lambda_2) \approx -(\lambda^*)^2 \left[\xi\left(n_{1,k}p_k + \frac{b_2}{R+\xi\lambda^*}\right) - \xi d_1 - 1\right] + \lambda_1 \left[\xi\left(n_{1,k}p_k + \frac{b_2}{R+\xi\lambda^*}\right)(2\lambda^* - 1)\right]$$
$$-\lambda_1 \left[2\lambda^*(\xi d_1 + 1) + d_1 R\right] - R\left[n_{1,k}p_k + \frac{b_2}{R+\xi\lambda^*}\right][1-\lambda_1] + \xi\frac{b_2}{(R+\xi\lambda^*)^2}[\lambda_2 - \lambda^*]\left[R - \lambda^*\left[\xi\lambda^* + (R-\xi)\right]\right] = 0 \tag{A.14}$$

$$g(\lambda_1, \lambda_2) \approx -(\lambda^*)^2 \left[\xi\left(n_{2,k}p_k + \frac{b_1}{R+\xi\lambda^*}\right) - \xi d_2 - 1\right] + \lambda_2 \left[\xi\left(n_{2,k}p_k + \frac{b_1}{R+\xi\lambda^*}\right)(2\lambda^* - 1)\right]$$
$$-\lambda_2 \left[2\lambda^*(\xi d_2 + 1) + d_2 R\right] - R\left[n_{2,k}p_k + \frac{b_1}{R+\xi\lambda^*}\right][1-\lambda_2] + \xi\frac{b_1}{(R+\xi\lambda^*)^2}[\lambda_1 - \lambda^*]\left[R - \lambda^*\left[\xi\lambda^* + (R-\xi)\right]\right] = 0 \tag{A.15}$$

We solve the previous system by equaling both expressions. After some algebra, we get:

$$\lambda_1 = \left[n_{1,k}\left[\xi(2\lambda^* - 1) + Rp_k\right] - d_1(\xi 2\lambda^* + R) - 2\lambda^* + b_2\left[\frac{1}{R+\xi\lambda^*}\left(R - \xi + \frac{\xi R}{R+\xi\lambda^*}\right) + \xi\lambda^*\left(2 - \frac{\xi\lambda^* + R - \xi}{R+\xi\lambda^*}\right)\right] + (b_2 - b_1)\left(\frac{\xi\lambda^*(\xi\lambda^* + (R-\xi)) - \xi R}{(R+\xi\lambda^*)^2}\right)\right]^{-1}$$
$$\left[\left(\xi\lambda^*\left[\lambda^*\left[(d_1 - d_2) - (n_{2,k} - n_{1,k})p_k + \frac{1}{R+\xi\lambda^*}(b_2 - b_1)\right] + \left[R - \lambda^*(\xi\lambda^* + R - \xi)\right]\frac{1}{(R+\xi\lambda^*)^2}(b_2 - b_1)\right]\right.\right.$$
$$\left. - R\left[(n_{2,k} + n_{1,k})p_k + (b_2 - b_1)\frac{1}{R+\xi\lambda^*}\right]\right) + \lambda_2\left(\left[Rp_k - 1\right]\left[n_{2,k} - n_{1,k}\right] + \left[Rs_k - \xi(2\lambda^* - 1)\right]n_{1,k}\right.$$
$$- (b_2 - b_1)\left(\frac{1}{R+\xi\lambda^*}\left[\xi(2\lambda^* - 1) + R\right]\right) - (d_1 - d_2)(\xi 2\lambda^* + R) - d_1(\xi 2\lambda^* + R) - 2\lambda^*$$
$$\left.\left. + b_2\left[\frac{1}{R+\xi\lambda^*}\left(\xi\left[2\lambda^* - 1\right] + R - \frac{\xi\lambda^*(\xi\lambda^* + R - \xi) - \xi R}{R+\xi\lambda^*}\right)\right]\right)\right] \tag{A.16}$$

By plugging this expression into equation A.15, for instance, we can get an expression of the form: $\lambda_2 = G(\lambda^*)$, that we can plug into expression A.14 in order to get $\lambda_1 = F(\lambda^*)$.[1]

[1] For the sake of simplicity, we are skipping this calculation.
Note that expression (A.69) is expressed in terms of differences of holdings among banks

A.2.3 Some particular cases.

- **Case: Identical exposure to external network, different interbank cross-holdings.**

  In this case we have:

$$n_{1,k} = n_{2,k} = n_k; d1 = d2 = d.$$

  and expression A.16 can be simplified as follows:

$$\lambda_1 = \left[n_{1,k}\left[\xi\left(2\lambda^* - 1\right) + Rp_k\right] - d\left(\xi 2\lambda^* + R\right) + 2\lambda^*(\xi b_2 - 1) + \frac{1}{R+\xi\lambda^*}\left[b_2(R-\xi) - b_1\frac{(\xi\lambda^*(\xi\lambda^* + R - \xi) - \xi R)}{R+\xi\lambda^*}\right]\right]^{-1}$$
$$\left[\frac{b_2 - b_1}{R+\xi\lambda^*}\left(\xi\lambda^*\left[\lambda^* + \frac{R - \lambda^*(\xi\lambda^* + R - \xi)}{R+\xi\lambda^*}\right] - R\right)\right.$$
$$+ \lambda_2\left(\left[Rs_k - \xi\left(2\lambda^* - 1\right)\right]n_k - \frac{b_2 - b_1}{R+\xi\lambda^*}\left[\xi\left(2\lambda^* - 1\right) + R\right] - d\left(\xi 2\lambda^* + R\right) - 2\lambda^*\right.$$
$$\left.\left. + b_2\left[\frac{1}{R+\xi\lambda^*}\left(\xi\left[2\lambda^* - 1\right] + R - \frac{\xi\lambda^*\left(\xi\lambda^* + R - \xi\right) - \xi R}{R+\xi\lambda^*}\right)\right]\right)\right] \quad \text{(A.17)}$$

  Then, as done previously, we could substitute λ_1 with this expression in equation A.15, for instance, and get an expression of the form: $\lambda_2 = G(\lambda^*)$, that we can plug into expression A.14 in order to get $\lambda_1 = F(\lambda^*)$.[2]

[2] Once again, for the sake of simplicity, we are skipping this calculation.

- **Case: One debtor, one creditor within the interbank network**

In this case we have:

$$b_1 > 0; b_2 = 0$$

and expression A.16 can be simplified as follows:

$$\lambda_1 = \left[n_{1,k}\left[\xi\left(2\lambda^* - 1\right) + Rp_k\right] - d_1\left(\xi 2\lambda^* + R\right) - 2\lambda^* - b_1\left(\frac{\xi\lambda^*\left(\xi\lambda^* + (R-\xi)\right) - \xi R}{\left(R+\xi\lambda^*\right)^2}\right)\right]^{-1}$$
$$\left[\left(\xi\lambda^*\left(\lambda^*\left[(d_1 - d_2) - (n_{2,k} - n_{1,k})\,p_k\right]\right) + \frac{b_1}{R+\xi\lambda^*}\left[R - \xi\lambda^*\left(1 + \frac{R - \lambda^*(\xi\lambda^* + R - \xi)}{R+\xi\lambda^*}\right)\right] - R(n_{1,k} - n_{2,k})p_k\right)\right.$$
$$+ \lambda_2\left(\left[Rp_k - 1\right]\left[n_{2,k} - n_{1,k}\right] + \left[Rs_k - \xi\left(2\lambda^* - 1\right)\right]n_{1,k}\right.$$
$$\left.\left. + b_1\left(\frac{1}{R+\xi\lambda^*}\left[\xi\left(2\lambda^* - 1\right) + R\right]\right) - (d_1 - d_2)\left(\xi 2\lambda^* + R\right) - d_1\left(\xi 2\lambda^* + R\right) - 2\lambda^*\right)\right] \quad \text{(A.18)}$$

Then, as done previously, we could substitute λ_1 with this expression in equation A.15, for instance, and get an expression of the form: $\lambda_2 = G(\lambda^*)$, that we can plug into expression A.14 in order to get $\lambda_1 = F(\lambda^*)$.[3]

- **Case: Identical exposure to external network; one debtor, one creditor within the interbank network.**

In this case we have:

$$n_{1,k} = n_{2,k} = n_k; d1 = d2 = d; b_1 > 0; b_2 = 0$$

and expression A.16 can be simplified as follows:

$$\lambda_1 = \left[n_{1,k}\left[\xi\left(2\lambda^* - 1\right) + Rp_k\right] - d\left(\xi 2\lambda^* + R\right) - 2\lambda^* - b_1\left(\frac{\xi\lambda^*\left(\xi\lambda^* + (R-\xi)\right) - \xi R}{\left(R+\xi\lambda^*\right)^2}\right)\right]^{-1}$$
$$\left[\frac{b_1}{R+\xi\lambda^*}\left(R - \xi\lambda^*\left[1 + \frac{R - \lambda^*(\xi\lambda^* + R - \xi)}{R+\xi\lambda^*}\right]\right)\right.$$
$$\left. + \lambda_2\left(\left[Rs_k - \xi\left(2\lambda^* - 1\right)\right]n_k + b_1\left(\frac{1}{R+\xi\lambda^*}\left[\xi\left(2\lambda^* - 1\right) + R\right]\right) - d\left(\xi 2\lambda^* + R\right) - 2\lambda^*\right)\right] \quad \text{(A.19)}$$

Then, as done previously, we could substitute previous expression in equation A.15, for instance, and get an expression of the form: $\lambda_2 = G(\lambda^*)$, that we can plug into expression A.14 in order to get $\lambda_1 = F(\lambda^*)$.[4]

[3] Once again, for the sake of simplicity, we are skipping this calculation.
[4] Once again, for the sake of simplicity, we are skipping this calculation.

- **Case: One strong bank, one stressed bank.**

In this case we have heterogeneous ξ, in the sense that:

$\xi_1 = 0$ *Mkt. perception of financial robustness* ; $\xi_2 > 0$ *Mkt. perception of financial weakness*

In this case, the equations for leverage are:

$$\lambda_1 = \frac{n_{1,k}p_k + \frac{b_2}{R+\xi_2\lambda_2}}{n_{1,k}p_k + \frac{b_2}{R+\xi_2\lambda_2} - \left(d_1 + \frac{b_1}{R}\right)} \tag{A.20}$$

$$\lambda_2 = \frac{n_{2,k}p_k + \frac{b_1}{R}}{n_{2,k}p_k + \frac{b_1}{R} - \left(d_2 + \frac{b_2}{R+\xi_2\lambda_2}\right)} \tag{A.21}$$

When solving equation A.21 for λ_2, we get a quadratic expression which results independent from λ_1:

$$\lambda_2^2\left[\xi_2(n_{2,k} + \frac{b_1}{R} - d_2) - 1\right] + \lambda_2\left[(n_{2,k} + \frac{b_1}{R})(R - \xi_2) - d_2R\right] - (n_{2,k} + \frac{b_1}{R})R = 0 \tag{A.22}$$

If we solve this quadratic expression, we can plug the resulting expression for λ_2 into equation A.20 and get the expression for λ_1.

Appendix B

Dynamics in the network.

An endogenous mechanism of links formation based on preferential attachment could be implemented: each bank enters a financial (credit) contract with peers with a probability proportional to potential neighbor's financial robustness. A limit the notional value of interbank debts would be set to . This won't not prevent banks from changing counterparties, but just limit lending capacities in the interbank market.

An agent's fitness could be expressed as a function of her probability of default. Therefore, an agent's attractiveness will depend on her financial robustness.

Each agent will form a new link with an agent j -and cut an existing one with agent k- with the following probability[1]:

$$P_{i,t} = \frac{1}{1 + e^{\mathbb{P}_j - \mathbb{P}_k}} \tag{B.1}$$

or will keep its existing link with probability $(1 - P_{i,t})$.

[1]This expresion for the probability of link formation is based on the one used in Lenzu and Tedeschi, 2011. Although we are not including a parameter capturing the signal credibility about other peers' financial conditions information.

Appendix C

An alternative approach to the analysis of leverage.

In this section we set up an scenario where we can study the dependency on leverage of variables derived from economic measures such as RoE or the market value of assets, symmetry assumed.

Rearranging expressions 3 and 11 , we have, respectively:

$$\lambda \left[p_k n_k + \frac{b}{R + \xi\lambda} - (d + b) \right] = p_k n_k + \frac{b}{R + \xi\lambda} \tag{C.1}$$

and

$$\lambda \left[p_k n_k - d \right] = p_k n_k + \frac{b}{R + \xi\lambda} \tag{C.2}$$

The left-hand side in both equations is equivalent to λ **Net Worth*, while the right-hand side is equivalent to the *market value of assets*.
Equations C.1 and C.2 capture the condition for the intersection of the curves $n(\lambda) = \lambda$ **Net Worth* and $m(\lambda) =$ *market value of assets*. Solving for λ we get the respective expressions for the abscissa of the equilibrium points -λ-, that are equal to the ones given by expressions 5 and 13 , and so are the conclusions about the equilibrium.

When evaluating the efficiency of a firm, two are the most important measures to be consider: Return on Equity (RoE) and Return on assets (RoA). The DuPont identity, a popular formula for dividing ROE into its core components, explains the relationship between both measures of management effectiveness as follows: $RoE = RoA * leverage$. ROE can be related to ROA by multiplying a factor of financial leverage. According to financial literature, gains in financial leverage leads to higher RoE - even if financial leverage benefits diminish as the risk of defaulting on interest payments increases -.

Appendix D

Derivation of an accounting rule based on leverage targeting.

Recalling the expression for leverage

$$\lambda_{it} = \frac{\sum_k n_{ik,t} p_{k,t} + \sum_j \omega_{ij} \frac{b_{j,t}}{R_t + \xi_{i,t}\lambda_{j,t}}}{\left(\sum_k n_{ik,t} p_{k,t} + \sum_j \omega_{ij} \frac{b_{j,t}}{R + \xi_{i,t}\lambda_{j,t}}\right) - (d_{i,t} + b_{i,t})} \tag{D.1}$$

Before deriving the accounting rule governing a bank's balance sheet management, we recall some of the assumptions made so far. At an aggregate level, notional value of interbank holdings remain constant over time ($b_{i,t} = b_i$), so , given that equity remains "sticky", leverage is adjusted through external positions.
Furthermore, we don't address the effect of changes in counterparties' leverage on this rule for the moment ($\mathcal{B}_{i,t} = \frac{b_{i,t}}{R_t + \xi_{i,t}\lambda_{i,t}} = \mathcal{B}_i$).

The balance sheet management process establishes a linkage between two adjacent periods in time: in each period stocks generates flows updating the stocks.

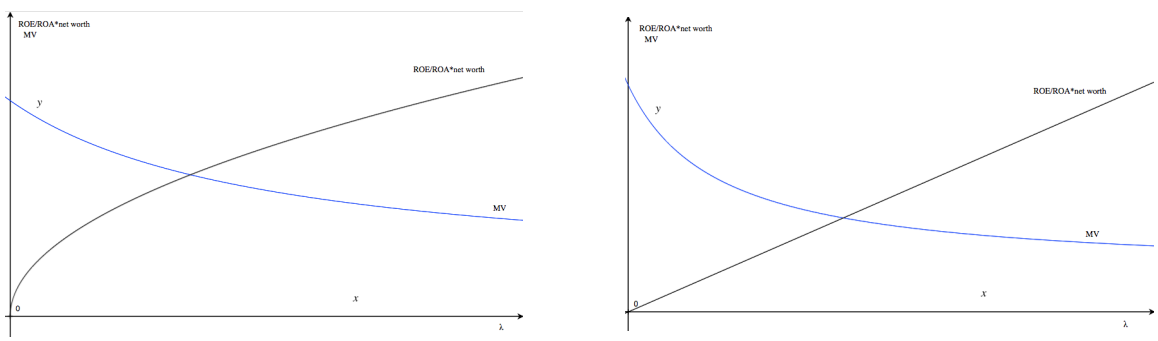
At time t=0

$$\lambda_i^T = \lambda_{i,0} = \frac{\sum_k n_{ik,0} p_{k,0} + \sum_j w_{ij}\mathcal{B}_j}{\sum_k n_{ik,0} p_{k,0} + \sum_j w_{ij}\mathcal{B}_j - (\mathcal{B}_i + d_{i,0})} \tag{D.2}$$

We define bank i's external assets as the sum of bank i's claims against agents in the real sector -private or public (sovereigns)-, being the market value of external assets held by an agent i in time t given by:

Expressions C.1 and C.2 result from substituting the expressions for RoE (*RoE= Net Income/ Equity*) and RoA (*RoE= Net Income/ Assets*) in the DuPont identity linking RoE and RoA. So the LHS of the equation λ*Net Worth is equivalent to $\frac{RoE}{RoA} * equity$, and it is nothing but a term proportional to the RoE-to-RoA ratio.

The effects of changes in leverage on the function on the left-hand side of equation C.1 seemed to be uncertain when calculating the partial derivative. But when looking at the asymptotic behaviour of the function, it's evident that the slope cannot be anything but positive. Regarding expression C.2, the sign of the derivative is unambiguously positive. Analogously, the responses of the market value of assets to variations in leverage are of the same sign, regardless of the valuation of interbank debt. The market value of assets is decreasing in leverage.
But when studying the concavity of both $m(\lambda)$ and $n(\lambda)$, we find that whilst the second derivative of function $m(\lambda)$ with respect to leverage has the same sign -positive- independently of the methodology underlying the valuation of interbank debt, the second derivative of function $n(\lambda)$ with respect to leverage can be either negative or zero, depending on whether we consider the book value or the market value of internal liabilities. So the representation of the curves in the plane would be as shown in figure C.1.



(a) Book value of liabilities

(b) Mkt value of liabilities

Figure C.1: λ*Net Worth* and *market value of assets* curves

According to the statement previously mentioned in regard to the relationship between ROE and leverage, a higher proportion of debt in the capital structure (and, therefore, a higher leverage) leads to higher ROE. This is captured by the positive slope of curve $\frac{RoE}{RoA} * equity$. Nevertheless, taking on too much debt may cause the cost of debt increases as creditors could demand a higher risk premium, and the benefits arising out of the gains in leverage would decrease. The negativeness of the second derivative of curve $\frac{RoE}{RoA} * equity$ with respect to leverage captures this fact when considering the book value of interbank liabilities, whereas we miss this evidence when valuing internal debt at market prices.

$$\sum_k n_{ik,t} p_{k,t} = \sum_m q_{im,t} p_m + \sum_h g_{ih,t} p_h, \quad \forall m \in [0, ..., M], h \in [0, ...H], k \in [0, ..., M + H]$$

where

$\sum_k n_{ik,t}$	the total amount of external assets held by agent i in time t
p_k	the market price (present value) of external assets.
$\sum_m q_{im,t}$	the total amount of bank i's claims against agents in the private sector held at time t.
p_m	the market price of those external assets.
$\sum_h g_{ih,t}$	the total amount of bank i's claims against agents in the public sector -governments' debt- at t
p_h	the market price of public external assets.

We consider the following in this setup:

$$\sum_k n_{ik,0} p_{k,t} = \sum_m q_{im,0} p_m + \sum_h g_{ih,0} p_h \quad \text{for} \quad t = 0$$

At time t=1 a shock in external asset n (sovereign debt) prices materialises: $p_{n,0} \to p_{n,1}$

$$\sum_k n_{ik,0} p_{k,1} = \sum_m q_{im,0} p_m + \sum_h g_{ih,1} p_h \quad \text{for} \quad t = 1$$

The previous shock changes the leverage as follows:

$$\lambda_{i,1} = \frac{\sum_k n_{ik,0} p_{k,1} + \sum_j w_{ij} \mathcal{B}_j}{\sum_k n_{ik,0} p_{k,1} + \sum_j w_{ij} \mathcal{B}_j - (\mathcal{B}_i + d_{i,0})} \neq \lambda_i^T \tag{D.3}$$

After the price shock, at time t=2, the volume of the external asset is adjusted to meet the target leverage, so are external liabilities. The new leverage at time t=2 is:

$$\lambda_{i,2} = \frac{\sum_k n_{ik,2} p_{k,1} + \sum_j w_{ij} \mathcal{B}_j}{\sum_k n_{ik,2} p_{k,1} + \sum_j w_{ij} \mathcal{B}_j - (\mathcal{B}_i + d_{i,2})} = \lambda_i^T \tag{D.4}$$

from equation D.3

$$\lambda_{i,1} \left[\sum_k n_{ik,0} p_{k,1} + \sum_j w_{ij} \mathcal{B}_j - (\mathcal{B}_i + d_{i,0}) \right] = \sum_k n_{ik,0} p_{k,1} + \sum_j w_{ij} \mathcal{B}_j$$

$$\lambda_{i,1} \sum_k n_{ik,0} p_{k,1} + \lambda_{i,1} \left[\sum_j w_{ij} \mathcal{B}_j - (\mathcal{B}_i + d_{i,0}) \right] = \sum_k n_{ik,0} p_{k,1} + \sum_j w_{ij} \mathcal{B}_j$$

$$(1 - \lambda_{i,1}) \sum_j w_{ij} \mathcal{B}_j = \lambda_{i,1} \left[\sum_k n_{ik,0} p_{k,1} - (\mathcal{B}_i + d_{i,0}) \right] - \sum_k n_{ik,0} p_{k,1}$$

$$\sum_j w_{ij}\mathcal{B}_j = \frac{(\lambda_{i,1}-1)}{(1-\lambda_{i,1})}\left(\sum_k n_{ik,0}p_{k,1}\right) - \frac{\lambda_{i,1}}{(1-\lambda_{i,1})}(\mathcal{B}_i + d_{i,0})$$

$$\sum_j w_{ij}\mathcal{B}_j = -\frac{\lambda_{i,1}}{(1-\lambda_{i,1})}(\mathcal{B}_i + d_{i,0}) - \sum_k n_{ik,0}p_{k,1} \tag{D.5}$$

By substituting D.5 into D.4

$$\lambda_{i,2} = \frac{\sum_k n_{ik,2}p_{k,1} - \frac{\lambda_{i,1}}{(1-\lambda_{i,1})}(\mathcal{B}_i + d_{i,0}) - \sum_k n_{ik,0}p_{k,1}}{\sum_k n_{ik,2}p_{k,1} - \frac{\lambda_{i,1}}{(1-\lambda_{i,1})}(\mathcal{B}_i + d_{i,0}) - \sum_k n_{ik,0}p_{k,1} - (\mathcal{B}_i + d_{i,2})}$$

$$\lambda_{i,2} = \frac{\Delta\sum_k n_{ik,2}p_{k,1} - \frac{\lambda_{i,1}}{(1-\lambda_{i,1})}(\mathcal{B}_i + d_{i,0})}{\Delta\sum_k n_{ik,2}p_{k,1} - \frac{\lambda_{i,1}}{(1-\lambda_{i,1})}(\mathcal{B}_i + d_{i,0}) - (\mathcal{B}_i + d_{i,2})}$$

where $\Delta\sum_k n_{ik,2}p_{k,1} = (\sum_k n_{ik,2} - \sum_k n_{ik,0})p_{k,1}$. Then:

$$\frac{\Delta\sum_k n_{ik,2}p_{k,1} - \frac{\lambda_{i,1}}{(1-\lambda_{i,1})}(\mathcal{B}_i + d_{i,0})}{\lambda_{i,2}} = \Delta\sum_k n_{ik,2}p_{k,1} - \frac{\lambda_{i,1}}{(1-\lambda_{i,1})}(\mathcal{B}_i + d_{i,0}) - (\mathcal{B}_i + d_{i,2})$$

$$\frac{\lambda_{i,1}}{(1-\lambda_{i,1})}(\lambda_{i,2}-1)(\mathcal{B}_i + d_{i,0}) + (1-\lambda_{i,2})\Delta\sum_k n_{ik,2}p_{k,1} = -\lambda_{i,2}(\mathcal{B}_i + d_{i,2}) \tag{D.6}$$

By assumption:

$$d_{i,2} - d_{i,0} = \Delta\sum_k n_{ik,2}p_{k,1} \Rightarrow d_{i,2} = d_{i,0} + \Delta\sum_k n_{ik,2}p_{k,1}$$

By substituting in D.6

$$\frac{\lambda_{i,1}}{(1-\lambda_{i,1})}(\lambda_{i,2}-1)(\mathcal{B}_i + d_{i,0}) + (1-\lambda_{i,2})\Delta\sum_k n_{ik,2}p_{k,1} = -\lambda_{i,2}(\mathcal{B}_i + d_{i,0} + \Delta\sum_k n_{ik,2}p_{k,1})$$

$$\frac{\lambda_{i,1}}{(1-\lambda_{i,1})}(\lambda_{i,2}-1)(\mathcal{B}_i + d_{i,0}) + \Delta\sum_k n_{ik,2}p_{k,1} = -\lambda_{i,2}(\mathcal{B}_i + d_{i,0})$$

$$\Delta\sum_k n_{ik,2}p_{k,1} = -\frac{\lambda_{i,1}}{(1-\lambda_{i,1})}(\lambda_{i,2}-1)(\mathcal{B}_i + d_{i,0}) - \lambda_{i,2}(\mathcal{B}_i + d_{i,0})$$

$$\Delta\sum_k n_{ik,2}p_{k,1} = (\mathcal{B}_i + d_{i,0})\left[\frac{\lambda_{i,1}}{(1-\lambda_{i,1})}(\lambda_{i,2}-1) - \lambda_{i,2}\right]$$

Given that

$$\frac{\lambda_{i,1}}{(1-\lambda_{i,1})}(\lambda_{i,2}-1)-\lambda_{i,2}=\frac{\lambda_{i,1}}{(1-\lambda_{i,1})}-\frac{\lambda_{i,2}\lambda_{i,1}}{(1-\lambda_{i,1})}-\lambda_{i,2}$$
$$=\frac{\lambda_{i,1}}{(1-\lambda_{i,1})}-\lambda_{i,2}\left(1+\frac{\lambda_{i,1}}{(1-\lambda_{i,1})}\right)=\frac{\lambda_{i,1}}{(1-\lambda_{i,1})}-\lambda_{i,2}\left(\frac{1}{(1-\lambda_{i,1})}\right)$$
$$=\frac{1}{(1-\lambda_{i,1})}(\lambda_{i,1}-\lambda_{i,2})=\frac{1}{(1-\lambda_{i,1})}(\lambda_{i,1}-\lambda_i^T)$$

then

$$\Delta\sum_k n_{ik,2}p_{k,1}=(\mathcal{B}_i+d_{i,0})\frac{1}{(1-\lambda_{i,1})}(\lambda_{i,1}-\lambda_i^T) \tag{D.7}$$

From equation D.5

$$\mathcal{B}_i+d_{i,0}=-\frac{1-\lambda_{i,1}}{\lambda_{i,1}}\left[\sum_j w_{ij}\mathcal{B}_j+\sum_k n_{ik,0}p_{k,1}\right]$$

By substituting in D.7

$$\Delta\sum_k n_{ik,2}p_{k,1}=-\frac{(1-\lambda_{i,1})}{\lambda_{i,1}(1-\lambda_{i,1})}\left[\sum_j w_{ij}\mathcal{B}_j+\sum_k n_{ik,0}p_{k,1}\right](\lambda_{i,1}-\lambda_i^T)$$
$$\Rightarrow\Delta\sum_k n_{ik,2}=-\frac{(\lambda_{i,1}-\lambda_i^T)}{\lambda_{i,1}}\frac{\left[\sum_j w_{ij}\mathcal{B}_j+\sum_k n_{ik,0}p_{k,1}\right]}{p_{k,1}}$$

and dividing by $n_{ik,0}$

$$\frac{\Delta\sum_k n_{ik,2}}{n_{ik,0}}=-\frac{(\lambda_{i,1}-\lambda_i^T)}{\lambda_{i,1}}\frac{\left[\sum_j w_{ij}\mathcal{B}_j+\sum_k n_{ik,0}p_{k,1}\right]}{n_{ik,0}p_{k,1}}$$

where $\frac{\Delta\sum_k n_{ik,2}}{n_{ik,0}}=\frac{n_{ik,2}-n_{ik,0}}{n_{ik,0}}$ and $\frac{\left[\sum_j w_{ij}\mathcal{B}_j+n_{ik,0}p_{k,1}\right]}{\sum_k n_{ik,0}p_{k,1}}$ is the ratio of the value of total assets to the external asset k value for bank i.

Defining $\alpha_{ik,1}$ as the ratio of external asset k value to total assets value, the rate of change for asset k for bank i is:

$$\frac{n_{ik,2}-n_{ik,0}}{n_{ik,0}}=-\frac{1}{\alpha_{ik,1}}\frac{(\lambda_{i,1}-\lambda_i^T)}{\lambda_{i,1}}=\frac{1}{\alpha_{ik,1}\lambda_{i,1}}(\lambda_i^T-\lambda_{i,1}) \tag{D.8}$$

We further modify this equation by including the parameter ϵ_i capturing the bank's reactivity to deviations in leverage, so changes in demand are proportional to the promptness in closing the leverage gap. Previous equation then becomes:

$$\frac{n_{ik,2} - n_{ik,0}}{n_{ik,0}} = -\frac{\epsilon_i}{\alpha_{ik,1}} \frac{(\lambda_{i,1} - \lambda_i^T)}{\lambda_{i,1}} = \frac{\epsilon_i}{\alpha_{ik,1}\lambda_{i,1}}(\lambda_i^T - \lambda_{i,1})$$

We now that $\sum_k n_{ik,0} = \sum_k n_{ik,1}$, so:

$$\frac{n_{ik,2} - n_{ik,1}}{n_{ik,1}} = -\frac{\epsilon_i}{\alpha_{ik,1}} \frac{(\lambda_{i,1} - \lambda_i^T)}{\lambda_{i,1}} = \frac{\epsilon_i}{\alpha_{ik,1}\lambda_{i,1}}(\lambda_i^T - \lambda_{i,1})$$

Then:

$$\frac{n_{ik,t} - n_{ik,t-1}}{n_{ik,t-1}} = -\frac{\epsilon_i}{\alpha_{ik,t-1}} \frac{(\lambda_{i,t-1} - \lambda_i^T)}{\lambda_{i,t-1}} = \frac{\epsilon_i}{\alpha_{ik,t-1}\lambda_{i,t-1}}(\lambda_i^T - \lambda_{i,t-1}) \tag{D.9}$$

Then, from the assumption of "sticky" equity:

$$d_{i,2} - d_{i,0} = \Delta \sum_k n_{ik,2} p_{k,1} \Rightarrow \text{from eq. D.7} \Rightarrow d_{i,2} - d_{i,0} = (\mathcal{B}_i + d_{i,0}) \frac{1}{(1 - \lambda_{i,1})}(\lambda_{i,1} - \lambda_i^T)$$

Dividing by $d_{i,0}$:

$$\frac{d_{i,2} - d_{i,0}}{d_{i,0}} = \frac{(\mathcal{B}_i + d_{i,0})}{d_{i,0}} \frac{1}{(1 - \lambda_{i,1})}(\lambda_{i,1} - \lambda_i^T)$$

where $\frac{(\mathcal{B}_i + d_{i,0})}{d_{i,0}}$ is the inverse ratio of the value of external debt to the value of total debt for bank i. Defining this ratio of external debt value to total debt value as $\beta_{i,0}$:

$$\frac{d_{i,2} - d_{i,0}}{d_{i,0}} = -\frac{1}{\beta_{i,0}(\lambda_{i,1} - 1)}(\lambda_{i,1} - \lambda_i^T)$$

having the expression for the relative change in debt for bank i:

$$\frac{d_{i,2} - d_{i,0}}{d_{i,0}} = \frac{1}{\beta_{i,0}(\lambda_{i,1} - 1)}(\lambda_i^T - \lambda_{i,1}) \tag{D.10}$$

After including the bank's commitment with the target leverage:

$$\frac{d_{i,2} - d_{i,0}}{d_{i,0}} = \frac{\epsilon_i}{\beta_{i,0}(\lambda_{i,1} - 1)}(\lambda_i^T - \lambda_{i,1})$$

And considering that $d_{i,0} = d_{i,1}$:

$$\frac{d_{i,t} - d_{i,t-1}}{d_{i,t-1}} = \frac{\epsilon_i}{\beta_{i,t-1}(\lambda_{i,t-1} - 1)}(\lambda_i^T - \lambda_{i,t-1}) \tag{D.11}$$

Chapter 2

International Expansion and Riskiness of Banks

We exploit an original dataset on 15 European banks classified as global systemically important banks (G-SIBs) by the BIS to assess whether expansion in foreign markets increases their riskiness, and through which channels that eventually happens. We find that there is a strong negative correlation between riskiness and foreign expansion. On the one hand, banks that expand abroad more have lower riskiness so that, given individual bank riskiness, their expansion reduced the (weighted) average riskiness of the banks' pool. On the other hand, foreign expansion of any given bank makes the bank and thus the banks' pool less risky. In terms of the channels, diversification, competition and regulation are all important. Expansion in destination countries with different business cycle co-movement and stricter regulations than the origin country decreases a bank's riskiness. As for competition, expansion decreases riskiness only when competition in the origin country is less intense than in the destination countries[1].

Keywords: banks' risk, global expansion, competition, diversification, regulation.

[1] This chapter is based on a joint work with Esther Faia (Goethe University of Frankfurt) and Gianmarco Ottaviano (London School of Economics). We are indebted with the European Commission for financial support within the MACFINROBODS project. We are grateful to Yona Rubinstein for helpful discussions and Sebastien Laffitte for outstanding research assistance.

2.1 Introduction

Using a newly collected dataset on global banks this paper examines from an empirical point of view a widely debated question, namely whether banks' internationalization has increased or decreased risk. Many attributed the emergence of the crisis to banks' globalization and/or more generally to financial globalization. In 2005 Rajan (35) highlighted the potential increase in risk contagion derived from finance and banking globalization. A growing empirical literature is emerging on the role that global banks have for credit expansion, liquidity management and competition. There is not yet a definite answer on the balance between the benefits and the dangers of the banking globalization. For instance, a recent IMF Financial Stability Report (32) showed that prior to the 2007 global risk had increased since much of the financial globalization took place through cross-border activity with little involvement of global banks into local retail activity. On the contrary, after the 2007 financial crisis there has been a shift in the business model of global banks, which currently tend to operate more through subsidiaries (occasionally through branches). Against this background, and leveraging an original panel dataset on the international expansion of the European banks classified as G-SIBs by the BIS from 2005 to 2014, we first study whether and how foreign expansion has affected these banks' riskiness. We then target the different forces at work, investigating whether and how the impact of foreign expansion on bank riskiness can be understood in terms of diversification, competition or regulation.

Our empirical analysis poses some methodological challenges related to reverse causation or to potential confounding factors. We focus on assessing the effects of exogenous shocks to foreign expansion on bank riskiness. However banks with different riskiness may have a different propensity to expand abroad so that any observed correlation between foreign expansion and bank riskiness may be due to the latter endogenously affecting the former. To deal with this problem, we follow the IV approach recently put forth by Goetz, Laeven and Levine ((29); GLL hereafter) and Levine, Lin and Xie ((33); LLX hereafter). The two papers are complementary. GLL assesses the impact of the geographic expansion of banks (in terms of assets) on their riskiness (proxied by the standard deviation of stock returns) by modelling the two stage estimation through an asset diversification channel. Instead LLX look at the impact of geographic expansion through diversification on banks' funding costs. Both papers are based on U.S. data and geographic expansion refers to the expansion in (metropolitan statistical areas in) states different from the one in which a bank is headquartered. The expansion decision itself, however, could be related to its risk position or its funding costs, especially so if the expansion changes bank's risk-taking incentives. To tackle this endogeneity problem, both studies instrument the observed geographic expansion of a bank with the forecast implied by a 'gravity equation'. The latter is estimated using

the characteristics of the bank's origin and destination markets as well as their distance.[2] The gravity estimation is an ideal candidate instrument. Indeed to the extent that the estimation does not include variables correlated with banks' risk, the estimated values will be correlated with the actual expansion, but not with banks' risk-taking behavior. Using this instrument, GLL and LLX find that geographic expansion reduces riskiness and funding costs respectively. GLL conjectured that this happened because of asset diversification. To test this hypothesis they examined how the impact of geographic expansion on riskiness varies upon the 'similarity' between the origin and the destination countries. They find that a key determinant of the negative relation between geographic expansion and banks' risk is the business cycle co-movement between the origin and the destination countries. Analogously, LLX find that the negative effect of the geographic expansion on funding costs is stronger when the origin state co-moves less with the rest of the US.

Differently from these papers, we look at international expansion, investigating three different transmission channels. We re-consider the diversification channel, but we also test the presence of a competition and of a regulation channel. The competition channel is motivated by results in the theoretical literature. Allen and Gale (1) had shown that higher competition in the deposit market tends to increase banks' risk-taking: as banks need to offer higher rates to entice investors into demand deposits, they also need to search for higher yield/risk assets. This result was challenged by Boyd and De Nicolo (6)), who showed that higher competition in the loan market tends to reduce banks' risk-taking. As more banks serve the loan markets, the rates shall decline and this brings about a decline in assets' risk. Recently Faia and Ottaviano (2016) have re-examined the link between banks' risk-taking and competition with a model featuring competition on both deposits' and loans' markets and allowing banks for the possibility to enter foreign markets, which are characterized by higher monitoring costs. They have shown that the link between competition and risk-taking depends on the balance between the relative strength of the deposit and the loans' markets competition, but that generally speaking for empirically relevant demand functions (for deposits and loans) banks' penetration in foreign market tends to reduce banks' risk-taking.

Our empirical findings can summarized as follows. First, there is a strong negative correlation between riskiness and foreign expansion. Using OLS with bank fixed effects to net out composition effects and to account for within variation, we find that regressing riskiness on foreign expansion produces a statistically significant and negative coefficient. Second, we test a selection channel (only low risk banks expand) by comparing OLS with and without

[2]The gravity equation has been extensively and successfully used to explain international flows of goods and services. See Appendix C for an overview.

bank fixed effects. Such comparison reveals negative selection effect, since we find a negative coefficient for the regression of openings on banks' risk. Third, to rule out the possibility of a reverse causality effect (banks' risk-taking behavior affects foreign expansion), we use a 2SLS with gravity-based IV. Under this specification the regression of riskiness on foreign expansion produces a larger (in absolute value) negative coefficient than with OLS.

To sum up, foreign expansion reduces the riskiness of the pool of banks in our sample. Banks that expand abroad have lower riskiness ('between effect') and foreign expansion renders any bank less prone to risk ('within effect'). The 'between effect' is, however, less robust than the 'within effect'.

Next, we test which of the above-described channels is responsible for the results. We find evidence that diversification, competition and regulation all play a role in understanding the 'within effect'. In line with the diversification channel, a bank's expansion in destination countries exhibiting different business cycle co-movement than the origin country decreases the bank's riskiness. With respect to regulatory arbitrage, expansion in destination countries with stricter regulation than the origin country decreases a bank's riskiness. As for competition, expansion has a lower impact on riskiness when competition in the origin country is less intense than in the destination countries.

The rest of the paper is organized as follows. Section 2 describes the novel dataset. Section 3 presents the empirical strategy and the results. Section 4 reports the findings related to the different transmission channels. Section 5 concludes.

2.2 Data

Our analysis exploits an original database on banks' geographic expansion that documents the evolution of banking globalization for a 10-year time period (2005 to 2014) and that captures recent trends in the international expansion of European banking groups. The data, related to banks' presence in Europe, cover a diversified range of European economies. Our dataset consists in panel data on foreign expansion decisions (i.e. decisions on entering a foreign market) for the European banks classified as G-SIBs by the BCBS by the end of 2015 ((9)). Based on this we have identified 15 banks in located 8 home countries and 38 potential destination countries (see appendix A for the complete list of countries included in the dataset). The panel is balanced, as we consider for each bank all potential host countries and years, even if the bank did not establish presence in a foreign country within a specific

year and despite the presence of not available information -missing values- in our sample.[3]

The data has been manually collected using *Bureau van Dijk's Bankscope*, *Zephyr*, *Bankers Almanac* dataset and *Bloomberg*. Several other complementary sources have been used, such as banks' annual reports, consolidated statements, websites, archives press releases, and report from national central banks, regulatory agencies, international organizations and financial institutions. Finally, the dataset is extended with geographic data that come from the CEPII's gravity dataset[4].

We measure international banking expansion by the count of global banks' entries in foreign economies by year, which are given by the number of foreign unit openings[5]. We define an opening in a host country as a parent bank applying one of the following growth strategies: 'Organic growth' by opening directly a new foreign branch or subsidiary or increasing the activity of already-existing units; 'Merger and Acquisition' through purchases of interest in local banks (ownership $\geq 50\%$) or takeovers; and 'Joint ventures'. Therefore, we consider that a bank enters a foreign market whenever it opens directly a branch or a subsidiary, or acquires, either directly or indirectly, a foreign entity, owning at least 50%. The opening would take place in this case either by increasing own ownership in an already-controlled institution or by acquiring a majority interest in a new one. We do not consider as an opening any new institution resulting from the merger among previously-owned group's entities. The establishment of representative offices, customer desks and the change of legal entity type (branch/ subsidiary) are disregarded as well. The parent bank is listed even if the opening was actually implemented by a foreign unit owned by the bank. Nevertheless, the count of openings that we use does not reflect the actual scale of events in each of the host countries, as we do not account for the branch network that an owned foreign unit may develop once it has entered the host economy. When the entering in the foreign market takes place through the acquisition of another institution, we count this opening as a single one, independently of the number of different entities belonging to the acquired one already present in that market. To maximize the precision we also obtained year-by-year detailed information on banking global strategies and ownership, extending the traditional sampling.

Our sample includes universal banks performing traditional retail and commercial banking services. But we also account for independent affiliates providing other banking services (private and investment banking, asset and wealth management), financial joint ventures,

[3]If the bank did not establish presence in a foreign country within a specific year the count of its openings is set equal to zero.

[4]This is available at: $http://www.cepii.fr/cepii/fr/bdd_modele/presentation.asp?id = 6$.

[5]Foreign units refer to incorporated foreign banks or financial companies with over a 50 percent ownership.

leasing companies holding the status of banks or MFI and factoring companies performing pure commercial credit-related activities. Consequently, the financial institutions in our sample are entities providing commercial and investment banking services (retail banking, private, banking, corporate and investment banking, asset management, etc). To sum up, our global banks are more akin to universal banks. This is understandable in light of the fact that large banks in Europe tend to operate as. Indeed our sample includes the top ten financial groups in Europe in terms of total assets. The banks considered are: *BNP Paribas*, *Crédit Agricole Group* and *Société Générale* in France; *Banco Santander* in Spain; *Unicredit* in Italy; *HSBC*, *Standard Chartered*, *RBS* and *Barclays* in the United Kingdom; *Deutsche Bank* in Germany; *ING Bank* in the Netherlands; *UBS* and *Credit Suisse* in Switzerland and *Nordea* in Sweden. We also consider *BPCE*, a banking group consisting of independent, but complementary commercial banking networks that provide also wholesale banking, asset management and financial services. Entities such as mutual and pension fund, trusts, financial holdings companies, instrumental corporations or affiliates performing activities related to private equity, advisory, real estate or insurance have been excluded from our sample. However, we consider joint ventures or leasing companies that hold the status of banks (according to *Bankscope* classification) or Monetary Financial Institutions (as defined by the European Central Bank), together with factoring companies, but only when these perform pure commercial-credit-related activities, as they can all be classified as consumer finance activities (retail banking).

We have focused on direct and indirect group's cross-border exposures, by considering both forms of penetration, namely branches and subsidiaries. Additionally, double counting has been avoided. Concerning take-overs, only the merged entity or the acquiring bank remained in the sample, while in terms of ownership, holding companies were excluded in countries where the banking group itself is present. As for ownership of a foreign unit, this has been determined based on both direct and indirect ownership structure. A bank or financial company is considered foreign-owned if at least 50% of shares are owned by the parent bank (see also Claessens, Demirguc-Kunt, and Huizinga(20); Clarke et al. (21)).

Based on the above-mentioned criteria, we identified 444 opening events in 38 host countries during the period $2005 - 2014$[6]. These events are listed in Table B.1 in Appendix B. This table shows that the largest number of events took place in Western Europe. Germany and Italy experienced the largest number of foreign bank units openings, while the smallest number is observed in CEE countries. Approximately half of the openings in the sample period occurred in the years prior to the crisis. The rate of growth of foreign-bank incor-

[6]Countries and opening events are listed in Appendixes A and B respectively.

poration shows a substantial decrease (almost a 80%) over the period considered. Even if annual decreases persisted from 2005 to 2012, the rate picked up in 2013 and 2014. Nevertheless, the number of openings in those last years was low in absolute terms compared to the number at beginning of the sample period. The largest drops in growth rates concentrated between 2008 and 2012, namely the period between the financial crisis of 2007-2008 and the euro area crisis of 2008-2012.

Figure 2.1 shows the evolution of foreign expansion by bank and year. The internationalization process was deeper during the pre-crisis period, with the exception of some financial groups such as *BNP Paribas* or *Crèdit Agricole*. The former's notable expansion in 2009 was principally due to the acquisition of the Dutch *Fortis*, whereas the latter's was essentially the result of an increase of retail banking activities (Consumer Finance) in several countries in 2008.

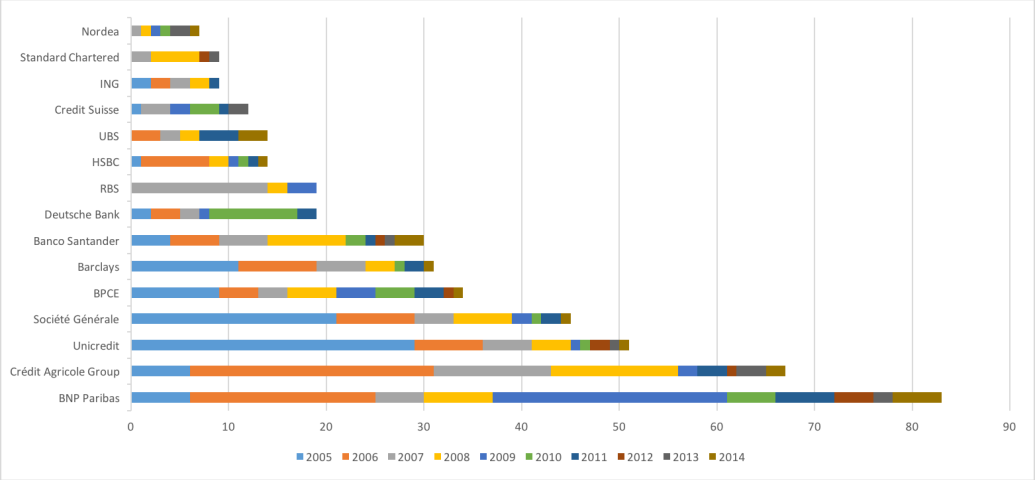


Figure 2.1: Foreign expansion of banks over the sample period.

Figure 2.2 illustrates the number of openings by origin country. Over the sample period the country that expanded the most was France, followed by the United Kingdom and Italy. From 2005 to 2014, French banks registered 229 events, while British and Italian ones 73 and 51, respectively. If the openings per bank are considered, France and Italy were by far the most globalizing origin countries in terms of banking expansion.

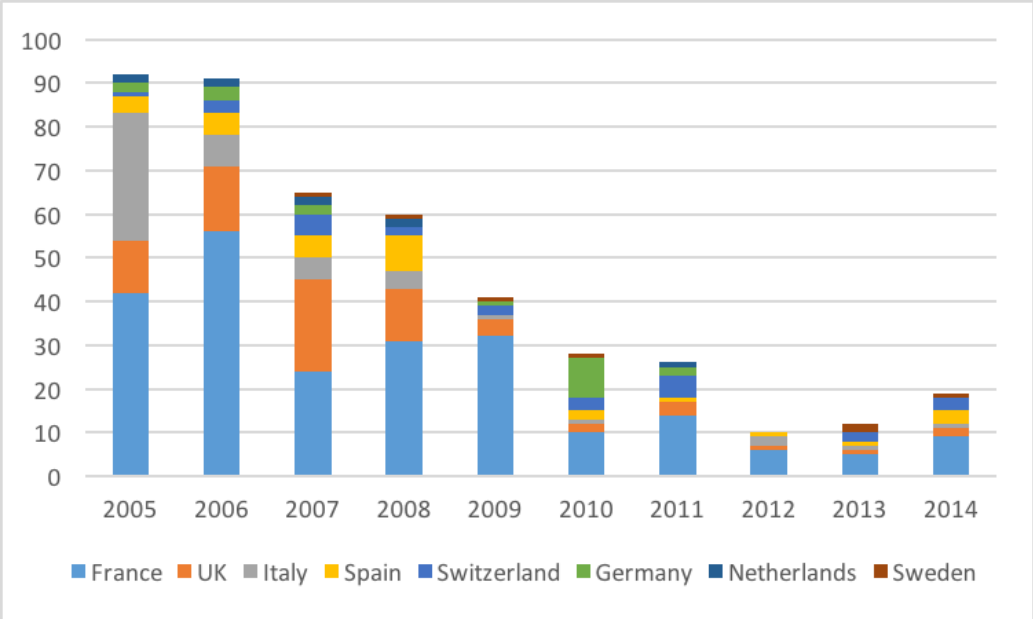


Figure 2.2: Openings of foreign bank units by home country and year.

We measure the riskiness of a bank using two different measures: a market-based variable, namely the Credit Default Swap price (CDS hereafter) and a book-based variable, namely the Loan-loss provisions to total loans.

GGL measure bank's riskiness using the volatility of banks' assets. However, we consider volatility captures only part of banks' risk and does not consider the systemic risk interrelation. CDS spread is a standard risk measure in current literature. Altavilla, Pagano and Simonelli (2016) analyse the impact of sovereign exposure in banks' risk. The latter is captured by 5-year CDS. The fact that CDS are derivatives highly sensitive to the default probability make this metric a good indicator of bank riskiness (Radev, 2016). Additionally, Acharya, Pedersen and Philippon (2017) identified CDS as a systemic risk measure. Specifically, they argue that ex post realized systemic risk is captured by the increase in credit risk estimated from CDS of large financial institutions during the crisis.

Pederzoli and Toricelli (2015) argue that systemic risk metrics can be based on market data, such as CDS spreads.

On the other hand, loan-loss provisions ratio is used as a measure of bank riskiness as well. Podpiera and Ötker (2010) refer to the loan-loss provision ratio as a measure of the credit risk from a bank's portfolio generated within one year. The degree of loan-loss provisioning is also considered an important quantitative indicator for assessing the health of the banking sector (default risk) (ECB Monthly bulletin, March 2004).

The CDS price corresponds to the price of the insurance against the default of a debt issuer. This is an overall measure of bank's risk (both on the asset and the liability side) as priced by the market. The higher the CDS price, the higher the defaulting probability as perceived by the market. The advantages of using this measure are two. First, it captures several aspects of banks' risk. Second, the assessment of risk is done by the market, hence it is not biased by possible banks' manipulations. The disadvantage of this measure is that it might be subject to market exuberance, hence it tends to be more volatile than other book-value metrics. In our case this disadvantage is addressed by taking the average CDS price over the year. The loan-loss provisions to total loans correspond to the portion of the expected loan repayments that the banks set aside to cover losses in the eventuality of defaulting borrowers. Hence the second metric captures the riskiness of loans and, therefore, it is related to the solvency of lending firms. For a given level of total assets, a higher level of loan loss provision indicates a higher probability of loss on loans (less solvent borrowers). The advantage of using this second metrics is that it is immune from market exuberance. On the other side it is a narrower indicator as it captures only loan-portfolio related risk while a bank might invest in other risky assets and/or hold a risky liability structure.

In any case it seems at first glance that the two metrics are highly correlated (see Figure 2.3 below). We will however see below that the chosen metric might provide different answers when we examine regressions without bank-year fixed effects.

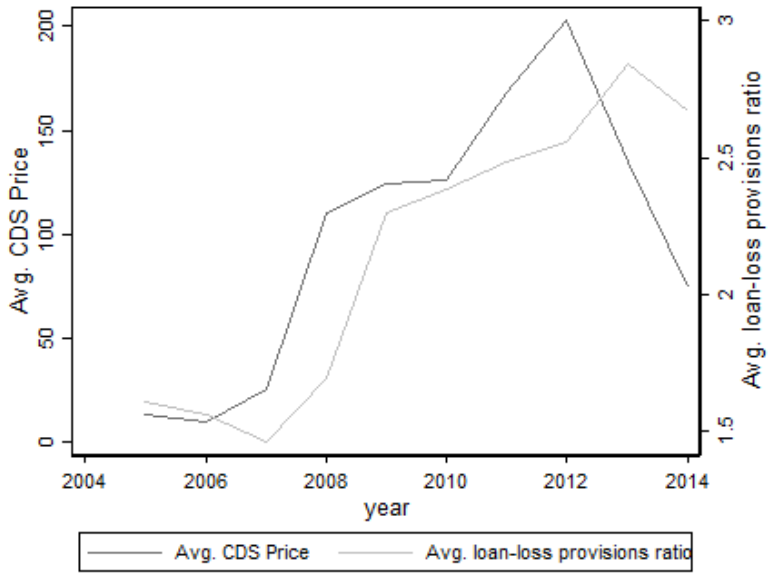


Figure 2.3: Average CDS Price in the sample

The correlation between average CDS price and loan loss-provision by bank is shown in table 2.1.

Table 2.1: CDS-LLP correlation by bank

Bank	Corr LLP-CDS
BPCE	-.531233
SOGE	-.4128902
BNPA	.1679065
BSCH	.3311116
RBOS	.3569053
BARC	.4880371
AGRI	.5030777
UBSW	.5120783
NDEA	.5270168
INGB	.66092
DEUT	.6860988
HSBC	.7208892
UNCR	.7349073
CRES	.7605467
SCBL	.7904543

In Figure 2.4 we display the yearly average CDS price of all banks, the minimum and the maximum CDS price in our sample (left axis) and the total number of openings (right axis). The latter is a proxy for the magnitude of bank's geographic expansion. The effect of the financial crisis on CDS prices is observed from 2008 and it is correlated with a drop in the total number of openings of G-SIB banks in Europe.

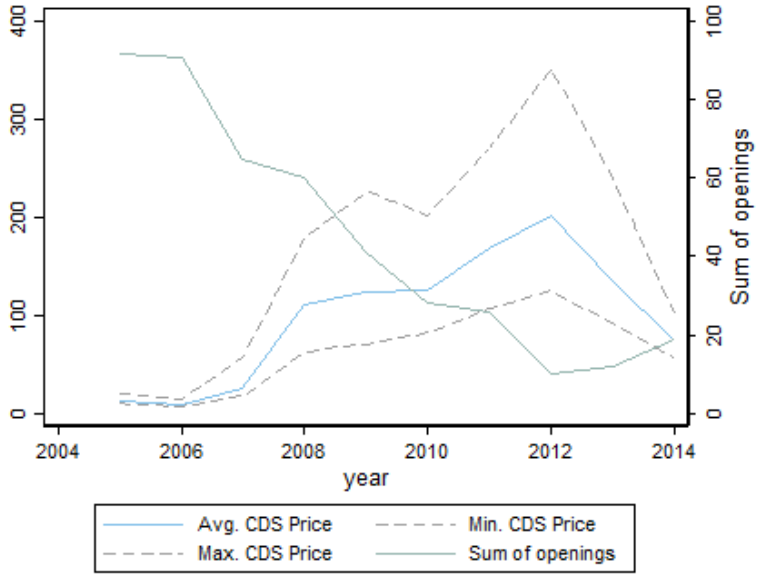


Figure 2.4: Average CDS Price in the sample

The dataset also contains a set of financial indicators. Bank's size proxied by total assets, overall financial health and strength (proxied alternatively by the loan-to-loss provisions to the loans' ratio, by the capital ratio and by the Tier1-to-assets ratio) and banks' profitability (proxied by the Return on Assets) are extracted from Bankscope and used as controls.

Next, following LLX(33) and GLL(29), we measure diversification by computing the following indicators of income diversity and asset diversity:

$$Income\ Diversity = 1 - \frac{|Interest\ inc. - noninterest\ inc.|}{Total\ income}$$

and

$$Asset\ Diversity = 1 - \frac{|Loans - Other\ assets|}{Total\ assets}.$$

At last, the degree of competition in banking is measured at country level by one minus the Herfindhal index and the Hirschman Index, both variables are provided by the European Central Bank. To gauge a country's degree of regulation, we include the *Macroprudential Index (MPI)* taken from Cerutti, Claessens and Laeven(11)). Finally, to control for particular links between countries, dyadic gravity variables are considered.

Table 2.2 summarizes some basic statistics[7] regarding the variables that will be used in our analysis[8].

The standard deviation of the logarithm of banks' annual average CDS prices indicates that this variable takes different values among banks and over time, as expected.
High volatility is observed on the loan-loss provisions ratio as well.
The average number of openings per year for a bank is almost 3, but it is very dependant on the bank and the year as shown by the very high standard deviation and can vary from 0 openings to 29 in a single year.
Other specific measures such as income diversity, asset diversity, capital ratio and asset ratio exhibit also relatively high standard deviations, showing that we have relatively different banks in our sample.

[7]The minimum for income diversity is -4.42. Even if it is expected that this value lies between 0 and 1 in absolute value according to the formula for income diversity, the negative value for income diversity is due to the fact that non-interest income takes negative values in our sample. Specifically, this affects the data for UBS in 2008 (the income diversity value for this year is the observed minimum -4.42).

[8]Issues of stationarity are not discussed at this stage, given that our panel is very short, i.e. we have a small N and T (so the asymptotic results may not be reliable). In such small panel, stationarity tests have a small power and systematically tend to not reject the unit root hypothesis. With a small temporal dimension, the potential non-stationarity is unlikely to be a problem. Besides, the problem of stationarity is very unlikely to emerge since we use year fixed effects and bank fixed effects.

Table 2.2: Descriptive statistics of the main variables used in the empirical analysis.

Variable	Obs.	Mean	Std. Dev.	Min	Max
ln(cds)	140	4.148594	1.077247	1.927346	5.861315
Loan loss provisions to total loans	138	2.118043	1.724864	.2	9.63
Expansion	150	2.96	4.768296	0	29
ln(tot assets)	150	13.97037	.4758832	12.27884	14.80599
ROA bank	139	.3582014	.4461254	-1.61	1.14
Income diversity	139	.7029369	.4935113	-4.418854	.9933677
Asset diversity	139	.7176454	.1773021	.2339715	.9990997
Capital ratio	130	14.33462	3.395106	8.87	25.6
Tier1/Assets	131	46.92355	14.7732	12.81485	81.11484
Deposits/Assets	139	665.2518	149.5965	331.7435	1257.695

2.3 Foreign Expansion and Riskiness

In this section we explore the impact of banks' expansion abroad upon their riskiness. As previously discussed the potential endogeneity problem is dealt through an instrumental variable approach. Our instrument will be given by the estimated gravity between the country of origin and the destination country. The channels through which this impact materializes will be investigated in the next section.

2.3.1 Endogeneity and Empirical Strategy

To assess the impact of foreign expansion on riskiness, we consider bank k headquartered in country i expanding to country $j \neq i$ in year t. We estimate the following regression by OLS:

$$Riskiness_{kt} = \alpha + \beta_1 \cdot Expansion_{kt} + Z_{kt} \cdot \Gamma + \mu_k + \mu_t + \epsilon_{kt}, \tag{2.1}$$

where $Riskiness_{kt}$ is measured by the (Naperian) logarithm of the bank's average CDS price over year t, $Expansion_{kt}$ corresponds to its total number of openings and Z_{kt} is a set of control variables. We include time fixed effects (μ_t) to control for a specific trend in the data (the crisis of 2007 and its consequences hereafter) and bank fixed effects (μ_k) to account for the constant bank-specific factors that influence the riskiness of the bank. In this specification, the results have thus to be interpreted as materialising *within* bank.

The OLS estimate could, however, be biased if the bank's expansion decision were related to its risk conditions, especially so if the bank expects that its geographic expansion could have an impact on its risk-taking. If the bank believes that expansion could reduce

its riskiness, then its decision to go abroad could be driven by an increase in riskiness. In this case the OLS estimate of β_1 would be biased upwards. To deal with this potential endogeneity bias, we use an IV strategy similar to GLL(29) and LLX(33). The observed geographic expansion of the bank will be instrumented with the one predicted by a gravity equation. This method is akin to the one used in Frankel and Romer (26) who study the impact of international trade on countries' economic performance by instrumenting the observed bilateral trade flows (which arguably depend on countries' economic performance) with the equivalent predicted by geographic variables and fixed country characteristics. To the extent that our gravity estimation does not include variables correlated with the risk-taking behavior of the bank, the instrument is correlated with actual openings but not with banks' risk.

Operationally, we proceed as follows. At first (stage zero), we compute the predicted bilateral openings from a gravity regression of actual openings in country j by bank k headquartered in country i at date t:

$$Openings_{kjt} = X_{kjt} \cdot \beta + \nu_{jt} + \nu_k + \varepsilon_{kjt} \tag{2.2}$$

where X_{kjt} are the standard dyadic gravity variables (e.g. distance, common border, common language, etc.), ν_{jt} is a country-time fixed effect and ν_k is a bank fixed effect. Second, we aggregate the bilateral predicted counts across destinations to obtain a prediction of the total number of openings of bank k at date t:

$$Expansion_{kt}^{pred} = \sum_{j \neq i} \left(X_{kjt} \cdot \widehat{\beta} + \widehat{\nu}_{jt} + \widehat{\nu}_k \right). \tag{2.3}$$

It is worth noting that we include fixed effects that are not correlated with changes in the bank's risk. If we had followed the structural trade gravity framework, we would have to include bank-time fixed effects and hosting-country-time fixed effects. However this inclusion would have made our instrument correlated with changes in the bank's risk. We also repeat the procedure above using no fixed effects at all. This will serve as an alternative instrument.

Equation (2.2) is estimated using Poisson Pseudo Maximum Likelihood (PPML hereafter). The OLS estimator is not appropriate for count data like ours for three reasons. First, assumptions on normality are not likely to be fulfilled by count models. Second, the OLS estimator could generate negative predictions in the case of count data. Third, the OLS estimator is less apt than a Poisson estimator to deal with the large number of zeros in our count data. Poisson regressions are, therefore, much better suited for our case. In addition note that we use the PPML estimator since this is robust to distribution mis-specification

(Cameron and Triverdi(10), Santos-Silva and Tenreyro(37)). As it is standard in gravity models, we cluster standards errors at the country-pair level (Head and Mayer (30) and Yotov et al. 2016).

Equation (2.2) does not account for the fact that different openings may have different size and thus different relevance for the bank. To take this into account, we also construct a weighted measure of predicted expansion, using the share of openings of all other banks in country j to proxy for the relative size of bank i's openings in that country. In this way the weights can be considered exogenous to bank k's choices. Specifically, we define the weight ω_{kjt} attached to $Openings_{kjt}$ as follows:

$$\omega_{kjt} = 1 + \frac{\sum_{h \neq k} openings_{hjt} \cdot total\_assets_{hjt}}{\sum_j \sum_{h \neq k} openings_{hjt} \cdot total\_assets_{hjt}} \in [1, 2]. \tag{2.4}$$

In our data ω_{kjt} ranges between 1 and 1.32, taking low (high) values for countries of little (great) importance for banks' total assets – which are likely to host small (large) openings. The countries with low values are Albania, Bosnia, Cyprus, Estonia or Iceland, the ones with high values are Germany, Luxembourg, Poland or Spain. The weighted predicted expansion can then be written as:

$$Expansion_{kt}^{wpred} = \sum_{j \neq i} \omega_{kjt} \left(X_{kjt} \cdot \widehat{\beta} + \widehat{\nu}_{jt} + \widehat{\nu}_k \right) \tag{2.5}$$

We will estimate two stage least squares for both the weighted and the unweighted expansion equation. Our two-stage approach consists of the following procedure. In the first stage we estimate the regression of actual openings on predicted ones. We will then use this estimate to instrument openings in the second stage when we will regress riskiness on expansion.

2.3.2 First Stage: Gravity Prediction

The results of the gravity estimation are reported in table 2.3. We employ three different specifications for the gravity equation. The first is more in line with standard estimations conducted in the gravity literature: we therefore compare the results of this model with previous ones in the literature. The second and the third specifications are however better suited to provide us with an instrument as we explain below. In all three specifications the regressors include log(distance), contiguity, the official common language, the common belonging to the European Union or to the Eurozone and the difference in the legal systems. The three specifications differ primarily in the full or partial inclusion of the fixed effects.

We display in column (1) the results of the gravity model estimated with the full set of fixed effects. This specification, which is more in line with the ones employed in the traditional gravity literature, allows us to account for multilateral resistance terms (see Head and Mayer (30)). Multilateral resistance between two countries is the average barrier of the two regions with all their partners (see Van Wincoop and Anderson (**?**)). Considering the opening of a new bank branch in Europe, multilateral resistance corresponds to the average barriers to the banking investment with all other countries. For given bilateral barriers between two countries, i and j, higher barriers between i and other countries is likely to raise the number of new branches that a bank headquartered in country j opens in country i. We do not use however the predicted gravity value from this specification as our instrument. Indeed the presence of the bank-year fixed effects, a factor which is likely to be correlated with bank risk, would make the predicted gravity correlated with the dependent variable of our second stage. Hence the endogeneity problem would remain. Nevertheless it is instructive to discuss the results of this specification. First, the estimation delivers an elasticity of openings to distance of -0.662. The magnitude of this coefficient is discussed and compared with other banking gravity papers in Appendix C. Second and surprisingly, sharing a common language has a negative impact on bilateral banks openings. This could be due to the fact that official common language is collinear to the distance or the continuity in our sample. Third, being members of the European Union and the Eurozone does not have any impact in this specification. At last and as expected, having a different legal system in the host country compared to the country of origin has an important negative impact on banks openings.

In column (2), we estimate the same gravity equation but without any fixed effects. The estimated gravity from this model is one of our candidate instruments. The elasticity to distance is a bit lower in this case. Contiguity or the common belonging to the European Union or the Eurozone now have a positive and significant impact on banking gravity.

Finally, we estimate the specification in column (3), which includes bank and host-year fixed effects. In our view this specification deliver the best instrument, albeit we also employ the predicted value implied by the second specification. Results for this case are very close to the ones with the full set of fixed effects. When the instrument is estimated with this set of fixed effects, it is generated using out-of-sample prediction, ignoring that observations that are always 0 for the couplet (source country, host country) are dropped from the estimation.

Table 2.3: Banking gravity

	PPML (1)	PPML (2)	PPML (3)
ln(distance)	-0.662***	-0.553***	-0.651***
	(0.170)	(0.149)	(0.173)
Contiguity	0.0367	0.910***	0.104
	(0.219)	(0.266)	(0.212)
Off. common lang.	-0.719*	-0.921***	-0.663*
	(0.391)	(0.271)	(0.360)
EU_{ij}	0.690	0.984*	0.932*
	(0.524)	(0.592)	(0.512)
$Euro_{ij}$	-0.382	0.714***	-0.294
	(0.277)	(0.201)	(0.276)
Diff. legal syst.	-0.629**	-0.123	-0.694**
	(0.310)	(0.171)	(0.275)
Observations	2,109	5,550	2,896
R-squared	0.296	0.026	0.193
Fixed effects	Bank-year host-year	No FE	Bank et host-year

Robust standard errors clustered at the bank-hosting-country level in parentheses
*** p<0.01, ** p<0.05, * p<0.1

2.3.3 Causal Effects of Expansion on Riskiness

We now test the impact of expansion on riskiness. We do so by comparing the OLS estimates with the two-stage using gravity prediction as an instrument. We also compare specifications with different assumptions on the fixed effects. Controls used in the various specifications include expansion, log(total assets), return on assets, income diversity, asset diversity, the ratios for the headquartered bank of capital, Tier 1 over assets and deposits over assets.

In Table 2.4 columns 1, 4 and 7 show OLS estimates, while the rest show 2SLS. All regressions in this Table do not include bank fixed effects. This allows us to provide a '*between*' interpretation of the results, as it reveals if high levels of openings are related to low bank riskiness. We keep time fixed effects to account for the common trend of CDS prices. Column (1) shows the OLS estimates by controlling only for the size of the bank in terms of assets. This baseline specification delivers a negative and significant correlation between expansion and riskiness. In other words, banks tend to expand abroad when they are less risky. We dissect the negative relation by dividing our CDS variable in quantiles. When we do so we observe a statistically significant difference in terms of openings among the quartiles. In the first quartile of CDS prices banks open on average 6.2 affiliates per year; banks in the second quartile open on average 3.7 affiliates per year; the remaining banks open on average 1.6 affiliates per year. This difference could be explained in our case by the economic crisis of 2007-2008 that increased banks' CDS price and reduced foreign expansion of banks. At last notice that the negative correlation holds when we control for bank-specific variables in columns (4) and (7).
The other columns of Table 2.4 account for the potential endogeneity bias using the instrument computed in the first stage. We must note that the instrument generated using a gravity model with fixed effects (column 3 of Table2.3) performs better (in terms of F-stat) than the one generated without fixed effects (column 2 of Table2.3), albeit both exhibit reasonable F-stats. Columns 2, 5 and 8 show results using the instrument estimated without fixed effects, while columns 3, 6 and 9 shows results using the instrument estimated with fixed effects. Overall first-stage-regression coefficients have the sign and the magnitude expected. For both instruments, there is a positive and almost unitary correlation between predicted and actual expansion. In columns (2) and (3) we do not find any causal effect from expansion to riskiness: banks that expand more are on average less risky but do not become less risky because they expand more. Controlling for more bank-level characteristics, we find in column (6) a negative and significant coefficient, but this effect disappears when we change some control variables. All in all, the 'between' causal effect of expansion

on riskiness is not very robust.[9]

In Table 2.5, we run exactly the same regression on the weighted expansion measure. Results are very similar to the ones of Table 2.4, thereby confirming results also when we account for the size of the openings.

In Table 2.6, we add bank-year fixed effects to our regressions in order to look at the results '*within*' the bank. These estimations are informative on the causal effect from geographic expansion to the riskiness of each bank. Once again in columns (1), (4) and (7), we show OLS estimates with different sets of controls **and instruments**. In all three cases, we find again a robust negative correlation between expansion and riskiness. A bank expands abroad when it is less risky. There is also a positive, albeit not robust, effect of bank size on riskiness. Turning to the 2SLS estimation (columns 2, 3, 5, 6, 8,9), we find a negative coefficient on expansion which is robust to different sets of controls. The geographic expansion of a bank tends to decrease its riskiness. The coefficient is larger (in absolute terms) than the one in the OLS estimation. In column (2) each new opening abroad decreases the price of the CDS by 3.5% (the other 2SLS columns can be interpreted analogously). If we consider the median number of openings by year, that is 1, expansion abroad reduces the CDS price by 3.5%. For banks that open 4 affiliates in a given year (corresponding to the fourth quartile), these openings contributes to a decrease of the CDS price by 14%. The results confirm our hypothesis that the OLS estimates are upwardly biased.
Several other results stand out. In column (2), the first-stage regression has a surprisingly large coefficient of 28.66. This is due to the use of fixed effects in the first stage compared with the 'zero stage' where we do not use fixed effects to generate the prediction. The results of columns (2) to (6) show a positive effect of size on riskiness, probably due to the fact that bigger banks were more exposed during the crisis. Larger income diversity (between interest and non-interest income) has a negative effect on the riskiness of the bank (columns (4) to (9)). Higher ratios of Tier1 capital to total assets and of deposits to total assets are consistently associated with lower riskiness of the banks as measured using CDS prices. This message is reasonable: well capitalized banks are priced better in terms of risk by the market. Both instruments (the one estimated with fixed effects and the one estimated without fixed effects) give similar and consistent results associated with a large F-stat.

In Table 2.7, we run the same estimation on the weighted expansion measure. Results are very similar to the ones of Table 2.4, confirming that they hold even when accounting for the size of the openings.

[9]It could be explained by the fact that when a bank is more risky (when the price of its CDS is higher), the probability of default is higher and expansion is likely to be limited. In our case, banks became more risky at the moment of the economic crisis of 2008 (see Figure 2.4), and they expanded less during this period.

Table 2.4: Dependent variable: CDS price. OLS and 2SLS regressions without bank-fixed effects. Unweighted metric of expansion.

	(1) OLS	(2) 2SLS	(3) 2SLS	(4) OLS	(5) 2SLS	(6) 2SLS	(7) OLS	(8) 2SLS	(9) 2SLS
First Stage									
Pred. expansion		1.246***	0.770***		2.052***	0.802***		1.754***	0.760***
		(0.419)	(0.150)		(0.583)	(0.167)		(0.461)	(0.151)
Second Stage									
Expansion	-0.0101***	0.0114	-0.0103	-0.0103***	-0.0198	-0.0206**	-0.00793**	0.000222	-0.0112
	(0.00373)	(0.0192)	(0.00746)	(0.00388)	(0.0131)	(0.00889)	(0.00372)	(0.0128)	(0.00786)
ln(Tot Assets)	-0.0239	-0.0583	-0.0235	-0.0765	-0.0575	-0.0558	-0.0745	-0.0884*	-0.0690
	(0.0422)	(0.0522)	(0.0425)	(0.0484)	(0.0557)	(0.0509)	(0.0464)	(0.0492)	(0.0461)
ROA				-0.0758	-0.0809	-0.0813	-0.109	-0.103	-0.112
				(0.0801)	(0.0757)	(0.0762)	(0.0812)	(0.0754)	(0.0761)
Income diversity				-0.0725	-0.0642	-0.0635	-0.0516	-0.0617	-0.0476
				(0.0481)	(0.0440)	(0.0428)	(0.0435)	(0.0452)	(0.0394)
Asset diversity				-0.262**	-0.236*	-0.234*	-0.270*	-0.314*	-0.252
				(0.125)	(0.125)	(0.123)	(0.158)	(0.162)	(0.155)
ratio_k				-0.0189**	-0.0210***	-0.0211***			
				(0.00737)	(0.00752)	(0.00696)			
Tier1/Asset							0.00205	0.00289	0.00171
							(0.00283)	(0.00287)	(0.00278)
Deposits/Asset							-0.000105	-0.000151	-8.66e-05
							(0.000196)	(0.000188)	(0.000193)
Observations	150	150	150	140	140	140	141	141	141
R-squared	0.949	0.942	0.949	0.953	0.952	0.952	0.952	0.951	0.952
Fixed effects	year	year	year	year	year	year	year	year	year
Instrument		Pred no fe	Pred k jt		Pred no fe	Pred k jt		Pred no fe	Pred k jt
F-Test 1st		8.870	26.39		12.37	22.95		14.47	25.35

Robust standard errors clustered at the bank-hosting-country level in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.5: Dependent variable: CDS price. OLS and 2SLS regressions without bank-fixed effects. Weighted metric for expansion.

	(1) OLS	(2) 2SLS	(3) 2SLS	(4) OLS	(5) 2SLS	(6) 2SLS	(7) OLS	(8) 2SLS	(9) 2SLS
Expansion w	-0.00963***	0.0111	-0.00998	-0.00983***	-0.0191	-0.0199**	-0.00752**	0.000302	-0.0108
	(0.00361)	(0.0185)	(0.00717)	(0.00375)	(0.0128)	(0.00852)	(0.00361)	(0.0123)	(0.00754)
ln(Tot Assets)	-0.0239	-0.0586	-0.0233	-0.0767	-0.0573	-0.0556	-0.0746	-0.0886*	-0.0688
	(0.0422)	(0.0524)	(0.0425)	(0.0484)	(0.0559)	(0.0508)	(0.0464)	(0.0493)	(0.0461)
ROA				-0.0754	-0.0803	-0.0807	-0.109	-0.103	-0.111
				(0.0801)	(0.0757)	(0.0763)	(0.0812)	(0.0754)	(0.0761)
Income diversity				-0.0726	-0.0641	-0.0633	-0.0517	-0.0618	-0.0475
				(0.0482)	(0.0441)	(0.0429)	(0.0436)	(0.0452)	(0.0394)
Asset diversity				-0.263**	-0.237*	-0.235*	-0.271*	-0.314*	-0.253
				(0.125)	(0.125)	(0.123)	(0.158)	(0.161)	(0.155)
ratio_k				-0.0189**	-0.0210***	-0.0212***			
				(0.00737)	(0.00755)	(0.00697)			
Tier1/Asset							0.00207	0.00290	0.00172
							(0.00283)	(0.00286)	(0.00277)
Deposits/Asset							-0.000106	-0.000151	-8.69e-05
							(0.000196)	(0.000188)	(0.000193)
Observations	150	150	150	140	140	140	141	141	141
R-squared	0.949	0.942	0.949	0.953	0.952	0.952	0.952	0.951	0.952
Fixed effects	year	year	year	bank year	year	year	year	year	year
Instrument		Pred no fe	Pred k jt		Pred no fe	Pred k jt		Pred no fe	Pred k jt
F-Test 1st		8.944	26.84		12.39	23.41		14.52	25.84

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.6: Dependent variable: CDS price. OLS and 2SLS regressions with bank-fixed effects. Weighted metric of expansion.

	(1) OLS	(2) 2SLS	(3) 2SLS	(4) OLS	(5) 2SLS	(6) 2SLS	(7) OLS	(8) 2SLS	(9) 2SLS
First Stage									
Pred expansion		28.66***	1.622***		33.06***	1.591***		32.66***	1.763***
		(8.455)	(0.451)		(10.22)	(0.440)		(8.901)	(0.406)
Second Stage									
Expansion	-0.0131***	-0.0351**	-0.0351***	-0.0109***	-0.0291**	-0.0362***	-0.0121***	-0.0454***	-0.0421***
	(0.00389)	(0.0153)	(0.0115)	(0.00404)	(0.0141)	(0.0119)	(0.00327)	(0.0142)	(0.0107)
ln(Tot Assets)	0.186*	0.214**	0.214**	0.162	0.203*	0.218**	-0.0356	-0.0490	-0.0477
	(0.102)	(0.100)	(0.0972)	(0.103)	(0.104)	(0.106)	(0.124)	(0.119)	(0.117)
ROA				-0.0472	-0.0461	-0.0457	-0.00391	0.00595	0.00496
				(0.0846)	(0.0809)	(0.0839)	(0.0720)	(0.0721)	(0.0705)
Income diversity				-0.0854**	-0.0795**	-0.0772**	-0.116***	-0.111***	-0.111***
				(0.0397)	(0.0358)	(0.0364)	(0.0354)	(0.0339)	(0.0327)
Asset diversity				-0.164	0.0551	0.140	0.239	0.681*	0.637*
				(0.283)	(0.317)	(0.311)	(0.309)	(0.364)	(0.354)
ratio_k				-0.0125	-0.00911	-0.00781			
				(0.00996)	(0.00925)	(0.00994)			
Tier1/Asset							-0.0109**	-0.0151***	-0.0147***
							(0.00472)	(0.00489)	(0.00477)
Deposits/Asset							-0.000588***	-0.000476	-0.000487*
							(0.000181)	(0.000289)	(0.000270)
Observations	150	150	150	140	140	140	141	141	141
R-squared	0.969	0.963	0.963	0.970	0.966	0.962	0.974	0.961	0.964
Fixed effects	bank year	bank year	bank year	bank year	bank year	bank year	bank year	bank year	bank year
Instrument		Pred no fe	Pred k jt		Pred no fe	Pred k jt		Pred no fe	Pred k jt
F-Test 1st		11.49	12.93		10.47	13.09		13.47	18.82

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.7: Dependent variable: CDS price. OLS and 2SLS regressions with bank-fixed effects. Weighted metric of expansion.

	(1) OLS	(2) 2SLS	(3) 2SLS	(4) OLS	(5) 2SLS	(6) 2SLS	(7) OLS	(8) 2SLS	(9) 2SLS
Expansion w	-0.0126***	-0.0340**	-0.0339***	-0.0104***	-0.0282**	-0.0349***	-0.0115***	-0.0441***	-0.0406***
	(0.00374)	(0.0148)	(0.0111)	(0.00389)	(0.0136)	(0.0115)	(0.00314)	(0.0138)	(0.0103)
ln(Tot Assets)	0.185*	0.213**	0.213**	0.162	0.202*	0.217**	-0.0355	-0.0492	-0.0477
	(0.102)	(0.100)	(0.0972)	(0.103)	(0.104)	(0.106)	(0.124)	(0.119)	(0.117)
ROA				-0.0471	-0.0458	-0.0453	-0.00385	0.00635	0.00526
				(0.0845)	(0.0808)	(0.0839)	(0.0720)	(0.0721)	(0.0705)
Income diversity				-0.0852**	-0.0789**	-0.0765**	-0.116***	-0.110***	-0.110***
				(0.0398)	(0.0359)	(0.0364)	(0.0354)	(0.0340)	(0.0328)
Asset diversity				-0.167	0.0514	0.134	0.236	0.676*	0.629*
				(0.283)	(0.317)	(0.311)	(0.309)	(0.365)	(0.354)
ratio_k				-0.0125	-0.00920	-0.00794			
				(0.00997)	(0.00927)	(0.00996)			
Tier1/Asset							-0.0108**	-0.0150***	-0.0146***
							(0.00472)	(0.00489)	(0.00478)
Deposits/Asset							-0.000589***	-0.000479*	-0.000491*
							(0.000181)	(0.000290)	(0.000269)
Observations	150	150	150	140	140	140	141	141	141
R-squared	0.969	0.963	0.963	0.970	0.966	0.962	0.974	0.961	0.963
Fixed effects	bank year	bank year	bank year	bank year	bank year	bank year	bank year	bank year	bank year
Instrument		Pred no fe	Pred k jt		Pred no fe	Pred k jt		Pred no fe	Pred k jt
F-Test 1st		11.59	13.43		10.52	13.53		13.52	19.47

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Next we test the robustness of our results by changing the risk metric. In the following tables, we move from a market-based measure of bank risk to a book-based measure, namely the loan-loss provisions to total loans. The first metric captures overall bank risk (both on the asset and the liability side) as measured by the market. The second metric captures more banks' asset risk. Both measures have similar trends, especially since the financial crisis impacted the two in a similar way (see figure 2.5). In Table 2.8, we run the estimation without any fixed effects. OLS regressions in columns (1), (4) and (7) illustrate that banks with higher loan loss provisions (hence riskier ones) expand more. This effect is opposite to the one found in Table 2.4. Accordingly, there is a selection in the expansion of banks that have low CDS price, but high loan loss provisions to total loans. This could be explained by the fact that the correlation between CDS and LLP is different across banks as illustrated by 2.5.

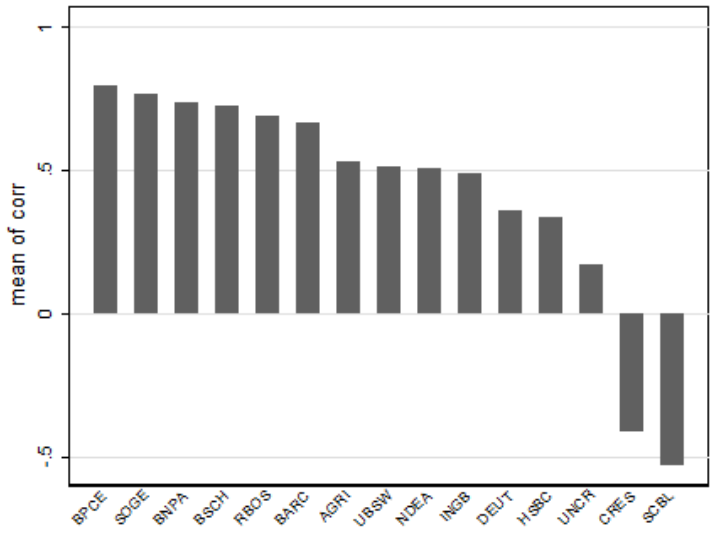


Figure 2.5: Correlation between LLP and CDS

Turning to the 2SLS estimations, we find a systematically strong positive impact of expansion on the loan loss provision ratio, which seems to confirm the selection effect just highlighted. Several other results stand out. We find a positive effect of the return on assets on the riskiness of the bank. This is intuitive and captures a search for yield effect: banks who invest in higher yield assets also exhibit a riskier asset portfolio. We also find a positive effect of the capital to asset ratio and the Tier1 to asset ratio on riskiness. This is well explained by the Basel II pro-cyclicality of the regulatory ratios. As asset risk raises the regulator imposes to the bank to increase the regulatory ratios. This does not contradict the result highlighted in Table2.6, namely that well capitalized banks are considered sound in terms of risk the market

Table 2.8: Dependent variable: loan loss provisions over loans. OLS and 2SLS regressions without bank-fixed effects. Unweighted metric of expansion.

	(1) OLS	(2) 2SLS	(3) 2SLS	(4) OLS	(5) 2SLS	(6) 2SLS	(7) OLS	(8) 2SLS	(9) 2SLS
First Stage									
Pred expansion		1.144***	0.694***		1.734***	0.737***		1.524***	0.694***
		(0.381)	(0.132)		(0.514)	(0.152)		(0.413)	(0.136)
Second Stage									
Expansion	0.0987***	0.608***	0.457***	0.0499**	0.311***	0.307***	0.102***	0.604***	0.403***
	(0.0186)	(0.209)	(0.105)	(0.0198)	(0.114)	(0.0795)	(0.0203)	(0.171)	(0.0933)
ln(Tot Assets)	0.0424	-0.823	-0.566*	-0.0328	-0.568	-0.559*	0.0462	-0.837*	-0.484
	(0.228)	(0.513)	(0.340)	(0.245)	(0.360)	(0.325)	(0.213)	(0.462)	(0.301)
ROA				-1.433***	-1.443***	-1.443***	-1.935***	-1.898***	-1.913***
				(0.489)	(0.444)	(0.444)	(0.512)	(0.440)	(0.429)
Income diversity				0.373*	0.203	0.205	0.608**	0.118	0.313
				(0.194)	(0.190)	(0.185)	(0.234)	(0.233)	(0.197)
Asset diversity				0.909	0.417	0.425	-0.285	-2.406*	-1.559
				(0.564)	(0.754)	(0.722)	(0.737)	(1.388)	(0.991)
ratio_k				-0.261***	-0.206***	-0.207***			
				(0.0412)	(0.0438)	(0.0420)			
Tier1/Asset							0.0623***	0.117***	0.0954***
							(0.0145)	(0.0302)	(0.0212)
Deposits/Asset							-0.00269***	-0.00605**	-0.00471***
							(0.000841)	(0.00262)	(0.00172)
Observations	148	148	148	139	139	139	140	140	140
R-squared	0.139	-1.196	-0.521	0.451	0.111	0.121	0.370	-0.861	-0.074
Fixed effects	year	year	year	year	year	year	year	year	year
Instrument		Pred no fe	Pred k jt		Pred no fe	Pred k jt		Pred no fe	Pred k jt
F-Test 1st		9.030	27.70		11.38	23.46		13.61	26.13

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

In Table 2.9, we run the same regressions on the weighted expansion measure. Results are very similar to the ones of Table 2.9, confirming robustness when we account for the size of the openings.

In Table 2.10 we re-estimate the above specifications, whose dependent variable is given by the loan loss provisions over loans, but add bank-year effects. Results have, therefore, a 'within' interpretation. In the three different OLS specifications (as usual columns 1, 4 and 7), the coefficient for expansion is always insignificantly different from zero. Accordingly, expansion seems to have no effect on the bank's asset risk. As for 2SLS estimation, we have again a very high coefficient on our first stage for the first instrument that is likely due to the inclusion of fixed effects. F-stats are again relatively high, and the first stage for our second instrument (the one estimated through a gravity with fixed effects) is as expected. In all cases, we find that expansion has an effect on riskiness. When a bank expands abroad, its riskiness measured by the loan-loss ratio decreases. With both instruments, coefficients are larger in absolute value than the OLS coefficients but, when the instrument is generated using a gravity model without fixed effects, the coefficient is twice as large. To quantify the impact of expansion on the bank's risk consider column 9. This is indeed our preferred specification since in this case the instrument is extracted from a gravity equation with fixed effects. In this case the median number of openings in a year (1 opening) decreases the bank's loan-loss provisions ratio by 0.08 percentage points. For 4 openings (corresponding to the fourth quartile of openings), geographic expansion reduces the loan loss provisions to asset ratio by 0.32 percentage points (the average ratio being 2.16). These findings suggests that expansion has an effect on the quality/risk of loans granted by the banks since the provisions for loan loss decreases after the expansion. The results are robust to different set of fixed effects.

In Table 2.11, we run the same estimation as before but on the weighted expansion measure. Results are very similar to the ones of Table 2.11. Our results hold also when we account for opening size.

Table 2.9: Dependent variable: loan loss provisions over loans. OLS and 2SLS regressions without bank-fixed effects. Weighted metric of expansion.

	(1) OLS	(2) 2SLS	(3) 2SLS	(4) OLS	(5) 2SLS	(6) 2SLS	(7) OLS	(8) 2SLS	(9) 2SLS
Expansion w	0.0953***	0.586***	0.436***	0.0485**	0.300***	0.292***	0.0980***	0.583***	0.384***
	(0.0178)	(0.201)	(0.100)	(0.0191)	(0.110)	(0.0756)	(0.0195)	(0.165)	(0.0891)
ln(Tot Assets)	0.0415	-0.826	-0.561*	-0.0340	-0.572	-0.553*	0.0449	-0.845*	-0.481
	(0.227)	(0.512)	(0.338)	(0.245)	(0.360)	(0.322)	(0.213)	(0.463)	(0.298)
ROA				-1.434***	-1.450***	-1.450***	-1.937***	-1.909***	-1.921***
				(0.489)	(0.444)	(0.444)	(0.512)	(0.443)	(0.430)
Income diversity				0.372*	0.199	0.205	0.607**	0.112	0.315
				(0.194)	(0.190)	(0.185)	(0.234)	(0.233)	(0.197)
Asset diversity				0.910	0.427	0.443	-0.277	-2.357*	-1.505
				(0.563)	(0.752)	(0.715)	(0.736)	(1.387)	(0.982)
ratio_k				-0.260***	-0.206***	-0.207***			
				(0.0412)	(0.0439)	(0.0419)			
Tier1/Asset							0.0622***	0.116***	0.0942***
							(0.0144)	(0.0300)	(0.0209)
Deposits/Asset							-0.00268***	-0.00602**	-0.00465***
							(0.000840)	(0.00261)	(0.00170)
Observations	148	148	148	139	139	139	140	140	140
R-squared	0.139	-1.192	-0.504	0.451	0.110	0.133	0.370	-0.864	-0.060
Fixed effects	year	year	year	bank year	year	year	year	year	year
Instrument		Pred no fe	Pred k jt		Pred no fe	Pred k jt		Pred no fe	Pred k jt
F-Test 1st		9.072	27.94		11.33	23.80		13.59	26.43

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.10: Dependent variable: loan loss provisions over loans. OLS and 2SLS regressions with bank-fixed effects. Unweighted metric of expansion.

	(1) OLS	(2) 2SLS	(3) 2SLS	(4) OLS	(5) 2SLS	(6) 2SLS	(7) OLS	(8) 2SLS	(9) 2SLS
First Stage									
Pred expansion		23.47***	1.400***		26.74***	1.406***		27.92***	1.582***
		(6.877)	(0.392)		(8.530)	(0.397)		(7.794)	(0.366)
Second Stage									
Expansion	-0.0235	-0.232***	-0.0939*	-0.00435	-0.218***	-0.0783*	-0.00244	-0.252***	-0.109***
	(0.0166)	(0.0806)	(0.0491)	(0.0134)	(0.0811)	(0.0426)	(0.0139)	(0.0748)	(0.0412)
ln(Tot Assets)	-0.307	-0.222	-0.278	-0.717	-0.442	-0.622	-0.712	-0.796*	-0.748**
	(0.387)	(0.363)	(0.332)	(0.440)	(0.433)	(0.384)	(0.469)	(0.440)	(0.378)
ROA				-0.818*	-0.760*	-0.798**	-0.753	-0.681	-0.723*
				(0.415)	(0.459)	(0.392)	(0.457)	(0.483)	(0.428)
Income diversity				0.111	0.155	0.126	0.0659	0.105	0.0824
				(0.168)	(0.225)	(0.170)	(0.195)	(0.252)	(0.199)
Asset diversity				-2.258***	-0.316	-1.586*	-1.411	0.805	-0.468
				(0.854)	(1.338)	(0.884)	(0.983)	(1.338)	(0.972)
ratio_k				-0.0693	-0.0400	-0.0592			
				(0.0483)	(0.0560)	(0.0448)			
Tier1/Asset							-0.00403	-0.0259	-0.0133
							(0.0128)	(0.0190)	(0.0129)
Deposits/Asset							-0.00226***	-0.000894	-0.00168
							(0.000724)	(0.00189)	(0.00109)
Observations	148	148	148	139	139	139	140	140	140
R-squared	0.822	0.660	0.803	0.851	0.683	0.831	0.857	0.631	0.816
Fixed effects	bank year	bank year	bank year	bank year	bank year	bank year	bank year	bank year	bank year
Instrument		Pred no fe	Pred k jt		Pred no fe	Pred k jt		Pred no fe	Pred k jt
F-Test 1st		11.64	12.77		9.825	12.57		12.83	18.72

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.11: Dependent variable: loan loss provisions over loans. OLS and 2SLS regressions with bank-fixed effects. Weighted metric of expansion.

	(1) OLS	(2) 2SLS	(3) 2SLS	(4) OLS	(5) 2SLS	(6) 2SLS	(7) OLS	(8) 2SLS	(9) 2SLS
First Stage									
Pred expansion		23.47***	1.400***		26.74***	1.406***		27.92***	1.582***
		(6.877)	(0.392)		(8.530)	(0.397)		(7.794)	(0.366)
Second Stage									
Expansion	-0.0235	-0.232***	-0.0939*	-0.00435	-0.218***	-0.0783*	-0.00244	-0.252***	-0.109***
	(0.0166)	(0.0806)	(0.0491)	(0.0134)	(0.0811)	(0.0426)	(0.0139)	(0.0748)	(0.0412)
ln(Tot Assets)	-0.307	-0.222	-0.278	-0.717	-0.442	-0.622	-0.712	-0.796*	-0.748**
	(0.387)	(0.363)	(0.332)	(0.440)	(0.433)	(0.384)	(0.469)	(0.440)	(0.378)
ROA				-0.818*	-0.760*	-0.798**	-0.753	-0.681	-0.723*
				(0.415)	(0.459)	(0.392)	(0.457)	(0.483)	(0.428)
Income diversity				0.111	0.155	0.126	0.0659	0.105	0.0824
				(0.168)	(0.225)	(0.170)	(0.195)	(0.252)	(0.199)
Asset diversity				-2.258***	-0.316	-1.586*	-1.411	0.805	-0.468
				(0.854)	(1.338)	(0.884)	(0.983)	(1.338)	(0.972)
ratio_k				-0.0693	-0.0400	-0.0592			
				(0.0483)	(0.0560)	(0.0448)			
Tier1/Asset							-0.00403	-0.0259	-0.0133
							(0.0128)	(0.0190)	(0.0129)
Deposits/Asset							-0.00226***	-0.000894	-0.00168
							(0.000724)	(0.00189)	(0.00109)
Observations	148	148	148	139	139	139	140	140	140
R-squared	0.822	0.660	0.803	0.851	0.683	0.831	0.857	0.631	0.816
Fixed effects	bank year	bank year	bank year	bank year	bank year	bank year	bank year	bank year	bank year
Instrument		Pred no fe	Pred k jt		Pred no fe	Pred k jt		Pred no fe	Pred k jt
F-Test 1st		11.64	12.77		9.825	12.57		12.83	18.72

Robust standard errors clustered at the bank-hosting-country level in parentheses
*** p<0.01, ** p<0.05, * p<0.1

2.4 Diversification, Competition and Regulation

In this section we dissect our previous results and explore the channels driving them. We consider three different channels: asset diversification, competition and regulation. The first channel has been already examined in past empirical literature based on U.S. data (see GLL(29) and LLX(33)). The second channel has been extensively explored in the theoretical literature with alternative results (see Allen and Gale(1),Boyd and De Nicolo'(6) and Faia and Ottaviano(24)), but has received little or no attention in the empirical literature. The third channel received also extensive attention by commentators who envisaged as motivation for banks' expansion an interest for regulatory arbitrage. In the years prior to the crisis regulations varied significantly across countries (despite the Basel suggestions which were common to all countries). Hence banks headquartered in countries with stricter regulations had an interest to expand in countries with laxer regulations. Below we explore the importance of those three channels.

Before turning to the regressions a few observations might give a first glance of how the channels operate (see also Appendix D for more details). Origin countries tend to be rather different from other countries in terms of diversification, competition and regulation. In particular, origin countries have on average higher business cycle co-movement with the rest of the Europe (0.92 against 0.8 in terms of growth correlation), more competition (0.92 against 0.87 in terms of Herfindahl-Hirschman Index) and more similar regulation[10]. Also destination countries exhibit even larger differences than origin countries with respect to the rest of Europe. In our sample, 75% of all openings take place in countries that have less co-movement with the rest of the Europe than the origin country, 54% in countries that have a stricter regulation and 59% in countries that are less competitive. This seems to suggest that banks tend to expand to countries whose business cycles are less correlated with the rest of Europe than in the origin country, to countries that are less competitive than the country of origin, and to countries with better regulation.

Several considerations might emerge from the observations above. We focus however on examining whether the negative impact of foreign expansion on a bank's riskiness varies when expansion involves countries with different degrees of business comovement, competition or regulation. In other words we exploit variations in destination countries with respect to several indicators (diversification, competition and regulations) and relatively to the country of origin to examine how the various channels contribute to the negative relation between expansion and risk.

[10]See Table D.1 in Appendix D for additional details.

2.4.1 Diversification

First, we test the impact of the diversification motive on the relation between risk and expansion. We do so by exploiting the variations in destination country's co-movement vis-á-vis the rest of the area and relatively to the country of origin co-movement. We therefore define a new variable that proxies business cycle co-movement (we label it with the acronym *cmv*) with the correlation of a country growth rate with the growth rate in all other countries in the sample, distinguishing between expansions to destination countries with higher and lower business cycle co-movement than the origin country. To address the problem of endogeneity with these two types of expansions, we then repeat our 2SLS with two new instruments: the predicted expansion to countries with higher co-movement than the origin country; and the predicted expansion to countries with lower co-movement than the origin country. Our initial baseline instrument is the one generated through the gravity estimation with bank and year fixed effects. This choice is motivated by the fact that the other instruments have a very low correlation with the actual openings. Except for the change in instruments all other controls remain the same as before.

The corresponding results are reported in Table 2.12. The dependent variable is the CDS price. We focus on this as it is an all-encompassing metric of risk. We use the unweighted instrument in the first two-columns and the weighted one in the second two columns. OLS estimates (columns 1 and 3) suggest that it is openings in countries with lower co-movement that drive the overall negative impact of foreign expansion on bank riskiness. However, once the endogeneity bias is removed, 2SLS estimates (columns 2 and 4) reveal that risk decreases when bank's expansion takes place in a country with either lower and higher business cycle co-movement vis-á-vis the rest of the destination countries. It is worth noting that expansion to countries featuring higher co-movement with the rest of Europe has a larger coefficient estimate than expansion to countries with lower co-movement. This can be explained by the possibility to hedge against global risk in line with GLL(29) and LLX(33). It could also be explained by the fact that in Europe the countries with high co-movement are countries with stronger economic fundamentals. Conversely, the countries with a low co-movement are generally countries with weaker economies, which may weaken the risk-reduction effect of expansion in these countries, even though it is still significant.
Be that as it may, diversification is an important channel through which international expansion reduces individual bank risk.

Table 2.12: Testing for the diversification channel.
Dependent variable: CDS prices. OLS and 2SLS regressions with bank-year fixed effects. Unweighted instrument in the first two-columns and weighted instrument in the second two columns.

	(1)	(2)	(3)	(4)
VARIABLES	OLS	2SLS	OLS	2SLS
Exp. when $cmv_j < cmv_i$	-0.0143***	-0.0393***		
	(0.00376)	(0.0124)		
Exp. when $cmv_j > cmv_i$	-0.000146	-0.104**		
	(0.0139)	(0.0471)		
Exp. w when $cmv_j < cmv_i$			-0.0137***	-0.0381***
			(0.00360)	(0.0119)
Exp. w when $cmv_j > cmv_i$			-0.000204	-0.101**
			(0.0134)	(0.0454)
ln(Tot Assets)	-0.0200	-0.122	-0.0204	-0.121
	(0.126)	(0.132)	(0.126)	(0.132)
ROA	-0.00632	0.0183	-0.00617	0.0184
	(0.0730)	(0.0810)	(0.0730)	(0.0808)
Income diversity	-0.119***	-0.0937**	-0.119***	-0.0923**
	(0.0366)	(0.0415)	(0.0367)	(0.0417)
Asset diversity	0.202	0.910**	0.200	0.896**
	(0.306)	(0.416)	(0.306)	(0.414)
Tier1/Asset	-0.00988*	-0.0202***	-0.00987*	-0.0201***
	(0.00515)	(0.00616)	(0.00514)	(0.00612)
Deposits/Asset	-0.000652***	-0.000168	-0.000652***	-0.000175
	(0.000206)	(0.000449)	(0.000206)	(0.000446)
Observations	141	141	141	141
R-squared	0.974	0.952	0.974	0.952
FE	bank year	bank year	bank year	bank year
Instr.		pred. k jt		pred. k jt
F-Test 1st		6.496		6.565

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

2.4.2 Competition

Next we test the impact of variations in competition for the usual relation. We now turn to competition, which we measure (inversely) through the canonical indicator of market concentration in the destination country, namely the Herfindahl-Hirschman Index of industry concentration (HHI). We then partition origin countries in two groups depending on whether their HHI is higher or lower than the median HHI among origin countries, and we define a 'low competition' dummy variable that takes value 1 for origin countries with higher-than-median HHI and 0 otherwise.[11] As in the case of diversification, having to deal with two types of expansions (from higher-than-median and from lower-than-median HHI countries), we use two instruments: the predicted expansion (with bank-year fixed effects); and the predicted expansion interacted with the low competition dummy.

The results are reported in Table 2.13 which is constructed in a similar way as Table 2.12. Results show that banks headquartered in countries with a higher level of competition (lower-than-median HHI) have lower riskiness. It also shows that expansion for banks headquartered in those countries reduces bank riskiness less than expansion from the other countries. Actually, on net the effect of expanding from higher competition countries is essentially null. Therefore, the overall negative impact of expansion on bank riskiness is entirely driven by banks expanding from countries that are less competitive (higher-than-median HHI). This finding can be rationalized in the wake of Faia and Ottaviano (24). In the logic of their model, more competition has two opposite effects in the markets of funds (deposits) and loans when banks have market power in both. On the one hand, tougher competition in the banks' funding market increases the interest rate banks pay as it reduces their oligopsonistic power. On the other hand, tougher competition in the market for loans decreases the spread ('markup') between the interest rate banks earn on loans and the interested rate they pay on funds due to weakened oligopolistic power. Due to moral hazard investors finance more risky projects when the interest rate on loans is higher, but the effect of competition on this interest rate is generally ambiguous depending on whether the oligopsonistic effect or the opposing oligopolistic effect dominates. In our sample expansion from less competitive markets drives the overall fall in bank riskiness. In light of the Faia and Ottaviano (24) model, this would be consistent with the negative oligopolistic effect of competition on the spread dominating its positive oligopsonistic effect on funding costs as long as expanding from less competitive countries increased the competitive pressure on banks.

[11] The median HHI among origin countries corresponds to the bottom 20% HHI among all countries in the sample.

Table 2.13: Testing for the competition channel.
Dependent variable: CDS prices. OLS and 2SLS regressions with bank-year fixed effects. Unweighted instrument in the first two-columns and weighted instrument in the second two columns.

	(1)	(2)	(3)	(4)
VARIABLES	OLS	2SLS	OLS	2SLS
Expansion	-0.0181***	-0.0673***		
	(0.00513)	(0.0204)		
Expansion * $\mathbb{1}_{low\ hhi\ in\ i}$	0.0137**	0.0617***		
	(0.00687)	(0.0205)		
Expansion w			-0.0173***	-0.0642***
			(0.00490)	(0.0196)
Expansion * $\mathbb{1}_{low\ hhi\ in\ i}$ w			0.0131*	0.0590***
			(0.00666)	(0.0198)
$\mathbb{1}_{low\ hhi\ in\ i}$	-0.0944	-0.379***	-0.0942	-0.378***
	(0.0956)	(0.120)	(0.0959)	(0.121)
ln(Tot Assets)	-0.0224	0.0184	-0.0227	0.0167
	(0.125)	(0.118)	(0.125)	(0.118)
ROA	0.00208	0.0227	0.00209	0.0228
	(0.0703)	(0.0610)	(0.0703)	(0.0608)
Income diversity	-0.114***	-0.105***	-0.114***	-0.104***
	(0.0358)	(0.0361)	(0.0359)	(0.0360)
Asset diversity	0.0935	-0.0289	0.0916	-0.0312
	(0.317)	(0.283)	(0.317)	(0.284)
Tier1/Asset	-0.00971*	-0.00846	-0.00969*	-0.00847
	(0.00495)	(0.00520)	(0.00495)	(0.00519)
Deposits/Asset	-0.000616***	-0.000635*	-0.000617***	-0.000638*
	(0.000187)	(0.000332)	(0.000187)	(0.000328)
Observations	141	141	141	141
R-squared	0.975	0.958	0.975	0.958
FE	bank year	bank year	bank year	bank year
Instr.		pred. k jt		pred. k jt
F-Test 1st		6.853		6.509

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

2.4.3 Regulation

At last we examine the impact of variations in regulation on the usual relation. We proxy regulation with the macroprudential index (MPI) of Cerutti, Claessens and Laeven (11). For each origin country i we partition destination countries in two groups depending on whether their regulation is stricter than the origin country ($mpi_j > mpi_i$) or less strict ($mpi_j < mpi_i$). As in the case of diversification, we instrument the two endogenous groups of openings with the corresponding predicted expansions.

From the estimates reported in Table 2.14, we see that a large part of the overall negative effect of geographic expansion on bank riskiness in the sample is (un-surprisingly) driven by the expansion to countries with stricter regulation. If banks expand in countries with stricter regulation the monitoring exerted by the supervisor is likely going to reduce risk.

Table 2.14: Channels: Regulation

VARIABLES	(1) OLS	(2) 2SLS	(3) OLS	(4) 2SLS
Exp. when $mpi_j < mpi_i$	-0.0171**	-0.0229		
	(0.00654)	(0.0172)		
Exp. when $mpi_j > mpi_i$	-0.00499	-0.0729***		
	(0.00875)	(0.0261)		
Exp. w when $mpi_j < mpi_i$			-0.0161**	-0.0222
			(0.00635)	(0.0165)
Exp. w when $mpi_j > mpi_i$			-0.00523	-0.0700***
			(0.00850)	(0.0252)
ln(Tot Assets)	-0.0369	-0.0426	-0.0368	-0.0427
	(0.125)	(0.122)	(0.125)	(0.122)
ROA	-0.00125	-0.00554	-0.00142	-0.00499
	(0.0731)	(0.0735)	(0.0730)	(0.0733)
Income diversity	-0.118***	-0.100***	-0.118***	-0.0994***
	(0.0355)	(0.0358)	(0.0356)	(0.0359)
Asset diversity	0.216	0.753*	0.215	0.741*
	(0.307)	(0.439)	(0.308)	(0.435)
Tier1/Asset	-0.0102**	-0.0177***	-0.0102**	-0.0176***
	(0.00497)	(0.00628)	(0.00497)	(0.00627)
Deposits/Asset	-0.000658***	-0.000191	-0.000654***	-0.000198
	(0.000207)	(0.000383)	(0.000208)	(0.000383)
Observations	141	141	141	141
R-squared	0.974	0.959	0.974	0.959
FE	bank year	bank year	bank year	bank year
Instr.		pred. k jt		pred. k jt
F-Test 1st		11.57		11.81

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

2.5 Conclusion

We have build an original dataset on 15 European banks classified as G-SIBs by the BIS to assess whether expansion in foreign markets increases their riskiness, and through which channels this eventually happens. We have distinguished a 'between effect' from a 'within effect'. According to the former effect, banks that expand abroad more have lower riskiness so that, given individual bank riskiness, their expansion reduced the (weighted) average riskiness of the banks' pool. According to the latter effect, foreign expansion of any given bank makes the bank and thus the banks' pool less risky.

We have found that there is a strong negative correlation between riskiness and foreign expansion. This is due to a robust 'within effect' as well as to less robust 'between effect'. In terms of the channels, we have found evidence that diversification, competition and regulation are all important in explaining the 'within effect'. The reduction in individual riskiness is associated with better asset diversification. Expansion in destination countries with stricter regulation than the origin country decreases a bank's riskiness as well. As for competition, expansion has a distinct impact on riskiness only when competition in the origin country is less intense than in the destination countries.

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Appendices

Appendix A

Countries

Origin countries of banks: France, United Kingdom, Switzerland, Italy, Germany, Netherlands, Spain and Sweden.
Host countries: All potential origin countries and Albania, Austria, Belgium, Bulgary, Bosnia and Herzegovina, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, Greece, Hungary, Ireland, Lituania, Luxembourg, Latvia, Malta, Montenegro, Norway, Poland, Portugal, Romania, Russia, Serbia, Slovakia, Slovenia, Turkey and Ukraine.

Appendix B

Openings

Table B.1: Number of openings of foreign units by host country and year

Countries	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	Total
Albania		1	1								2
Austria	2	3	1	5	3	1				1	16
Belgium	4	4	2	2	2				1	1	16
Bulgary	3	2	1	1				1			8
Bosnia and Herzegovina	2	1									3
Switzerlnad	4	3	2	2	3	2					16
Cyprus	1										1
Czech Republic	2	1	1		2	1		1			8
Germany	5	8	4	5	2	1	5	1	2	1	34
Denmark	1	2	2	4	2	2		1	1	2	17
Spain	2	3	1		2	2	1	1	1	1	14
Estonia	2		1								3
Finland	1	1	1	2	1			1		1	8
France	3	2		2		1					8
UK	5	9	4	5	2	1	3		1		30
Greece	1	3	1		1				1		7
Croatia	2	1	1	1							5
Hungary	2	2		2	1	1				1	9
Ireland	4	1	7		1	1					14
Italy	5	7	5	7	2		4		1	1	32
Lithuania	2					1					3
Luxembourg	4	7	6	1	4	3	5		1		31
Latvia	2										2
Malta		1									1
Montenegro	1										1
Netherlands	6	8	4	2	4	3	1		1	1	30
Norway	3		2	2	1			1	1	1	11
Poland	2	2	2	3	2	3	2	2		3	21
Portugal	3	2	2	1	1	1	1		1	1	13
Romania	4	2		3	1					1	11
Russia	3	4	4	2		1	1				15
Serbia	2	3	1								6
Slovakia	2	1	1	1	1						6
Slovenia	1										1
Sweden	2	1	1	2	1	2	1			3	13
Turkey	3	2	7	3	1	1	2	1			20
Ukraine	2	3		2	1						8
TOTAL	**93**	**90**	**65**	**60**	**41**	**28**	**26**	**10**	**12**	**19**	**444**

Appendix C

Gravity Literature

The gravity framework has originally been used to describe trade flows (Tinbergen, 1962 being the first to apply this framework) and a large literature now exist to provide strong theoretical and empirical basis to this framework. One of the first idea for gravity is that trade flows are decreasing with the distance, because distance raises transport costs, all other things being equal. According to the meta analysis of Head and Mayer (2014), the distance elasticity of trade is between 0.89 and 1.14 according to the estimation methodology. This framework has also been applied to intangibles flows such as FDI or financial variables, showing that the geographical distance raises other costs than transportation costs (information costs for instance). In this case, the distance elasticity is lower, but significantly different from 0.

More specifically, a few papers are interested in the impact of geographical variables on cross-border banking and banks international expansion. Galindo et al. (2003) show that the bank penetration measured by the sum of assets of banks of the host country held by banks in the source country decreases with the distance between the two countries. They measure a distance elasticity of 0.32. Buch (2005) confirms this result using data of foreign asset holdings of banks located in France, Germany, United Kingdom and USA. She finds an elasticity of 0.65 in 1999 that varies between 0.31 in France to 1.13 in Italy. Focarelli and Pozzolo (2003) show that bank foreign investment is also consistent with the gravity framework. According to the method used, the find an elasticity of bank foreign investment to distance between 0.3 and 0.47 according to their fixed effects specification. Berger et al. (2004) propose a gravity analysis on bank expansion through M&A. They find a distance elasticity of 0.88 when they include host country and source country fixed effects. Claessens and Van Horen (2014) study the foreign location decisions of banks in a large number of countries in 2009. In order to have an estimation close to the gravity theory developed in the trade literature, they include the competitor remoteness as a regressor. This regressor

is intended to absorb multilateral resistance terms (see Anderson and Van Wincoop, 2003). They find a small distance elasticity of foreign bank ownership that varies between 0.032 and 0.115 according to the methodology used.

The difference between our gravity model and these ones is that we explicitly take into account multilateral resistance factors, that are theoretically needed, by adding exporter-time and importer-time fixed effects in our first estimation. Nevertheless, we do not construct our instrumental variable using this specification because bank time-varying fixed effects are likely to be correlated with bank's riskiness.

Banking litterature

Paper	Year	Dependant variable	Estimation strat.	Dist. coef	Alternative strat.	Alt. coef.
Portes and Rey, JIE	2000	Gross purchases plus sales of portofolio equities (1989-1996)	OLS, no FE	-0.881	OLS bilateral FE	-0.646
Aviat and Coeurdacier, JIE	2007	Financial claims in country j from banks located in country I in 2001	OLS, no FE	-0.445	OLS bilateral FE	-0.74
Coeurdacier and Martin, JoJIE	2009	log of aggregate equity holdings (1), the log of banking claims (3) in 2001	(1) bilateral FE	-0.42	(3) bilateral FE	-0.49
Buch, JMCB	2003	Log of foreign assets 2009	OLS Country FE	-0.29		
Buch, RIE	2005	Log of assets of banks (1983-1999)	OLS Country FE	-0.65	Log of liabilities, OLS, country FE	-0.72
Galindo et al., WP	2003	Sum of assets of banks of the host country in which the source country owns 50 percent or more of their equity in 2001	OLS, bilateral FE	-0.318		
Focarelli and Pozzolo, JoBusiness	2005	Dep var = 0 if the bank has no foreign branches/subsidiaries in j, 1 if it has a foreign branch and 2 if it has a foreign subsidiary at the end of 1998	Multinomial logit for branches	-0.31	Multinomial logit for subsidiaries	-0.30
Berger et al., JIMF	2004	Number of M&A in year t in which a country i financial institution purchased a country j financial institution divided by the product of the GDP of i and j (1985-2000)	Tobit, i, j and t FE	-0.88	Tobit, t FE	-0.64

Claessens and Van Horen, JMCB	2014	Number of banks from country i in country j in 2009	Tobit, no FE	-0.115	Poisson (no FE, set of controls + trade)	-0.033
Vlachos, WP	2004	Portfolio holdings by country i in country j in 2001	OLS, bilateral FE	-0.29		
Faruqee et al., IMF WP	2004	Stock of country j equity held by residents of country i at the end of 1997	Standard gravity variables, no FE	-0.559		
Salins and Benassy-Quere, WP	2006	Portfolio investment stocks from country i to country j	Tobit, no FE	-0.802		
FDI literature						
Head and Ries, JIE	2008	FDI flows	PPML	-0.592		
Trade literature						
Head and Mayer, Handbook of IE	2014	Trade	Meta-analysis - All gravity	-0.89	Meta-analysis - Structural gravity	-1.14

Note: Zeros are generally treated using log(1+variable). FE stands for Fixed effects. "Bilateral FE" means source and host country fixed effects.

Appendix D

Comovement, Regulation and Competition

Table D.1: Descriptive statistics on comovement, regulation and competition

	Comovement		Competition		Regulation	
	Source countries	Host countries	Source countries	Host countries	Source countries	Host countries
Mean	0.92	0.80	0.92	0.87	1.44	1.49
Sd	0.07	0.16	0.06	0.07	1.13	1.41
Min	0.78	0.31	0.98	0.97	0.00	0.00
Max	0.97	0.98	0.80	0.69	3.00	4.89

Note: Data is averaged over all years in the sample. Source countries are excluded from the host countries statistics