Abstract Pasinetti’s (1981) book — and, in general, Pasinetti’s vertically (hyper-) integrated framework — has been the subject of many reviews, and of many criticisms. Some of these criticisms are actually due to some ambiguity, in Pasinetti’s exposition — or to the fact that Pasinetti’s (1981) framework has been developed starting from simplifying assumptions that, though being functional to the development of the main idea at the core of this approach to economic analysis, are quite unrealistic.

The aim of the present paper is that of replying to such criticisms by taking advantage of the conceptual excursus made in Garbellini & Wirkierman (2010b) through Pasinetti’s (1981) work, and of the generalisation carried out — starting from the hints provided by Pasinetti (1988) — in Garbellini (2010b) and Garbellini (2010a). The task is that of arguing that Pasinetti’s device of vertical hyper-integration is a powerful tool to study economic reality, and therefore that it is necessary to further develop it, in order to provide economic analysis with an alternative approach — rooted in the Classical-Sraffian tradition but overcoming its difficulties — and therefore able to deal with the most important characteristics of modern industrial societies: structural change, technical progress and economic growth.

Keywords Natural system, vertically integrated sectors, vertically hyper-integrated sectors, functional income distribution, natural rates of profit, natural prices.

JEL classification B51, O41

1 Introduction

Pasinetti’s (1981) book — and, in general, Pasinetti’s vertically (hyper-)integrated framework — has been the subject of many reviews, and of many criticisms. Some
of these criticisms are actually due to some ambiguity, in Pasinetti’s exposition — or to the fact that Pasinetti’s (1981) framework has been developed starting from simplifying assumptions that, though being functional to the development of the main idea at the core of this approach to economic analysis, are quite unrealistic.

These criticisms can be grouped into two categories.

The criticisms in the first category can be overcome by simply clarifying some points that have not been grasped, or have been grasped only partially — that is to say, such criticisms originate from a failure in fully understanding Pasinetti’s analytical framework.

The criticisms in the second category, on the other hand, are due to the failure in understanding the aim of Pasinetti’s (1981) book, as opposed to that of Pasinetti’s (1988) paper (which is likely much less known than Pasinetti (1973)): while the latter provides a first step to formulate the whole analytical framework in a really general way — with all inter-industry relations and with the more complex description of the technique in use — the former’s aim is that of showing the mechanism at the basis of vertically hyper-integration. In order to do so, many simplifying assumptions have been introduced, to get rid of analytical complications, and focus attention on the peculiarities of this kind of approach.

In other words, Pasinetti’s (1981) book is the first building block for the understanding of the whole vertically hyper-integrated approach — actually, as stressed in Garbellini & Wirkierman (2010b), it also is the intermediate step leading Pasinetti himself to the complete and explicit formulation of this approach. From an analytical point of view, it is built on a number of simplifying assumptions which make it unsuitable for an immediate implementation for empirical investigations.

The starting point towards the accomplishment of this latter task is Pasinetti (1988); from there on, the way is open and waits to be explored. Anyhow, it is in Pasinetti (1981) that the really deep and novel theoretical and conceptual implications are drawn, and the problems affecting economic analysis since the time of the Classics faced. Many hints are given on how to go straight to the accomplishment of the above-mentioned task, too.

It is therefore my contention that Pasinetti’s work has to be seen as a unitary corpus, providing both deep and thought theoretical insights and clear indications of the way to be followed for empirical investigations.

The aim of the present paper is therefore that of taking advantage of the conceptual excursus made in Garbellini & Wirkierman (2010b) through Pasinetti’s work.

---

1As explained in detail in Garbellini & Wirkierman (2010b), all Pasinetti’s work, starting from his doctoral dissertation, can be seen as a series of steps finally leading to Pasinetti (1988), i.e. to the explicit presentation of vertically hyper-integrated sectors as a tool for economic analysis.
(1981) work — in order to reply to the first category of criticisms — and of the generalisation carried out starting from the hints provided by Pasinetti (1988) in Garbellini (2010b) and Garbellini (2010a) — in order to reply to the second category of criticisms. The task is that of showing that Pasinetti’s device of vertical hyper-integration is a powerful tool to study economic reality, and therefore that it is necessary to further develop it in order to provide economic analysis with an alternative approach — rooted in the Classical-Sraffian tradition but overcoming its difficulties — and therefore able to deal with the most important characteristics of modern industrial societies: structural change, technical progress and economic growth.

All the criticisms that will be considered in what follows come from two kinds of sources: review articles, in particular on Pasinetti (1981), and discussions emerged during conferences or meetings in which Pasinetti’s work has been the object of discussion. In some cases, the discussions have been directly stimulated, or caused, by the presentation of papers by myself (in particular Garbellini & Wirkierman 2010b, Garbellini & Wirkierman 2010a).

2 Structural Change and Economic Growth

2.1 Normative analysis as opposed to positive analysis

We can summarise very effectively this first criticism to Pasinetti’s (1981) framework by quoting a review article — actually a very critical one — written in 1982 by Harris:

Pasinetti’s ‘structural dynamics’ is constrained within the requirements of his especial conditions of equilibrium, albeit a moving equilibrium, and, oddly enough, equilibrium is itself conceived as a kind of ‘natural’ state. Here one runs up against a problematical feature of this analysis that needs to be pursued further.

(Harris 1982, p. 29)

Harris — but he is not alone — had clearly read the book as an attempt to perform a positive analysis, that is to say to describe what actually happens in a specific, capitalist, economic system: “it is presumed that the analysis is applicable to real functioning capitalist economies” (Harris 1982, p. 40).

Probably, the misunderstanding partially flourishes from the often made association between Pasinetti’s and Kaldor’s names, due to the well-known contributions they gave to the Classical/Keynesian theory of income distribution, sometimes not very accurately collected under the collective name of ‘Kaldor-Pasinetti’ theory.

\[\text{See Kaldor (1955) and Pasinetti (1962).}\]
This assertion is also confirmed by the words of another commentator:

Pasinetti (like Kaldor) goes further [...] by also postulating that there is full employment in the natural system. Output is then fixed by labour supply.

(Taylor 1995, p. 699)

Kaldor’s analysis, actually, was a positive one: he tried to provide a ‘Keynesian’ theory of income distribution, where profits were considered as a prior claim on the share of the net output of an economic system, and then arriving at the so-called Cambridge equation, relating the rate of profit and the rate of growth of the economic system. Or, to state it using Pasinetti’s own words:

A second and separate problem concerns the interpretative value of the model. When Mr Kaldor presented his theory of income distribution, he pointed out that the interpretative value of the theory depends on the Keynesian hypotheses on which it is built. [...] But this is not the approach that I should like to take here. Whether we are or whether we are not prepared to accept the model in this behavioural sense, there are important practical implications which are valid in any case. I should look, therefore, at the previous analysis simply and more generally as a logical framework to answer interesting questions about what ought to happen if full employment is to be kept over time, more than as a behavioural theory expressing what actually happens.

(Pasinetti 1974, pp. 118-119, emphases added)

As explained at length in Garbellini & Wirkierman (2010b, sections 3.5 and 5), Pasinetti’s (1981) framework is intended to find out those physical requirements that, if met, guarantee full employment of the labour force, full expenditure of national income and full utilisation of (vertically hyper-integrated) productive capacity. The dynamic capital accumulation conditions are precisely those requirements that must be met for the amount of new investment to drive capital accumulation in line with the pace of the dynamics of final demand for consumption commodities. “They are true whatever individual behaviour may be; as a simple matter of logical necessity” (Pasinetti 1974, p. 119; emphasis added).

No automatism is implied: reaching the objective in one period does not guarantee to be again in a situation of equilibrium in the following one. Physical requirements for new investment must be met period after period, in a constant actively pursued search for the new equilibrium situation. “If full employment is to be maintained [once reached], that amount of investment must be undertaken” (Pasinetti 1974, p. 119). No reference to a capitalist economic system is made: the institutional set-up is left outside the analysis. The way in which such conditions

---

3That is, the set of social relations of (re-)production. See Garbellini & Wirkierman (2010b, section 2).
can be met within different institutional frameworks is a subject to be discussed separately, in a different stage — the institutional one — of the analysis.

Therefore, equilibrium is not conceived as “a kind of ‘natural’ state”. On the contrary. Such a “state” can be reached if, and only if, it is actively pursued as an agreed end of the existing institutions.

The ‘natural economic system’ is therefore defined as an ideal, normative dynamic system in which the set of physical requirements described above — the macroeconomic condition for full employment of the labour force and full expenditure of the national income and the sectoral capital accumulation conditions — are met; the ‘natural’ rates of profit single out that amount of investment that must be undertaken if we want such conditions to actually be met period after period. The structure of such an ‘ideal’ system is continuously changing through time, due to technical progress, changes in individuals’ real income, shifting of per capita demand for consumption commodities.

Once these fundamental characteristics and aims of Pasinetti’s (1981) framework are fully grasped and understood, it is therefore hard to maintain that the natural economic system is essentially a golden-age equilibrium of a very special kind. It is one in which all of the structural changes which the author believes it is important to analyse unfold in full view as time goes by. These are the changes which, if we are to accept the stylised facts, do happen in real life and sometimes with disastrous consequences. But in the Pasinetti-golden-age they happen without any disturbance, specifically in regards to the condition of full-employment. Whereas in the ordinary golden age nothing happens, at least as far as changes in the ‘structure’ are concerned, in this particular golden age all sorts of changes occur and still there is full employment. We could just as well call it, therefore, a super-golden-age.

(Harris 1982, pp. 40-41)[4]

To conclude, equilibrium is not imposed by Pasinetti (1981); the requirements for its realisation period after period are singled out, in order to stress the physical new investments — i.e. capital accumulation — necessities of the economic system.

2.2 Over-determination of the equation systems

Closely connected to the criticism mentioned above in section 2.1, we may add that some commentators objected that the effective demand condition is a rank

[4] Incidentally, this means that, with constantly changing rates of growth of final demand for consumption commodities, the natural rates of profit are themselves continuously changing through time too.

[5] Clearly enough, when talking about golden ages Harris is referring to Joan Robinson. See Robinson (1958) and Robinson (1962).
condition over-determining the system by imposing full employment. An example of such a criticism has been put forward by Parrinello:

[. . .] in an unpublished paper in 1967 to Pasinetti’s multi-sectoral model (Pasinetti, 1965, 1993). In fact, this model imposes a rank condition that guarantees the persistence of full employment in the presence of technical progress, which is assumed to be exogenous but obeying that condition.

(Parrinello 2004, p. 319)

As explained in detail both in Garbellini & Wirkierman (2010b) and in Garbellini (2010b), the effective demand condition is a macroeconomic condition that, if satisfied, guarantees full employment of the labour force and full expenditure of the national income.

From a strictly mathematical point of view, it is obtained by Pasinetti (1981) as a condition for getting non trivial solutions out of both the physical quantity and the relative price system. When the two systems are reformulated as eigenproblems (see Garbellini 2010b, sections 3 and 5), it is a condition for having a unitary eigenvalue, to which the solution vector we are looking for is associated.

At the end of the previous section we stressed that Pasinetti (1981) does not assume equilibrium, but looks for the conditions to achieve it. In the same way, he does not impose full employment — which is one requirement coming from Pasinetti’s (1981) very definition of an equilibrium situation — but looks for the conditions that, if satisfied, imply a state of full employment in the system.

Therefore, what he does when formulating the quantity and price systems, is closing them with two expressions — one for each — describing a situation of full employment of the labour force and of full expenditure of national income. If such equations hold, the resulting systems describe a situation of flow equilibrium, and the corresponding solutions are ‘equilibrium’ solutions; their mathematical condition of existence provides us with a formal relation indicating the requirements that must be met, from an economic point of view, for such an equilibrium situation to be realised.

The criticism again involves the misunderstanding of the normative, rather than positive, nature of Pasinetti’s (1981) framework. He is not assuming equilibrium in order to describe what actually happens in a concrete economic system. He is describing equilibrium in order to arrive, by means of formal logic alone, to the conditions that must be realised if an equilibrium situation is to be achieved.

Going back to the mathematical aspect of the problem, saying that the macroeconomic condition is a rank condition amounts to saying that the last equation of the quantity and price systems, respectively, are over-determining the corresponding system. As shown elsewhere (see Garbellini & Wirkierman 2010b, pp. 6-7), these last equations can be modified in order to allow for the non-realisation of full
employment of the labour force — within the quantity system — and full expenditure of the national income — within the price system. When such a procedure is adopted, the expression for the macroeconomic condition changes from:

$$\sum_i a_{in}(t)a_{ni}(t) + \sum_i T_i^{-1}a_{in}(t)a_{nk_i}(t) + \sum_i a_{k_i}(t)a_{nk_i}(t) = 1 \quad (2.1)$$

to

$$\sum_i a_{in}(t)a_{ni}(t) + \sum_i T_i^{-1}a_{in}(t)a_{nk_i}(t) + \sum_i a_{k_i}(t)a_{nk_i}(t) = \alpha \gtrless 1 \quad (2.2)$$

Expression (2.1) is a macroeconomic condition for flow equilibrium, i.e. a ‘normative’ relation; expression (2.2) is, by contrast, obtained from two equation systems written down to describe a contingent situation — in which flow equilibrium is not realised — and therefore is a ‘positive’ relation describing what happens at the ‘macroeconomic’, but it would be better to say aggregate, level. That is to say, by adopting such a procedure we exit the foundational stage of the analysis, entering the institutional one, which was not Pasinetti’s (1981) aim.

Incidentally, Parrinello (2004) also adds that

[. . . ] still maintain[s his] previous critical assessment of Pasinetti’s model from a theoretical point of view: normal prices are not associated with persistent full employment. However, we cannot charge with inconsistency a model because its system of equations becomes over-determined in the absence of a constraint that the model builder explicitly imposes on its parameters.

(Parrinello 2004, footnote 13, p. 321)

In this respect, it is worth stressing that Pasinetti’s notion of ‘natural prices’ is different from that of ‘normal prices’ coming from the so called ‘surplus approach’. It is therefore out of place criticising the former for not being consistent with the characteristics of the latter.

### 2.3 Pre-institutional theory of income distribution

A further, quite spread, criticism can also be very effectively summarised by using the reviewer’s own words:

[Pasinetti’s] attempt to develop his production system independently of institutional features runs into difficulties as soon as he deals with prices and distribution. ‘Profits’ is a term with meaning only in a capitalist society. His ‘natural’ rates of profit, that in equilibrium model provide the finance required in each sector to maintain the sectoral equilibrium rates of growth, would be ‘natural’ rates of tax in a socialist economy. The only category of income in the latter would be wages.

(Asimakopulos 1982, p. 1566)
The argument concerning the characteristic of Pasinetti’s (1981) pre-institutional theory of income distribution has been carried out in detail in Garbellini & Wirkierman (2010b, section 4.1) and it is therefore not necessary to go deep into this issue. Suffice here to recall that “at a pre-institutional stage of the analysis, a theory of the rate of profit is not a theory of income distribution among income recipients, […] because the very definition of the categories among which the purchasing power generated in the process of production is to be distributed essentially depends on the social relations of production of a particular institutional set-up” (Garbellini & Wirkierman 2010b, p. 21). Anyway, “prices of production provide for the purchasing power both to self-replace and expand productive capacity and to consume those commodities not re-entering the circular flow. Consider that profits and wages just establish the amount of purchasing power that must be channeled to demand for means of production to expand productive capacity and to demand for final consumption commodities, respectively” (Garbellini & Wirkierman 2010b, p. 22).

Moreover, it is worth spending a few lines on a further issue emerging from Asimakopulos’s (1982) critique quoted above, i.e. that “[p]rofits’ is a term with meaning only in a capitalist society”. I would say that ‘profits’ is a term which would not make sense if referring to a pre-industrial society; it was born together with the capitalistic6 mode of production, at the time of the Industrial Revolution. Of course, the first industrial societies were capitalist ones; in fact, we have to acknowledge that capitalistic economic systems with different institutional set-ups did in their turn emerge as a reaction to the capitalist social relations of production.

Therefore, while we could not use the term ‘profits’ when dealing with a pre-industrial economic system — we could not talk about capital accumulation actually, and therefore the necessity of using such a term would not even arise — we can perfectly think of profits within any kind of capitalistic economic system. The necessary and sufficient condition for the term ‘profits’ to make sense is the existence of a process of physical capital accumulation, whatever the social relations of production within which it takes place — and therefore whoever appropriates profits themselves, be them the capitalists, a central authority, or someone else.

A uniform rate of profit is a characteristic of capitalist societies; the natural ones, being different from sector to sector — and once the description of the technique in use is generalised, also leading to different prices for the very same commodities according to the growing subsystem they belong to — could not be realised within this institutional framework. But ‘profits’ are simply a component of the production prices, exceeding the labour costs and the costs associated to the

---

6For an explanation of the difference between the term ‘capitalist’ and the term ‘capitalistic’, see Garbellini & Wirkierman (2010b, section 2).
reproduction of used-up intermediate commodities; such a component is computed in proportion to the stock of accumulated capital, evaluated at current prices.

### 2.4 Labour as the only non-reproducible factor of production

The last criticism to Pasinetti’s (1981) *Structural Change and Economic Growth* that I want to consider here is that “Dr. Pasinetti assumes that labour is the only scarce factor of production” (Champernowne 1964, p. 660). More precisely,

> [t]he analysis ignores the role of natural resources. This approach seems reasonable as a first step, because it properly assigns conceptual priority to reproducible commodities. But it would seem necessary to grant, even at this level, as long as technical progress is the main focus of analysis, that the rate and direction of such technical progress may be significantly conditioned by the economic stimulus that comes from the dynamics of natural-resource utilisation. Consumption patterns may also be similarly influenced. […] There are significant aspects of the process of uneven development and disproportionality of growth that are not captured in this analysis.

(Harris 1982, p. 39)

This kind of criticism is not an isolated one, especially nowadays that the issues of ‘sustainable development’ and management of exhaustible natural resources have become very fashionable in economic analysis, both in the mainstream and among heterodox economists — input-output analysis, particularly, seems pursuing more and more this research line, being used for environmental applications such as ‘industrial ecology’.

The fact that natural resources are important is not denied, of course, as Pasinetti himself stated in his Doctoral Thesis:

> My impression is that the problems of scarcity are theoretically very exciting; and yet in practice have not had the importance which our theories have tended to give them. The bulk of contemporary economic theory has started from the investigation of the optimum allocation of scarce resources in an absolutely stationary world; and has then tried to extend the same concepts to a growing economic system. I am proposing a theoretical model which starts from the opposite end; namely from an economic system in which there is no scarcity but there is learning and thus economic growth. Later on — I am hoping — it may well turn out to be easier to introduce scarce resources into a model for learning and growth than it has been so far to introduce learning and growth into a model of scarce resources.

(Pasinetti 1965, p. 695)
However, there is a difference between using a theoretical framework for studying a particular, concrete, institutional problem — e.g. accounting for greenhouse gas emissions associated to the production of a particular final commodity — and introducing a particular, concrete, institutional problem into the foundational basis of the framework itself.

Scarce resources cannot be the basis of the production paradigm, as Pasinetti calls it. Labour is considered as the only primary factor of production because without labour, without human effort, no commodities can be produced:

Nothing in the present theoretical scheme has any economic relevance — i.e. value — other than in relation to the activity and wants of the members of the community. What nature offers is a datum — it is taken for granted. Any commodity, by itself, has no personality: it has no right or claim. Of course, commodities do physically produce other commodities — machines produce machines, animals reproduce animals — but this ‘physical’ productivity must be correctly interpreted. Commodities cannot appropriate the commodities that come out of them. Only Man can. The physical productivity of commodities simply is a part of their technical or biological properties, which for Man is a datum. What becomes relevant, for economic purposes, which means for the process of pricing, is only the amount of human activity which is required, whether directly or indirectly [or hyper-indirectly], to make a technological or biological process work.

(Pasinetti 1981, p. 131)

On the contrary, all other non-produced factors of production can be substituted with different ones — e.g. oil, by finding alternative fuels — or exploited in a more efficient way to overcome the problem of its scarcity — e.g. land, thanks to technical progress. The argument can once again be better presented by borrowing someone else’s words; in this case, Sraffa’s:

But how are we going to replace these natural things? There are 3 cases: a) they can be reproduced by labour (land properties, with manures and so on; b) they can be substituted by labour (coal by hydroelectric plant: or by spending in research and discovery of new source and new methods of economising) c) they cannot be either reproduced nor substituted and in this case they cannot find a place in a theory of continuous production and consumption: they are dynamical facts, that is a stock that is being gradually exhausted and cannot be renewed, and must ultimately lead to destruction of the society. But this case does not satisfy our conditions of a society that just manages to keep continuously alive.


7This is one of the above mentioned environmental applications of input-output analysis, called ‘carbon footprint’.
Up to the Mercantilist era, “the wealth of a nation was identified with the wealth of its king” (Pasinetti 1977, p. 2). But with the breakthrough of the Industrial Revolution, there has been a change of emphasis — started by the Physiocrats and then taken up by the Classical economists — from the problems concerning the scarcity of natural resources to those concerning produced commodities: “it was no longer, or not so much, the distinction between wealth as a stock and wealth as a flow that was seen as important, but rather the contrast between produced wealth (whether as an annual flow or as an accumulation of means of production) and exogenously given natural resources” (Pasinetti 1977, p. 3).

This theoretical shift reflects, according to Pasinetti (see for example Pasinetti 1965, Pasinetti 2007) the historical shift from the phase of trade — i.e. the pre-industrial era, “perceived even as early as at the turn of the first millennium” (Pasinetti 1965, p. 573) — and the phase of industry; the former is based on exchange, “by a better spatial allocation of existing resources and products” (Pasinetti 1965, p. 573); the latter on production, i.e. “a process of augmenting wealth through a material increase in the quantity and number of products, to be reached by the practical application of the advantages of science, division and specialisation of labour, better organisation, invention and utilization of new resources of energy and new materials” (Pasinetti 1965, p. 573).

To show how this historical shift caused the theoretical one, Pasinetti stresses that during the phase of trade — an intrinsically static concept — the economists’ concern was “the problem of how to reach the best allocation of given resources” (Pasinetti 1965, p. 574); on the contrary, the phase of industry — an intrinsically dynamic concept — brought about a whole series of new challenges, connected to the necessity of re-organising society and finding new and better methods of production: “[t]he economist is faced here no longer with a problem of rationality, but with a process of learning” (Pasinetti 1965, p. 575).

Pasinetti’s conclusion is therefore that

these are two distinct series of problems. A particularly important difference between the two, for theoretical analysis, is that they acquire an opposite practical relevance in relation to time, the former being relevant (in the short run) just when the latter is practically irrelevant and the latter becoming relevant just when (in the long run) the former becomes irrelevant.

(Pasinetti 1965, p. 575; emphasis added)

I did recall here — even if very briefly (for details, see Pasinetti 1965, pp. 572-575) — this historico-theoretical excursus because it gave rise to a further criticism, concerning the whole argument but particularly the above-quoted conclusion:

The classical surplus theories are characterized by some authors as being concentrated on reproducible commodities, and hence “production”, as opposed to the concentration on commodities of the scarcity type and hence on
“exchange” which would be the hallmark of the dominant marginalist theories. Accordingly the two kinds of theory would deal with two distinct series of problems, with an opposite practical relevance in relation to time, the classical theory becoming relevant just when (in the long run) the marginalist theory becomes irrelevant [(cfr. e.g. Pasinetti 1965, pp. 573-575)]. Whereas it aptly describes some differences between the two approaches, this distinction seems not to go to the roots of the difference, which lies in the way in which both “production” and “exchange” are treated in each approach.

(Garegnani 1984, p. 298, footnote 15)

This is a distorted interpretation of Pasinetti’s passage. In that passage, Pasinetti is referring to two different set of problems, not about two different theoretical paradigms. The contention is that, when dealing with problems related to optimal allocation of given resources, one deals with an essentially static problem, which can be faced as the rational choice of how to allocate an already existing endowment of ‘wealth’ in order to reach a certain objective. The relevance of these problems is therefore confined to the short run.

On the contrary, the analysis of problems related to industrial production is intrinsically dynamic: there are changes, induced by technical progress, that result from slow but persistent processes that can be perceived only in the long run, i.e. in a dynamic context; the temporal dimension cannot be disregarded, but has to be considered as the standpoint of the analysis. The Physiocrats first, and the Classical economists afterwards, had perfectly understood the importance of developing a production paradigm; Marx, in particular, was precisely working in such a direction.

The marginalist revolution — or better, counter-revolution — happened to take place precisely in the middle of these great historical, social struggles that brought modern industrial economic systems into existence. Nonetheless, marginal theory has been developed only with reference to the rational problem of optimal allocation of scarce resources, which quite obviously sounds as a contradiction: a new theoretical paradigm emerging after an unprecedented social and historical change should drive attention to the new problems, not bring it back to the old ones.

This is to say that Pasinetti asserts that the focus of marginal theory is on scarce resources, and that, as a consequence, its method of analysis allows to deal only with static problems. But he did not say that the marginalist way of doing so is the correct one to deal with the issue of exchange. He did not say that marginalist theory is relevant in the short run while Classical theory is relevant in the long run. He did say that the problems on which Classical theory is focused are relevant in a dynamic framework; while the problems marginal theory deals with — though not in the correct way — are relevant in a static framework.
3 Vertical hyper-integration and growing sub-systems

3.1 Pasinetti and Sraffa

Some criticisms raised against Pasinetti’s framework concern the fact that the use of vertical (hyper)-integration adds nothing to what can be already concluded by using Sraffa’s sub-systems or a standard multi-sectoral (industry-level) model. To a greater extent, it is sometimes asserted that vertical (hyper-)integration disregards inter-industry relations, whose description is one of the major achievements of multi-sectoral analysis. In sum, Pasinetti’s framework does not have very much to add to economic theory, and comes to be an elegant but not very useful elaboration.

We will concentrate on the issue of inter-industry relations, and on the consideration of the circular flow, later on, in section 3.3. Let us therefore start by considering the first part of such criticism, analysing the relation between Sraffa’s sub-systems and Pasinetti’s vertically integrated and hyper-integrated sectors.

As Pasinetti (1973) points out, the notion of vertical integration is very widely used in economic analysis — even if often without full awareness — and not only within non-neoclassical frameworks, but in a multiplicity of contexts of very different nature:

> The notion of vertical integration is implicit in all discussions on the theory of value of the Classical economists. The same thing can be said of the marginalist economists. When, for example, Léon Walras adopted the device of eliminating intermediate commodities from his analysis of production, he was making use of the logical process of vertical integration. Keynesian macroeconomic analysis is also generally carried out in terms of vertically integrated magnitudes (net national income, net savings, new investments, consumption, and so on). Very rarely, however, is the logical process of vertical integration explicitly discussed. Generally it is simply taken for granted.

(Pasinetti 1973, p. 1)

Clearly, identifying the core — and the originality — of Pasinetti’s contribution with the device of vertical integration is not simply reductive, but inadequate, as such a device has been used by a great number of economists, in a great number of different periods, situations, and within different theoretical frameworks.

I have already discussed at length, elsewhere, the difference between vertically integrated and hyper-integrated sectors. Suffice here to recall some basic points.

First of all, Pasinetti’s (1973) vertically integrated sectors represent an attempt at analytically formulating Sraffa’s (1960) sub-systems, as it should be clear by reading Sraffa’s own words:

8See Garbellini (2010b, section 4).
Consider a system of industries (each producing a different commodity) which is in a self-replacing state.

The commodities forming the gross product [...] can be unambiguously distinguished as those which go to replace the means of production and those which together form the net product of the system.

Such a system can be subdivided into as many parts as there are commodities in its net product, in such a way that each part forms a smaller self-replacing system the net product of which consists of only one kind of commodity. These parts we shall call ‘sub-systems’.

[... ] Although only a fraction of the labour of a sub-system is employed in the industry which directly produces the commodity forming the net product, yet, since all other industries merely provide replacements for the means of production used up, the whole of the labour employed can be regarded as directly or indirectly going to produce that commodity.

(Sraffa 1960, p. 89, emphases added.)

Actually, at a single point in time, sub-systems and vertically integrated sectors are the same thing; or better, the latter are a compact way of describing the former. Both sub-systems and vertically integrated sectors are a way of re-classifying the production processes that take place in the economic system — alternative to the more usual and directly observable one based on industries — aimed at identifying and isolating all the direct and indirect processes that allow the production of the net output, i.e. final demand.

This essential coincidence, within a single period of time and in a static framework, of Sraffa’s sub-systems and Pasinetti’s vertically integrated sectors could lead to draw the conclusion that the latter has nothing to add to what the former has already said. But Pasinetti went further.

While Sraffa, as he explicitly said, limited his analysis to “taking a ‘photograph’ of an economic system, as this actually can be observed at a certain point of time” (Pasinetti 2007, pp. 189-190) Pasinetti tries to overcome this limitation, analysing the dynamics of economic systems.

In order to do so, he redefines the notion of net output, in order to be able to treat extended reproduction avoiding the breaking up of the circular flow caused by the introduction of growth into the picture (See Garbellini 2010b, section 4.2). In particular, he separates that part of the net output that does not re-enter the circular flow, i.e. consumption commodities — from the one which does re-enter it in the following period, as additional productive capacity: new investments. Therefore, even if we are still in front of a way of re-partitioning the productive activities taking place in the economic system as a whole, the way in which such re-partitioning is effected is entirely different. It is a full generalisation of Sraffa’s idea.

9Pasinetti is citing the Sraffa papers, C294/2.
More specifically, the gross product of vertically integrated sector $i$ is given by two components. The first one is a quantity $y_i = x_i + j_i$ of the \textit{homogeneous} commodity $i$, sold at the end of the production period either to be consumed consumed ($x_i$) or be part of the \textit{whole} economic system’s new investments ($j_i$), i.e. to become means of production in the following period(s). The second one is the set of \textit{heterogeneous} commodities (re-)produced as the used up — both directly and indirectly — means of production. At the end of the production period, in order for the economic system as a whole to be provided with an increased productive capacity for the following one, each sector has to buy new investment goods from, and to sell a part of its net output to, the others. Hence, when we consider growth, the sub-systems are no more in a “self-replacing state”.

On the contrary, the gross product of vertically hyper-integrated sector $i$ is made up by two components, but defined in an entirely different way. The first part is a quantity $x_i$ of the homogeneous commodity $i$ which is produced in order to be consumed. The second part consists of heterogeneous commodity produced in order to become means of production. They include the \textit{whole set} of new investments commodities that are necessary to expand the sector’s productive capacity — in line with the evolution of final demand for the corresponding consumption commodity — as well as that set of intermediate commodities that have to replace those used up during the production process. In this way, thus, each vertically hyper-integrated sector produces all the new productive capacity it needs: it does not need to buy part of their net output from, and sell part of its net output to, the others. The “self-replacing state” is recovered.

By going into dynamics, Pasinetti can analyse changes in the structure of physical quantities of the economy, instead of considering “[n]o changes in output and [. . .] no changes in the proportions in which different means of production are used by an industry” (Sraffa 1960, p. v), and hence overcome the second great criticism which has been raised against Sraffa’s system, i.e. that of being only a ‘half-system’. Vertical hyper-integration is the tool allowing to put together Leontief’s concerns with the quantity, physical side of the production re-process and Sraffa’s concern with the price, value side.

### 3.2 Fixed coefficients and exogenous technical progress

After having considered the relation between Pasinetti and Sraffa, it is worth devoting some time to stressing the analogies between Pasinetti and Leontief. This will open up the way to clarify a methodological characteristic of the whole framework developed by Pasinetti which has not been grasped in its full relevance, and therefore has given rise to a series of criticisms.

Pasinetti himself points out
the similarity of approach, from an empirical point of view, of the previous dynamic (vertically [hyper-integrated]) analysis and the static input-output analysis. Both of them share the characteristic of being built on coefficients which are intended to represent actual outcomes and which can therefore [...] be given an empirical content, simply by recording the actual performance of an economic system. [...] The coefficients that appear both in the input-output analysis and in the present (vertically [hyper-integrated]) analysis must, therefore, be interpreted as representing those physical quantities which can actually be observed.

(Pasinetti 1981, pp. 109-110; emphases added)

This excerpt is of fundamental importance for the understanding of Pasinetti’s methodological approach. He departs from the very same statistical conception as Leontief. The coefficients appearing in the whole analysis are precisely, period after period, those magnitudes that can actually be observed and measured.

After acknowledging such analogy, one can therefore be tempted to criticise Pasinetti’s framework with the same arguments used to criticise Leontief, by saying that he takes fixed coefficients, and makes the implicit assumption of constant returns to scale, because dealing with changing coefficients according of the scale of output would not be possible, or would be too difficult.

But this is not what Pasinetti does; he did not do so in 1981, and he did not do so in 1988.

As explained at length both in Garbellini & Wirkierman (2010b, sections 3.2 and 3.5) and in Garbellini (2010b, section 5), Pasinetti uses a particular unit of measurement for intermediate commodities, i.e. the units of vertically hyper-integrated productive capacity. In this way, it is possible to deal with capital accumulation by simply studying the dynamics of the stock of units of productive capacity, leaving aside the issue of their changing physical composition. The two problems are therefore separated so that each one can be analysed independently of the other:

the notion of a physical unit of productive capacity, by being defined with reference to the commodity that is produced, continues to make sense, as a physical unit, whatever complications technical change may cause to its composition in terms of ordinary commodities. (Pasinetti 1973, p. 24)

Therefore, the fact that matrix $A$ is continuously changing through time is not disregarded. In each period, the specific matrix considered is the one that can be obtained from national accounts.\[10\]

\[10\]In the analytical formulation of the framework, both matrix $A$ and all the derived matrices ($H$, $M$, etc) are not dated so as not to make notation and calculations too complicated. But this does not entail any implicit assumption on the dynamic behaviour of inter-industry coefficients.
As to returns to scale, Pasinetti makes no specific assumption about them. The argument goes along the same line as above; it may well be that, were the gross quantities produced different, the coefficients would not be the same. In other words, it is not maintained that the coefficients we observe would be the same whatever the scale of production. Coefficient $a_{ij,t}$ does not represent the amount of commodity $i$ which, at time $t$, is necessary for the production of one unit of commodity $j$; it is the quantity of commodity $i$ that has actually been used, in period $t$, for producing each unit of the total quantity of commodity $j$ that has actually been produced. This is a crucial difference. We do not care about what could have happened in a different situation. We record what has actually happened, and measure the corresponding relevant magnitudes.

This argument brings about a further criticism concerning Pasinetti’s (1981) book specifically, but can be better replied by also considering the more general formulation provided by Pasinetti (1988) and attempted in Garbellini (2010b) and Garbellini (2010a). Such a criticism concerns the ‘feasibility’ of the $\varrho's$, i.e. of the rates of growth of labour productivity at the level of each vertically hyper-integrated sector:

I am not entirely convinced that it is legitimate to express technical progress generally in terms of reductions of the inputs to these integrated sectors [...]. For technical change takes place at the industry level so that the rates of productivity growth in the different integrated sectors can not be thought of as being independent of each other. Moreover, it is easy to see that rates of productivity growth which are arbitrarily assumed at the level of integrated industries do not necessarily correspond to feasible (positive) rates of productivity growth at the level of ‘ordinary’ industries.

(Schefold 1982, p. 549)

Besides the fact that, unfortunately, the rates of productivity growth can also be negative, Schefold is absolutely right in saying that “the rates of productivity growth in the different integrated sectors can not be thought of as being independent of each other”. But in Pasinetti’s framework, such rates are not “arbitrarily assumed at the level of integrated industries”; in the same way as all other derived magnitudes, they are computed from the actual rates of change of labour requirements at the industry level. Therefore, it is not necessary to ask ourselves about the feasibility of such rates. In Pasinetti’s (1981) simplified framework, of course, the rate of productivity growth in the production of the consumption commodities ($\varrho_i, i = 1, 2, \ldots, m$) and of the ‘capital goods’ ($\varrho_k, i = 1, 2, \ldots, m$) are industry-level ones, since the technique is such that each vertical hyper-integrated sector is made up by to industries, one producing the consumption commodity and the other producing the corresponding intermediate commodity. Only the rate of change of productivity at the level of the sector as whole ($\varrho', i = 1, 2, \ldots, m$) is a
derived magnitude, being a weighted average of $g_i$ and $g_k$. But as soon as we introduce the most general description of the technique, it is easy to see how $g_i'$ is the weighted average of the rate of change of labour requirements in all the industries constituting the economic system as a whole (see Garbellini 2010a, section 5).

### 3.3 Vertical (hyper-)integration: circular flow and empirical relevance

Closely connected to the topic discussed above, in section 3.2 we have a further criticism concerning vertical (hyper-)integration and its connection to empirical facts:

> For the analysis of structural change [...] the relevant question is: does one lose useful information with this vertical integration manoeuvre? Unfortunately, the answer is yes. (Taylor 1995, p. 700)

I completely disagree with Taylor’s (1995) conclusion. Also in this case, it is not an isolated opinion; it is not uncommon to hear reviewers objecting that with vertical (hyper-)integration the circular flow is lost — while using Sraffa’s subsystems it is preserved.

Both vertical integration and hyper-integration are ways of repartitioning economic activities in a specific way: according to the single commodities composing the whole net product (i.e. consumption and new investment commodities) in the former case, according to the single consumption commodities in the latter. In both cases, such a re-partition is effected through a linear transformation, that can be easily reverted and thus preserving, in both directions, all the original information, since “once we possess the inverse matrix, all relations between the two approaches at a given point of time take the form of one-to-one correspondences” (Pasinetti 1981, p. 115).

Inter-industry relations, therefore, are not disregarded. On the contrary: they are still considered in all their importance. Not only: with respect to traditional inter-industry analysis, they are considered in a more complex way, as not only direct, but also indirect — and in Pasinetti (1988) also hyper-indirect — relations are taken into account.

---

12 The vector of vertically integrated productive capacity “contains the series of heterogeneous commodities that are directly and indirectly required in the whole economic system to obtain one physical unit of commodity $i$ as a final good”. (Pasinetti 1973, p.5; emphasis added). Therefore they take into account the fact that (part of) the output of an industry is used by another industry as an input, and vice versa.
In the specific case of vertical integration — that, incidentally, is a way of formalising Sraffa’s subsystems; if it is maintained that these latter preserve the circular flow, it cannot be maintained that vertically integrated sectors do not — the linear transformation is effected through the Leontief inverse matrix.

For vertical hyper-integration, the procedure, from an algebraic point of view, is precisely the same, with the only difference that the matrix we use for the linear transformation is not \((I - A)^{-1}\) but \((I - H_{ci})^{-1}\) (with \(i = 1, 2, \ldots, m\); see Garbellini 2010b, section 4.2).

The [...] inverse matrix appears, therefore, as the linear operator which may be applied to an inter-industry classification of labour and capital goods, in order to reclassify them according to the new type of (vertically [hyper-]integrated) sectors.

In this way, each vertically [hyper-]integrated sector is reduced to one flow-input of labour and one stock-quantity of capital goods; or, more specifically, to one vertically [hyper-]integrated labour coefficient and to one vertically [hyper-]integrated unit of productive capacity. [...] Formally, the new coefficients are, therefore, derived concepts (derived from the consolidation of inter-industry coefficients) but they have a deeper economic meaning and possess [...] much more favourable characteristics for dynamic analysis.

(Pasinetti 1981, pp. 113-114)

The emphasis put by Pasinetti on the fact that vertically (hyper-)integrated coefficients are derived magnitudes with “a deeper economic meaning” brings about another remark made by Schefold in his review of Pasinetti’s (1981) book:

This does not mean that the concept of vertically integrated sectors is meaningless — on the contrary, it is very helpful —, but it illustrates the point that we have yet to examine the interdependence between different rates of productivity growth in integrated sectors and that the input/output structure retains its factual and conceptual priority over the derived concept of integration.

(Schefold 1982, p. 549)

In this respect, it is worth stressing that Leontief’s input-output model can be considered as the static counterpart of the vertically hyper-integrated framework. This means that there is no logical priority of one of them over the other: they are instruments to be applied to two different, complementary, problems:

Over time, and as the conditions of production and of consumption change (owing to technical progress, economies and diseconomies of scale, etc.) the inter-industry relations break down and become different from one moment to the next, so that a particular input-output table is needed for
each stage in the evolution of the economy under consideration. These tables can be compared, [...] but they cannot be analytically linked to one another [...]. The continuity in time is kept, on the other hand, at the vertically [hyper-]integrated level, where the relations which can be set up possess [...] a higher degree of autonomy. This means that the permanence of these relations in time is independent of technical change. In this context, the vertically [hyper-]integrated technical coefficients acquire a meaning of their own, independent of the origin of the single parts which compose them. The movements of these coefficients through time, and the various consequences thereof, can be investigated and followed as such. When more information is needed about the industrial structure at a particular point of time, the vertically integrated coefficients can be split and analysed into inter-industry coefficients particular to that point in time.

In this way, static input-output analysis and dynamic vertically [hyper-]integrated analysis appear as mutually complementary and completing each other. Inter-industry relations, referring to any particular point of time, represent a cross-section of the vertically [hyper-]integrated magnitudes, whose movements through time express the structural dynamics of the economic system.

(Pasinetti 1981, p. 117; second and third emphases added)

As to the factual priority of input-output relations, it would clearly not make sense to directly collect data on vertically hyper-integrated sectors instead of on industries, the latter being immediately observable and therefore easier to be recorded; whether to use a traditional input-output approach or the vertically hyper-integrated one, once again, depends on the kind of problems that we want to investigate.

Very closely connected to what we have been saying in the first part of the present section — i.e. how to fit actual data into a vertically hyper-integrated model, and the relation between Pasinetti’s and standard input-output analysis — there comes a further criticism, concerning the empirical relevance of vertically hyper-integrated analysis. It is maintained that vertically integrated and vertically hyper-integrated analyses have no empirical relevance, especially when dealing with technical change and productivity measures, because any conclusion drawn could be equally drawn by using the standard input-output model, the only difference being that the latter would have an immediately clear economic meaning, while a meaningful economic explanation of the former could hardly be given.

Having already stressed the complementarity of vertically (hyper-)integrated and traditional input-output analysis, a first hint at what the empirical relevance of the former is can be given by providing some examples of its application in the literature.
3.3.1 Changes in labour productivity

A very interesting example of the use of vertical (hyper-)integration in empirical analysis concerns the construction of productivity measures alternative to the traditional measures of multifactor productivity obtained from a neoclassical aggregate production function.

The literature concerning productivity measures and their empirical evaluation is very rich, starting from Solow’s (1957) well-known paper and the one side, and therefore from Pasinetti’s (1959) critique and further exchanges\footnote{See Pasinetti (1998) and Solow (1998).} on the other side:

There have been some attempts by economists to complete [evaluations of technical change] and to introduce capital into the picture, by making use of theoretical notions as the production function, but these attempts — in the writer’s opinion — have neglected an important characteristic of capital — that it is reproducible and that its process of production is also subject to technical change. It is my purpose in this paper to go into these problems. I shall try to give a short economic interpretation of technical change and suggest a procedure for evaluating it, with respect to all factors of production.

(Pasinetti 1959, p. 270; emphasis added)

This excerpt stresses an issue which is very important when dealing with changes in productivity: all the, reproducible, intermediate means of production are themselves subject to technical progress. Therefore, when measuring productivity changes, or changes in capital intensity, “the changes which occurred in the production of physical capital itself, i.e. the changes in productivity in capital goods industries” must be “explicitly taken into account” (Pasinetti 1959, p. 274).

Pasinetti’s (1959) paper opened up a line of research based on empirical applications trying to compute changes in labour productivity in vertically integrated terms, explicitly acknowledging for the role of vertically integrated sectors in taking into account technical progress not only in the very production of each final commodity, but also in the production of all the intermediate commodities used up during the production process itself. The contention is that the phenomenon of technical change, and its consequences on the economic system as a whole, cannot be adequately understood but by considering all its effects on the production process; not only direct, but also indirect ones.

Without going into details here, let me mention some works which adopt a standpoint connected with vertically integrated analysis: Gossling & Dovring (1966), Gupta & Steedman (1971), Gossling (1972), Rampa (1981), Rampa &

3.3.2 Analysis of tertiarisation

Another field of application of vertically integrated analysis has been the analysis of the so-called process of tertiarisation in advanced industrial economies. The great majority of applied papers studying this topic have their starting point in the work by Siniscalco (1982) and Momigliano & Siniscalco (1986).

The topic of tertiarisation started to gain popularity in Italy between the end of the ‘70s and the beginning of the ‘80s, when data suggested that the manufacturing industries were losing importance — in terms of produced output and employment — with respect to the services industries.

In the meanwhile, however, almost all great firms were bringing about a radical change in their productive structure, i.e. an outsourcing of all those activities other than the core one, that were previously vertically integrated and therefore carried out within the firm itself.

The coexistence of these two phenomena brought the authors to the conclusion that there was the possibility that at least part of the increased relative importance of services with respect to manufacture could be due to this process of outsourcing. In the most extreme of all hypotheses, the growth of tertiary activities could even be the result of an increase, rather than a decrease, in industrial activity, therefore requiring a greater and greater amount of — externalised — services. But even in the smoothest case, industry-level data could be misleading, producing an over-estimation of the phenomenon.

Performing the analysis in terms of vertically integrated sectors — or sub-systems — could allow the authors to overcome this bias, and thus to obtain a more precise idea of the relative changes of those two ‘macro-sectors’: the results of such an empirical study led to the conclusion that, in fact, the phenomenon had been strongly over-estimated by traditional, industry-level, analysis, and that tertiarisation was not a strong tendency of the Italian economy up to that time.

In order to carry out this empirical application, they made use of a linear operator that, being independent of prices, applied to a vector of whatever magnitude classified by industries — both in real and nominal terms — could transform it in a vector classified by vertically integrated sectors. Such a linear operator was originally developed by Gossling, first in a paper (Gossling & Dovring 1966) and then in a book (Gossling 1972).

The debate went on for some years. It started with Siniscalco’s (1982) article, analysing the productive structure of the Italian economy by industries and sectors,
and presenting the Gossling operator; it continued with Rampa’s (1985) paper on
the study of the industry and the services sectors in Italy in the period 1965-1983;
it was then channeled in a book, edited by Pasinetti himself, titled “Structural
Change in the Productive System. Integration between Industry and Service Sec-
tor”\footnote{This is my own translation of the title of the book which has not been edited in
English.} (Mutamenti strutturali del sistema produttivo. Integrazione tra industria e
settore terziario). The main chapter of the book was the essay by Momigliano
and Siniscalco (Momigliano & Siniscalco 1986), followed by a series of comments.
Among the others, there was a comment by Giorgio Rampa (on methodological
issues: Rampa 1986) and the authors’ reply (Siniscalco & Momigliano 1986).

This kind of vertically integrated analysis has been more recently resumed by
Montresor and Vittucci Marzetti, again for the study of tertiarisation, considering
various groups of OECD countries, with quite interesting results\footnote{See Montresor & Vittucci Marzetti (2006), Montresor & Vittucci Marzetti (2007a),
Montresor & Vittucci Marzetti (2007b) and Montresor & Vittucci Marzetti (2008).}

As it can be seen, Pasinetti’s (1973) formalisation of Sraffa’s subsystems has
been applied in a relatively small set of empirical works, concerning an even smaller
scope of problems. No doubt that traditional input-output models had a much
wider application. But the result of these few empirical exercises constitute a
clear example of how vertically integrated analysis can give different answers with
respect to those which would be obtained by using traditional, industry-level,
input-output models.

The potential fields of applications, however, are much more than these.
First of all, measures of changes in (labour) productivity can be computed
in terms of vertically hyper-integrated sectors also. As maintained elsewhere
(Garbellini & Wirkierman 2010a), the effects of technical progress on the pro-
duction process cannot be summarised by a single measure. A set of measures,
to be interpreted together in their reciprocal relation, are necessary for a com-
plete understanding of the phenomenon. In this respect, I think that having both
vertically integrated and hyper-integrated measures, with their decompositions in
direct, indirect, and for the latter, also hyper-indirect labour, can be useful for
providing a more complete picture of technical change through time.

The dynamics of technical progress, and thus of labour productivity, also in-
fluence international trade relations, according to what Pasinetti has called the
Therefore, once defined a satisfactory set of vertically integrated and vertically
hyper-integrated measures, the same kind of empirical exercise performed in Gar-
bellini & Wirikierman (2010a) for the case of Italy can be performed for different countries, or groups of countries — and for longer periods of time — in order to analyse the joint dynamics of labour productivity and the patterns of international trade.

Moreover, Pasinetti’s (1981) framework, and its analytical generalisation, can also be useful for the study of other aspects of technical progress, such as the process of capital accumulation and physical investment — due to the advantage, mentioned above, of allowing to study the dynamics of new investment, and thus of capital accumulation, independently of that of the composition of productive capacity — and the dynamics of capital intensity and degree of mechanisation characterising modern industrial systems.

To conclude, all issues related to dynamics can be fruitfully studied by using vertical hyper-integration: “over time, the input-output coefficients change and the inter-industry system breaks down. [...] Then it is only the vertically [hyper-]integrated model that allows us to follow the vicissitudes of the economic system through time” (Pasinetti 1981, p. 115). No doubt that there is much work to do in fully generalising the theoretical framework. I have tried to do a first step in Garbellini (2010b) and Garbellini (2010a); it is my conviction that an adequate way of introducing fixed capital into the picture, a complication that I have avoided in this first stage, is necessary to make the model better equipped to suit reality.\[16\]

### 3.4 Simplifying assumptions

The last two criticisms to Pasinetti’s (1981) framework that I want to consider here concern the adoption of simplifying assumptions regarding the description of the technique in use and the laws of movement of the relevant economic magnitudes, respectively.

Let us start from Pasinetti’s (1981) description of the technique in use. In the simplified setting of the book, there are $2 \times m$ produced commodities: $m$ consumption commodities and $m$ capital goods. Each consumption commodity $i$ ($i = 1, 2, \ldots, m$) is produced by means of labour and by a specific capital good $k_i$, which enters only that particular production process; i.e. the industry producing capital good $k_i$ provides inputs to the industry producing consumption commodity $i$ only. Capital goods are produced by means of labour alone.\[17\] Each vertically

\[16\] It is also worth saying that, unfortunately, it is quite difficult to find proper data from national accounts, be them from the various national statistical offices, from Eurostat, OECD, etc., especially concerning physical capital, with the necessary disaggregation.

\[17\] I am considering here the ‘intermediate case’. Pasinetti (1981) considers also a more complex case, in which capital goods are produced by means of labour and capital goods too. But such a complication does not change in a significant way the description of the
hyper-integrated sector $i$ is therefore made up by two industries ($i$ and $k_i$) that are constituent components of sector $i$ only.

Clearly, this is a crude simplification; the criticism often made is that there are no basic commodities, and no inter-industry relations. However, though being analytically very convenient, this assumption is also conceptually very easy to be generalised: each intermediate commodity $k_i$ can be thought of as a particular composite commodity, constituted by all the physical commodities actually produced in the economic system in different sectoral proportions. The set of intermediate commodities used up — directly, indirectly and hyper-indirectly — for the production of consumption commodity $i$ are a unit of (vertically hyper-integrated) productive capacity, and can be called capital (composite) commodity $k_i$.

The analytical generalisation follows straightforward: by eliminating these simplifying assumptions, the whole set of inter-industry relations is reintroduced into the picture. Each vertically hyper-integrated sector is made up by all the industries of the economic system, according to the inputs they provide for the production of the corresponding consumption commodity (see Pasinetti 1988, Garbellini 2010b).

By means of this generalisation, the input-output data coming from national accounts can be fitted into the model and used for empirical applications. All inter-industry relations are taken into account. Each vertically hyper-integrated sector is a growing subsystem “repeatedly going through the whole intricate pattern of inter-industry connections” (Pasinetti 1981, p. 110).

As to the movements through time of the relevant economic magnitudes assumed by Pasinetti (1981), the main argument behind the criticisms can again be summarised by an excerpt taken from Harris’s (1982) review of *Structural Change and Economic growth*:

Pasinetti makes good use of this idea [of the presence of a learning process] on the consumption side of his model. But he does not exploit the full potential on the production side, insofar as he assumes that technical change is a smoothly recurring process taking place at a constant (but non-uniform) rate in all sectors.

(Harris 1982, p. 38)

Pasinetti (1981), in sketching his *General multi-sector dynamic model* (Pasinetti 1981, Chapter V) assumes that time is continuous and that all relevant economic magnitudes, namely population, direct labour requirements, and demand technique in use; moreover, the intermediate case is the one Pasinetti himself considers at length, and it is my contention that it is the most convenient one. For details on this point, see Garbellini & Wirkierman (2010b).
coefficients, change through time exponentially (an assumption borrowed from Harrod 1948) at steady — though different from sector to sector — rates\[\text{footnote}18\].

The choice of assuming this kind of dynamic movements has been the object of criticisms. The core of the problem lies in the consideration of continuous time. Once this choice is made, it makes not much sense to take non-steady rates of growth. Continuous time was chosen for a matter of simplicity, since it allows to keep the dynamic analysis mathematically as simple as possible, and thus to focus attention on the aspects that Pasinetti (1981) wanted to stress. Introducing the complication of non-steady rates would have made things much more complicated, and therefore the choice of continuous time would have become pointless: “[a]ny other types of movements — continuous or discontinuous — may be hypothesised, though with some obvious complications”, (Pasinetti 2007, p. 285n).

The only consistent way of introducing non-steady rates of growth is that of reformulating the whole framework using discrete, rather than continuous, time, which is precisely what I have done in Garbellini (2010a). In this way, the rates of change of the above-mentioned economic magnitudes is different from time period to time period, and a whole series of further consideration can be made concerning dynamics. This clearly is a choice which becomes compulsory, so to speak, when one wants to perform empirical applications using this framework. National accounts data are discrete, not continuous, and the degree of realism — and therefore the possibility of fitting real data — improves if the analytical formulation is made in the same terms.

Before concluding, it is however worth stressing that the choices of the simplifying assumptions made by Pasinetti at that time had very clear reasons. The 1981 book was intended to be the exposition of a new framework for analysing “the dynamics of the wealth of nations”. The task was quite ambitious, especially when the great number of issues touched upon by Pasinetti (1981) is taken into account. It was therefore necessary to avoid all possible further complications, in order to make the basic idea and the main results of the book immediately understandable — even in this way, the accomplishment of this objective has not been an easy one. The following passage, though having been written for a different purpose, develops the argument much better than I could do:

\[\text{footnote}18\]Such rates of growth are not arbitrarily fixed, but simply considered as exogenous with respect to the kind of analysis which is carried out at the fundamental level. As such, they represent an equal number of degrees of freedom, that one can close by using actual data or trying to explain from a theoretical point of view. In principle, therefore, any theoretical or empirically consistent explanation of the behaviour of such rates of growth can be introduced at the ‘institutional level’. The meaning of the term ‘pre-institutional’, and therefore the scope of foundational analysis as opposed to that of institutional one, has been analysed and discussed in Garbellini & Wirkierman (2010b, section 2).
The economists of early centuries set themselves the rather ambitious task of studying economic reality in all its complexity, using, however, somewhat crude methods of analysis.

Today economists are more conscious of the complexity of real economic relationships and adopt the procedure of initially assuming a simplified economic system. This simplified economic system is, however, studied in a rigorous way, with analytical methods which, in principle at least, should leave no room for any ambiguity. It is only after having studied a simplified economic system that the attempt is then made to introduce, one at a time, more complex hypotheses. This procedure is of course followed in the present analysis.

(Pasinetti 1977, p. 35)

The task of studying a simplified economic system has been accomplished in an excellent way by Pasinetti (1981); the introduction of a first set of more complex hypotheses has been achieved by Pasinetti (1988). I hope to have been able to do a further step forward with Garbellini (2010b) and Garbellini (2010a). The remainder of the path is still awaiting for future research.
3 VERTICAL HYPER-INTEGRATION AND GROWING SUB-SYSTEMS
References


REFERENCES


REFERENCES


