The Single-Mindedness Theory: Micro-foundation and Applications to Social Security Systems

"We work in order to have leisure" (Aristotle)

Introduction

The participation in the labour force of older persons in the U.S. labour market has been steadily declining over the last century. If the labour force participation of men aged 65-69 was around 60 per cent in the 1950s, the same figure had fallen to 26 per cent in the 1990s (Diamond (1997)) (Figure 1). In many OECD countries, workers withdraw from the labour market well before the official retirement age. Eventually this long-term decline, associated with an increase in life expectancy, has led to a considerable increase in years spent in retirement. Also, Government expenditure on social security has been skyrocketing and so has the percentage of workers covered by the system (Figure 2). This situation risks becoming financially unsustainable over the next years, unless governments undertake structural reforms as suggested by many economists (see Feldstein and Liebman (2001) amongst the others).

Over the last few years, the economic literature has been trying to give plausible explanations for this strong change in old workers' lifestyles. According to an OECD survey (OECD (2005)) financial incentives embedded into public pensions and other assistance schemes pull old workers into retirement. Nevertheless, the OECD makes a distinction between *pull factors of retirement* and *push factors of retirement*. The former include all those financial benefits that incentivise workers to anticipate their retirement, whilst the latter refer to negative perceptions by old workers about their ability or productivity and to socio-demographic characteristics.

In this paper I will distance myself from the OECD's view, which considers financial benefits as a *pull factor* which reduces the amount of work. I suggest that preference of workers for leisure shapes the characteristics of modern social security systems. Thus, generosity of governments' transfers is not exogenously given but it is rather the effect of a precise political mechanism; this is driven by old workers who use their political power to obtain what they need to finance their retirement years.

To explain the early retirement phenomenon, I will use an overlapping generation model (OLG) which considers a society divided into two groups of workers: the old and the young. I will assume that there is political competition between two candidates who must choose effective marginal tax rates on labour in order to maximize the probability of winning elections.

The core assumption of the model is based on the idea of "single-mindedness", introduced by Mulligan and Sala-i-Martin (1999). They assumed that the old prefer leisure more than the young; this structure of preferences would explain why the old require (and eventually obtain) more generous transfers from the government and why social security expenditures have been increased so much in recent years. They adopted an OLG model where society is divided into old and young workers and showed that

> retired elderly can concentrate on issues that relate only to their age such as the pension or the health system

while the young have to choose amongst

> age-related and occupation issues

Eventually, they concluded,

> the elderly are politically powerful because they are more single-minded and (. . .) more single-minded groups tend to vote for larger social security programs that benefit them

According to this theory the group of old workers, because more single-minded, would have a greater leverage over politicians and they are more able to influence policy outcome (a sort of tyranny of the elder or "Gerontocracy", to quote authors).

Indeed, neither Demographics nor the need for assistance would explain the skyrocketing increase in the governments' expenditure for social security systems and the broad reduction in retirement age over the last decades, but preferences of the old for leisure would provide a more suitable explanation to this upward trend. In a recent work, Diamond (Diamond (2005)), attempting to describe the linkage between the social security system and the retirement in the U.S., wrote in his conclusions:

> there is clear evidence from both previous work (. . .) that the broad structure of the SS program influences retirement timing. Evidence on the effects of variation in the benefits provided by this program is less clear, however.

In particular, I will assume that the Government has to decide how to divide the revenues generated by the taxation of the two groups. In doing this, it exploits a balanced budget constraint which is based on (distortionary) labour income taxation. Eventually, I will demonstrate that the older generations obtain a higher tax credit (or a reduction in the effective marginal tax rate) than the younger generations and that they get a higher amount of leisure. This is a situation which is consistent with the old's needs, since their preferences are more oriented toward retirement than toward work. The work also explains the importance of the single-mindedness of social groups and the role of preferences of individuals in political competition. The more single-minded a group, the higher is its political power, captured by a density function which is assumed to be monotonically increasing in the level of leisure. Since more single-minded groups are, other things being equal, more politically powerful, they are more able to obtain favourable policies from political candidates in equilibrium.

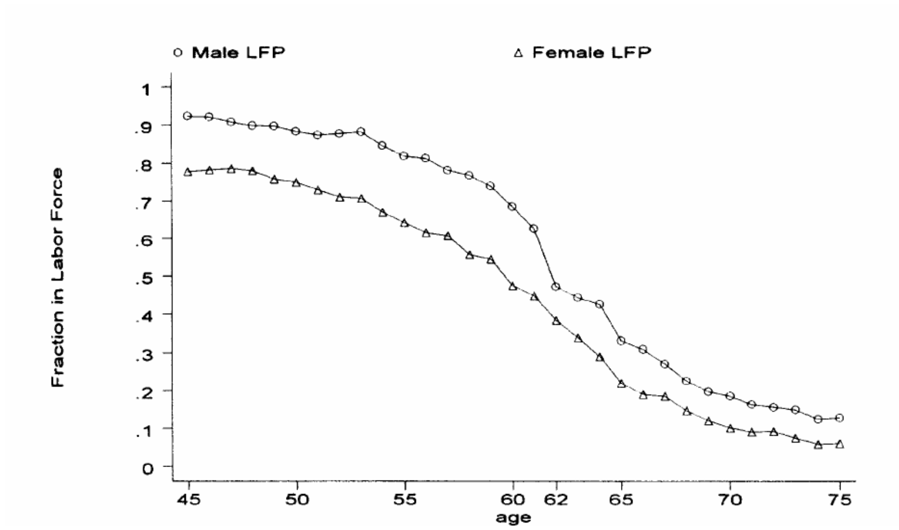

Figure 1 - Partecipation Rates by Age and Sex in the U.S.

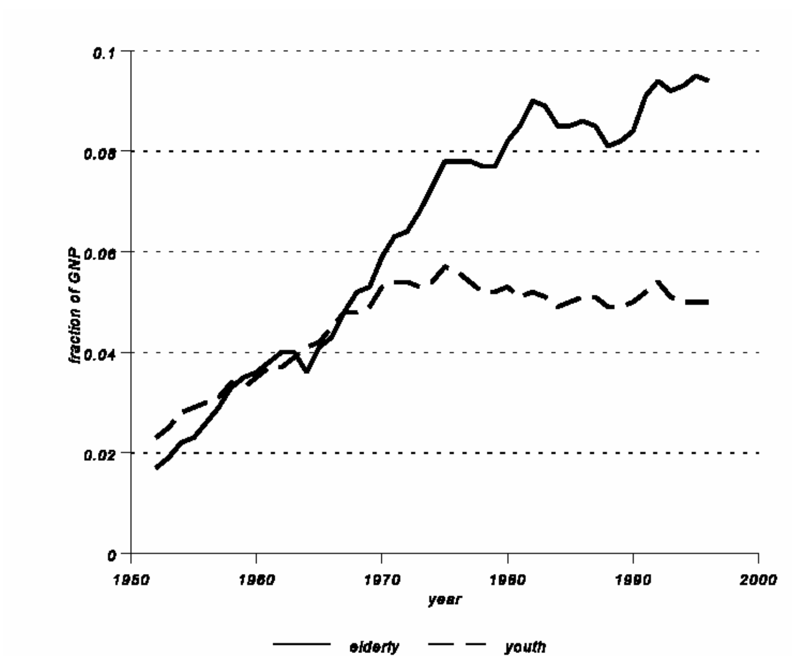

Figure 2 - Expenditures in social security programmes by cohort in the U.S.

The basic model

I consider an overlapping generation model, where each generation lives for two periods: *youth* and *old* age. At any period of time, the generation of youths coexists with the generation of the elderly. At the beginning of the next period, the elderly die, the young become old and a new generation of youths is born. As a consequence, there are two overlapping generations of people living at any one time. Generations are unlinked, meaning that for whatever reason, a generation does not leave any bequest to another generation. Individuals consume all the available income earned at a given period of time; thus, it is not possible either to save or to borrow money.

Then, at time $t = 0, ..., +\infty$, let a continuum of voters of size one be partitioned into two generations of workers $I = t-1, t$. The *old* represent the generation born at time $t-1$ and it is denoted by $t-1$ whilst the *young* represent the generation born at time t and it is denoted by t. The two generations have same size, which does not change over time[1]. A single worker is denoted by $i \in \left[0, \frac{1}{2}\right]$.

Each worker i has to decide how to divide his total endowment of time T between work, $L_t^i > 0$ and leisure, $l_t^i > 0$. If leisure is equal to the total endowment of time, I assume that the worker retires.

Every voter's welfare depends both on fiscal policies chosen by two political candidates $j = A, B$ which affect his consumption, known by both parties, and on personal attributes of candidates, which are only imperfectly observed by rivals. Both candidates have an ideological label (i.e. they are seen as Democrats or Republicans), exogenously given. In other words, I assume that individuals' preferences for consumption are perfectly observable, whilst other political features, such as ideology, are not (Linbeck and Weibull's *stochastic heterogeneity* (1987)). The deterministic component of workers' welfare is captured by a quasi-linear utility function in consumption and leisure, whilst the stochastic component is captured by the expression $\mathbf{D}^B \cdot (\xi_t^{i,I} + \zeta)$, where

$$\mathbf{D}^B = \begin{cases} 1 & if\ B\ wins \\ 0 & if\ A\ wins \end{cases}$$

The term $\zeta \lesseqgtr 0$ reflects B's general popularity amongst the electorate and it is only realized between the announcement of parties' policy vector and elections. It is not idiosyncratic, it is known by candidates, and it is uniformly distributed as

$$\zeta \sim U\left[-\frac{1}{2}, \frac{1}{2}\right]$$

with mean zero. Otherwise, the term $\xi_t^{i,I} \lesseqgtr 0$ represents an individual component of preferences for B. It is known by candidates and uniformly distributed as

$$\xi_t^{i,I} \sim U\left[-\frac{1}{2s^I}, \frac{1}{2s^I}\right]$$

with mean zero and density s^I.

A representative old worker at time t has the following utility function:

$$U_t^{i,t-1} = c_t^{t-1} + \psi^{t-1} \log l_t^{t-1} + \mathbf{D}^B \cdot (\xi_t^{i,t-1} + \zeta) \tag{1}$$

where c_t^{t-1} is consumption and $\psi^{t-1} \in [0,1]$ is a parameter representing the intrinsic preference of the old worker for leisure.

The old worker consumes all his income:

$$c_t^{t-1} = w^{t-1}(1 - \tau_t^{'t-1})(T - l_t^{t-1}) \tag{2}$$

where w^{t-1} is the unitary wage per hour worked, $\tau_t^{'t-1} := \tau\left(1 - a_t^{t-1}\right)$ the effective tax rate on labour income equal to the nominal tax rate $\tau \in \left[\tau^{\min}, \tau^{\max}\right]$ net of the tax credit $a_t^{t-1} \in \left[a_t^{t-1\min}, a_t^{t-1\max}\right]$, with $a_t^{t-1\min} < 1$ and $a_t^{t-1\max} > 1$.

I assume that τ is equal for every generation and steady over time. $\tau^{\min}$ and $\tau^{\max}$ denotes the minimum and maximum legal tax rates, whilst $a^{\min}$ and $a^{\max}$ the minimum and maximum tax credits, both written in a budget law.

Similarly, preferences of a representative young worker t are given by the following utility function:

$$U_t^{i,t} = c_t^t + \psi^t \log l_t^t + \beta(c_{t+1}^t + \psi^{t-1} \log l_{t+1}^t) + \mathbf{D}^B \cdot (\xi_t^{i,t} + \zeta) \tag{3}$$

subject to

$$c_t^t = w^t(1 - \tau_t^{'t})(T - l_t^t) \tag{4}$$

$$c_{t+1}^{t} = w^{t}(1 - \tau_{t+1}^{'t})(T - l_{t+1}^{t}) \tag{5}$$

where β is a discount factor and $a_t^t \in \left[a_t^{t\,\min}, a_t^{t\,\max}\right]$ the tax credit, with $a_t^{t-1\,\min} < 1$ and $a_t^{t-1\,\max} > 1$.

Conditions $a_t^{I\,\min} < 1$, $a_t^{I\,\max} > 1$ make a redistribution programme feasible because, as we will see later in studying the budget constraint of the Government, it allows a generation to obtain positive transfers which are paid by the other generation.

Definition of Single-Mindedness

I introduce two essential definitions:

Definition 1 *generation A is said to be more single-minded than generation B with respect to leisure if its marginal utility of leisure is greater than B's. That is if* $\psi^A > \psi^B$.

This definition states that generations are not focused (single-minded) on leisure in the same way. They attribute different weights to leisure and thus are less or more prone to substitute it with consumption goods. I will provide later some empirical results which demonstrate that a difference between the young and old for leisure exists.

Definition 2 *generation A is said to be more politically powerful than generation B if its density is higher than B's. That is if* $s^A > s^B$.

The political power of a generation is represented by its ability to influence candidates' choices, when they have to take decisions about the optimal policy vector. In traditional probabilistic voting models this power is expressed by a density function which captures the distribution of the constituency.

Axiom 3 *the density function of a generation is monotonically increasing with the level of leisure. That is* $s^I = s(l)$*, with* $\frac{\partial s}{\partial l} > 0$.

Note, that this axiom brings something new with respect to previous probabilistic voting models, where the density function was only a constant and did not depend on anything.

In the resolution it will be demonstrated that $l^I = l(\psi)$ and $\frac{\partial l}{\partial \psi} > 0$; that is, leisure in monotonically increasing in preferences for leisure. This result enable us to show that, *ceteris paribus*, *an increase in the single-mindedness of a generation entails an increase in its political power*. To demonstrate this, it is sufficient applying the chain rule to obtain $\frac{ds^I}{d\psi^I} = \overbrace{\frac{\partial s^I}{\partial l^I}}^{>0} \cdot \overbrace{\frac{\partial l^I}{\partial \psi^I}}^{>0} > 0$.

This result says that the linkage between preferences of a generation and its political power passes through an increase in the level of leisure which the density depends upon. In other words, it must be the case where over leisure, different generations have different preferences for political parties. A greater level of single-mindedness entails higher values of the density function which tends to give to the distribution a ticker shape.

Different preferences for leisure

I introduce an axiom which refers to a fundamental difference between the young and the old.

Axiom 4 *the old are more single-minded for leisure than the young; that is,* $\psi^{t-1} > \psi^t$.

This axiom is certainly strong but it is supported by robust empirical evidence. As a matter of fact, Economics has produced many works providing possible explanations of the existence of a difference in preferences. Besides, recently other social sciences like Sociology and Psychology have added very useful contributions. In summarizing the most important achievements, I will make a distinction between economic reasons and non-economic reasons.

The **economic reasons** are contained mainly in works by Mulligan and Sala-i-Martin (1999).

1. `Differences in labour productivity`. Since labour productivity declines with age, the old are less productive than the young and, as a consequence, they earn a lower wage. This theory would explain the willingness of the old to retire early: less productive workers in the labour market find it profitable to devote relatively more of their time and effort to political activity, in order to gain monetary transfers that they would not obtain if they relied on the labour market. Nevertheless, for the theory to hold it is important to assume that leisure devoted to political activities is a *normal good*. That is, an increase in total leisure time provokes an

increase in leisure devoted to political activities, due to the income effect. Of course these assumptions are not unanimously accepted in literature. In particular, evidence about the effects of age on productivity and wages does not lead to clear-cut conclusions. For example, a work by Skierbekk (2003) found that individual job performance decreases from around 50 years of age and that productivity reductions at older ages are particularly strong for work tasks where problem solving, learning and speed are needed, while in jobs where experience and verbal abilities are important, older individuals' maintain a relatively high productivity level.

2. `Differences in Human Capital Accumulation`. The young are more engaged in self-financed human capital accumulation, while they work, than the old. As a consequence, the value of time for the young may be higher than their average hourly wage (see Stafford and Duncan (1985)).

3. `Long-term employment contracts`. The empirical evidence shows that due to Lazear-type contracts, labour productivity for workers aged 60+ is significantly lower than wages.

As for the **non-economic reasons**, I refer to a work by Hershey, Henkens and Van Dalen (2006). In comparing the Dutch with the U.S. social security system, the authors discovered that "the Americans had significantly longer future time perspectives, higher level of retirement goal clarity and they tended to be more engaged in retirement planning activities". Thus, these findings are able to explain the existence of sociocultural differences in the preferences for retirement. They go on affirming that "American workers think, prepare and save more for retirement... beginning in early adulthood", focusing on differences between societies, where there exists a major difference in financial responsibility, different level of uncertainty for future pension payouts and different psychological pressures. Finally, in concluding that the success of political initiatives depends in part on "changing the dimensions of the psyche that motivate individuals to adaptively prepare for old age", they implicitly recognize that preferences of individuals for leisure may endogenously change over time, again due to cultural and psychological issues.

A graphical illustration

In order to ease the comprehension of the relation between single-mindedness and political power I provide a graphical illustration. Figure 3 shows an example of different distributions amongst cohorts.

The figure shows how distributions of generations depend on leisure and that the old generation (red) has a thicker distribution than the young generation (orange). The distribution is assumed to be uniform. The broadness of the interval $\left[-\frac{1}{2s^I}, \frac{1}{2s^I}\right]$ is changeable, because s is a monotonically increasing function of leisure, and higher levels of leisure increase s, reducing the broadness of the interval. As a result, we obtain a higher concentration of "swing voters", those voters who are indifferent to the two candidates, around ζ.

Figure 4 shows the effects of an increase in ψ in a generation. A change in ψ (from ψ to ψ', with $\psi' > \psi$) entails an increase both in l and s. Since s stands at the denominator of the expression representing the endpoints of the interval, the broadness of the interval reduces and the distribution becomes thicker.

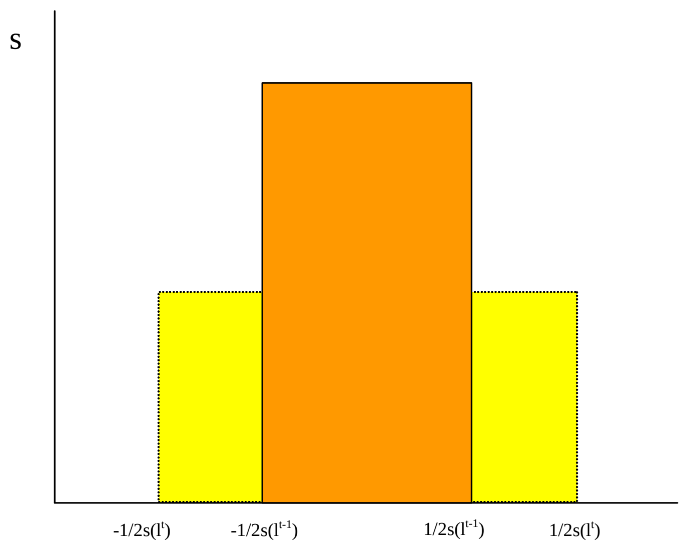

Figure 3 - Distribution functions of single-minded generations

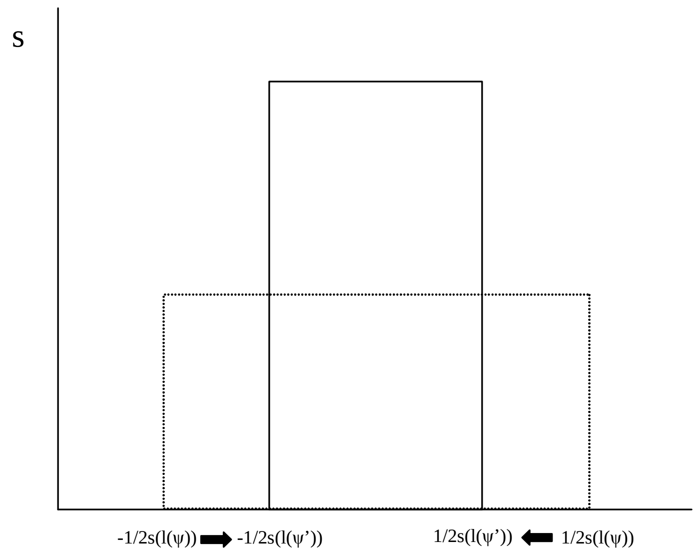

Figure 4 - Effects of a change in a generation' preferences on distribution

The Government

I consider two self-interested candidates $j = A, B$ who choose an element $\mathbf{q}_t^j = \left\{\tau_t^{'t-1j}, \tau_t^{'tj}\right\}$, encompassing the two effective tax rates $\tau_t^{'t-1j}$ and $\tau_t^{'tj}$, from the (common) strategy set $\mathfrak{Q}\subset \mathbb{R}^2$.

Furthermore, I introduce the budget constraint of the Government at time t:

$$\Upsilon_t^j := \frac{1}{2}\tau_t^{'t-1j}(T - l_t^{t-1})w_t^{t-1} + \frac{1}{2}\tau_t^{'tj}(T - l_t^t)w_t^t = 0 \tag{6}$$

where $\frac{1}{2}\tau_t^{'t-1j}(T - l_t^{t-1})w_t^{t-1}$ represents the total revenues collected by the taxation of the old and $\frac{1}{2}\tau_t^{'tj}(T - l_t^t)w_t^t$ the total revenues collected by the taxation of the young.[2]

Since revenues are proportional to the amount of labour, taxation entails inefficiencies, as it distorts workers' decisions on the amount of labour supplied.

As suggested by Lindbeck and Weibull (1987), I assume the existence of a *balanced-budget redistribution* where the government cannot redistribute more resources than those available in the economy, and cannot use tax revenues for any other purpose than redistribution – so that the condition $\Upsilon_t^j = 0$ says that revenues collected via labour taxation are only used to redistribute wealth amongst cohorts. To avoid the case in which a difference in wage levels is the sole reason for early retirement I assume that wages are equal for every generation: $w_t^{t-1} = w_t^t = w$. Furthermore, without loss of generality, I normalize the wage rate to the unity.

The advantage of adopting a budget constraint with distortionary taxation like 6 is realism. Economists like Profeta (2002) and Mulligan and Sala-i-Martin (1999) formalized models attempting to explain the linkage between inter-generational redistribution and early retirement; nevertheless, they seem to be affected by a fundamental problem due to the use of lump-sum transfers. In Mulligan and Sala-i-Martin "an interest group may tax its members with a labour income tax and distribute the proceeds to them in a lump-sum fashion"; Profeta used lump-sum taxation to transfer wealth both within and amongst cohorts. Finally, Linbeck and Weibull studied a redistribution model with political competition where gross incomes are fixed and known and, "first-best (individual) lump-sum redistributions are in principle feasible". Unfortunately, a redistributive system with the presence of lump-sum taxation does not exist in the real world. All the most recent studies on features of social security systems around the world show that the taxation on income is the only source of financing social expenditures. For instance, Diamond (2005) found out that "The Social Security system in the U.S. today is financed by a payroll tax which is levied on workers and firms equally", whilst Mulligan and Sala-i-Martin, adopting a cross-section analysis of 89 countries, recognized that the 96 per cent of social security programs are financed via payroll taxes.

The political competition

The Lindbeck and Weibull framework

As said before voters' welfare depends both on a deterministic and a stochastic component. The presence of uncertainty, captured by variables related to preferences for political nominees, guarantees the existence of a NE in a multi-dimensional space (see Lindbeck and Weibull (1987) and Dixit and Londregan (1994)). In the absence of uncertainty candidates would be perfectly able to observe how voters cast their ballots and then each voter would abruptly switch backing toward the candidate who promises him the most favourable policy. In such a case the non-existence of an equilibrium is due to the fact that any chosen policy would be beaten by another policy. Therefore, traditional Downsian electoral competition models lead to a negative result where no Condorcet winner exists. Probabilistic voting models, instead, smooth out this discontinuity because a small change in the policy chosen entails only a small change in the probability of backing from voters and not a total loss of backing. Smoothing out the discontinuity in the probability of winning reopens the possibility that an equilibrium returns.

Each voter i in generation I votes for candidate B if and only if B's policy vector provides him with a greater utility than that provided by candidate A's policy vector. That is i votes for B if and only if:

$$V^I(\mathbf{q}_t^B) + \zeta + \xi_t^{i,I} \geq V^I(\mathbf{q}_t^A) \qquad \forall i \tag{7}$$

where $V^I(\mathbf{q}_t^j)$ represents the indirect utility function which generation I obtains under the policy vector chosen by candidate j, $\mathbf{q}_t^j$.

The role of swing voters

In each generation there is a fraction of *swing voters* ι, represented by all of those individuals who are indifferent between voting for A or B. For these voters the condition 7 holds with equality:

$$\xi_t^{\iota,I} = V^I(\mathbf{q}_t^A) - V^I(\mathbf{q}_t^B) - \zeta \tag{8}$$

Otherwise, all voters i with $\xi_t^{i,I} < \xi_t^{\iota,I}$ vote for A and all voters with $\xi_t^{i,I} > \xi_t^{\iota,I}$ vote for B. Swing voters are pivotal, since even a small change in the policy vector may force them to vote for a candidate. Suppose we start from a situation where A's policy, $\mathbf{q}_t^A$, is exactly equal to B's policy, $\mathbf{q}_t^B$. A candidate knows that, should it move away from that policy, some swing voters would be better off (and vote for him) and some others would be worse off (and vote for the other candidate). Thus, in choosing a policy, a candidate must calculate the number of swing voters which he gains and compare it with the number of swing voters he loses; a change in policy should be made if and only if a candidate evaluates that the number of swing voters gained outweighs the number of swing voters lost. For example, suppose that A increases its transfer to group $t-1$ by a small amount, due to an increase in the tax credit. Each old worker receives $\frac{\partial \Upsilon_t^A}{\partial a_t^{t-1A}}$. But since a change in the tax credit modifies the level of leisure, and the density of the group accordingly, as a result the cut-off point represented by equation 8 modifies as well. The magnitude of the shift is equal to the extra consumption that an individual obtains multiplied by the marginal utility of consumption $\frac{\partial \Upsilon_t^A}{\partial a_t^{t-1A}}\left(\frac{\partial IUF_t^{t-1}}{\partial a_t^{t-1A}}\right)$, where IUF_t denotes the Indirect Utility Function. Nevertheless, since candidates must stay within the budget, candidate A increases the taxation of the other group by $\frac{\partial \Upsilon_t^A}{\partial a_t^{tA}}\left(\frac{\partial IUF_t^{t}}{\partial a_t^{tA}}\right)$. Therefore, A finds the shift in a policy which favours the group of the old to its advantage if and only if

$$\frac{\partial \Upsilon_t^A}{\partial a_t^{t-1A}}\left(\frac{\partial IUF_t^{t-1}}{\partial a_t^{t-1A}}\right)s^{t-1} \geq \frac{\partial \Upsilon_t^A}{\partial a_t^{tA}}\left(\frac{\partial IUF_t^{t}}{\partial a_t^{tA}}\right)s^{t} \tag{9}$$

This condition states that the group which is more ready to switch its vote in response to a change in policy is treated more favourably by political candidates.

I denote the expected share of votes for candidate A in generation I at time t by $\pi_t^{A,I}$:

$$\pi_t^{A,I} = \frac{1}{2}s^I[\xi_t^I + \frac{1}{2s^I}] = \frac{1}{2}s^I\xi_t^I + \frac{1}{4} \tag{10}$$

and substituting (8) into (10) I obtain:

$$\pi_t^{A,I} = \frac{1}{2}s^I[V^I(\mathbf{q}_t^A) - V^I(\mathbf{q}_t^B) - \zeta] + \frac{1}{4} \tag{11}$$

The total number of expected votes which A obtains must sum the expected number of votes of the two groups:

$$\pi_t^A = \overbrace{\frac{1}{2}s^{t-1}[V^{t-1}(\mathbf{q}_t^A) - V^{t-1}(\mathbf{q}_t^B) - \zeta] + \frac{1}{4}}^{\pi_t^{At-1}} + \overbrace{\frac{1}{2}s^{t}[V^{t}(\mathbf{q}_t^A) - V^{t}(\mathbf{q}_t^B) - \zeta] + \frac{1}{4}}^{\pi_t^{At}} \tag{12}$$

Notice that π_t^A is a random variable since it depends on ζ which is also random and how, *ceteris paribus*, an increase in B's general popularity amongst the electorate reduces π_t^A. Candidate A's probability of winning is simply the probability to obtain the simple majority of votes:

$$p_t^A\left(\mathbf{q}_t^A,\mathbf{q}_t^B\right)=\Pr[\pi_t^A\geq\frac{1}{2}]=\Pr[\sum_{I=t-1}^{t}s^I[V^I(\mathbf{q}_t^A)-V^I(\mathbf{q}_t^B)-\zeta]\geq 0]$$

Rearranging terms we obtain:

$$p_t^A\left(\mathbf{q}_t^A,\mathbf{q}_t^B\right)=\Pr[\pi_t^A\geq\frac{1}{2}]=\Pr[\sum_{I=t-1}^{t}s^I[V^I(\mathbf{q}_t^A)-V^I(\mathbf{q}_t^B)]\geq\ \zeta\sum_{I=t-1}^{t}s^I]$$

Denoting $\sum_I s^I=s$ and $\frac{1}{s}\sum_I s^I[V^I(\mathbf{q}_t^A)-V^I(\mathbf{q}_t^B)]=\widehat{\zeta}$ we obtain:

$$p_t^A\left(\mathbf{q}_t^A,\mathbf{q}_t^B\right)=\Pr[\pi_t^A\geq\frac{1}{2}]=\Pr[\zeta\leq\widehat{\zeta}]:=\mathcal{F}\left(\widehat{\zeta}\right)$$

where $\mathcal{F}\left(\widehat{\zeta}\right)$ denotes the comulative density function. Finally, we also take into account the distribution of the random variable ς to write a final expression for the probability of winning:

$$p_t^A\left(\mathbf{q}_t^A,\mathbf{q}_t^B\right)=\Pr[\pi_t^A\geq\frac{1}{2}]=[\widehat{\zeta}+\frac{1}{2}]$$

Candidate B wins with probability $p_t^B=1-p_t^A$.

Notice that $p_t^j\left(\mathbf{q}_t^A,\mathbf{q}_t^B\right)$ may be written as the sum of probability of winning with respect to each generation, weighted by the size of the generation, equal to $\frac{1}{2}$; that is $p_t^j\left(\mathbf{q}_t^A,\mathbf{q}_t^B\right)=\frac{1}{s}\left(\frac{1}{2}p_t^{j,t}\left(\mathbf{q}_t^A,\mathbf{q}_t^B\right)+\frac{1}{2}p_t^{j,t-1}\left(\mathbf{q}_t^A,\mathbf{q}_t^B\right)\right)$, where $p_t^{j,I}\left(\mathbf{q}_t^A,\mathbf{q}_t^B\right)$ denotes the probability of winning for candidate j for generation I. I will use this decomposition in the following propositions.

Each candidate maximizes the probability of winning[3]; that is a candidate wants either to maximize the expected margin of victory or to minimize the expected margin of loss, given the other candidate's policy vector.

We now have all the elements to define a two-person, constant-sum and symmetric game Γ where the two candidates $j=A,B$ are players, the two policy vectors $\mathbf{q}_t^j\in\mathfrak{Q}\subset\mathbb{R}^2$ the strategies and the probabilities of winning $p_t^j\left(\mathbf{q}_t^A,\mathbf{q}_t^B\right)$: $\mathfrak{Q}\times\mathfrak{Q}\longrightarrow\mathbb{R}$ the payoffs. Γ is written as $\left(\mathfrak{Q},\mathfrak{Q};p_t^A\left(\mathbf{q}_t^A,\mathbf{q}_t^B\right),p_t^B\left(\mathbf{q}_t^A,\mathbf{q}_t^B\right)\right)$. It is also useful to remind that in a two-person, constant-sum game a pair of policies $\left(\mathbf{q}_t^{A*},\mathbf{q}_t^{B*}\right)\in\mathfrak{Q}\times\mathfrak{Q}$ is an equilibrium if and only if it is a saddle point for the game

$$\Gamma = \left(\mathfrak{Q}, \mathfrak{Q}; p_t^A \left(\mathbf{q}_t^A, \mathbf{q}_t^B\right), 1 - p_t^A \left(\mathbf{q}_t^A, \mathbf{q}_t^B\right)\right)$$

Definition 5 *A Pair* $\left(\mathbf{q}_t^{A*}, \mathbf{q}_t^{B*}\right) \in \mathfrak{Q} \times \mathfrak{Q}$ *is called a (pure strategy) Nash equilibrium (NE) of* Γ *if and only if* $p_t^j \left(\mathbf{q}_t^A, \mathbf{q}_t^{B*}\right) \leqslant p_t^j \left(\mathbf{q}_t^{A*}, \mathbf{q}_t^{B*}\right) \leqslant p_t^j \left(\mathbf{q}_t^{A*}, \mathbf{q}_t^B\right)$, $\forall\ \mathbf{q}_t^A, \mathbf{q}_t^B \in \mathfrak{Q}$ *which satisfy the budget constraint.*

Timing of the game

The game has three stage. In the first the two candidates, simultaneously and independently, announce (and commit to) their policy vectors.

In the second stage elections take place. A candidate wins if and only if she obtains the majority of votes; in the case of a tie a coin is tossed in order to decree the winner. Finally, in the third stage, workers choose their leisure, given the level of tax credits chosen by the Government.

Calculate the equilibrium

I solve the game by backward induction, starting from the final stage.

A representative old worker solves the following optimization problem:

$$\max_{\left\{l_t^{t-1}\right\}} \ c_t^{t-1} + \psi^{t-1} \log l_t^{t-1} + \mathbf{D}^B \cdot \left(\xi_t^{i,t-1} + \zeta\right)$$

$$s.t.\ c_t^{t-1} = \left(1 - \tau_t^{\prime t-1}\right)\left(T - l_t^{t-1}\right)$$

The optimal amount of leisure which solves the problem is:

$$l_t^{t-1*} = \frac{\psi^{t-1}}{1 - \tau_t^{\prime t-1}} \tag{13}$$

Substituting (13) into (1) we obtain an expression for the Indirect Utility Function:

$V_t^{t-1} = T\left(1 - \tau_t^{\prime t-1}\right) - \psi^{t-1} + \psi^{t-1} \log \psi^{t-1} - \psi^{t-1} \log\left(1 - \tau_t^{\prime t-1}\right) + \mathbf{D}^B \cdot \left(\xi_t^{i,t-1} + \zeta\right)$

with $1 - \tau\left(1 - a_t^{t-1}\right) > 0 \Longrightarrow a_t^{t-1} > 1 - \frac{1}{\tau}$ for the existence of the logarithm.

I do the same for the representative young worker:

$$\max_{\{l_t^t, l_{t+1}^t\}} c_t^t + \psi^t \log l_t^t + \beta \left(c_{t+1}^t + \psi^{t-1} \log l_{t+1}^t\right) + \mathbf{D}^B \cdot \left(\xi_t^{i,t} + \zeta\right)$$

$$s.t. \;\; c_t^t = \left(1 - \tau_t^{'t}\right)\left(T - l_t^t\right)$$

$$c_{t+1}^t = \left(1 - \tau_{t+1}^{'t}\right)\left(T - l_{t+1}^t\right)$$

The resolution of the problem yields the optimal level of leisure at time t and $t+1$ and the Indirect Utility Function:

$$l_t^{t*} = \frac{\psi^t}{1 - \tau_t^{'t}} \tag{14}$$

$$l_{t+1}^{t*} = \frac{\psi^{t-1}}{1 - \tau_{t+1}^{'t}} \tag{15}$$

$$V_t^t = T\left(1 - \tau_t^{'t}\right) - \psi^t + \psi^t \log \psi^t - \psi^t \log\left(1 - \tau_t^{'t}\right) \tag{16}$$

$$+\beta\left(T\left(1 - \tau_{t+1}^{'t}\right) - \psi^{t-1} + \psi^{t-1} \log \psi^{t-1} - \psi^{t-1} \log\left(1 - \tau_{t+1}^{'t}\right)\right) + \mathbf{D}^B \cdot \left(\xi_t^{i,t} + \zeta\right)$$

Comparative statics shows that the optimal level of leisure is increasing in preferences of groups for leisure and decreasing in the amount of tax credits. That is $\frac{dl_t^{I*}}{d\psi^I} = \frac{1}{1-\tau_t^{'I}} > 0$ and $\frac{dl_t^{I*}}{da_t^I} = -\frac{\tau\psi^I}{\left(1-\tau_t^{'I}\right)^2} < 0$.

Analysing the indirect utility functions with respect to $\tau_t^{'I}$ we may notice that two effects coexist: a `tax effect`, $T\left(1 - \tau_t^{'I}\right)$, and a `leisure effect`, $-\psi^I \log\left(1 - \tau_t^{'I}\right)$.

What is the effect of an increase in the optimal tax credit on the wealth of an individual? At a glance, one would be likely to answer that an increase in tax credits increases the individual's utility because the effective marginal tax rate is reduced and the net-of-taxes labour income is increased. But leisure effect says that an increase in tax credits reduces the amount of leisure, and eventually increases the utility. Therefore, the total effect on the welfare of an individual depends on which effect prevails.

In the second stage of the game

Proposition 6 *the political equilibrium is a tie.*

Proof. Candidates $j = A, B$ solve the following problem:

$$\max_{\left\{a_t^{t-1j}, a_t^{tj}\right\}} \quad p_t^j \left(\mathbf{q}_t^A, \mathbf{q}_t^B\right)$$

$$s.t.\ \Upsilon_t^j = 0$$

$$a^{\min} \leq a_t^{Ij} \leq a^{\max}$$

The set of Kuhn-Tucker Conditions may be written as follows:

$$\begin{cases} \frac{\partial \Upsilon_t^A}{\partial a_t^{tA}} \lambda^A = \frac{\partial p_t^A}{\partial a_t^{tA}} \\ \frac{\partial \Upsilon_t^A}{\partial a_t^{t-1A}} \lambda^A = \frac{\partial p_t^A}{\partial a_t^{t-1A}} \end{cases} \tag{17}$$

$$\begin{cases} \frac{\partial \Upsilon_t^B}{\partial a_t^{tB}} \lambda^B = \frac{\partial p_t^B}{\partial a_t^{tB}} \\ \frac{\partial \Upsilon_t^B}{\partial a_t^{t-1B}} \lambda^B = \frac{\partial p_t^B}{\partial a_t^{t-1B}} \end{cases} \tag{18}$$

$$\Upsilon_t^j = 0 \tag{19}$$

$$-a_t^{Ij} \leq -a^{\min} \qquad \mu_1^j \geq 0 \qquad \mu_1^j \left(a_t^{Ij} - a^{\min}\right) = 0 \tag{20}$$

$$a_t^{Ij} - a^{\max} \leq 0 \qquad \mu_2^j \geq 0 \qquad \mu_2^j \left(a_t^{Ij} - a^{\max}\right) = 0 \tag{21}$$

where λ^A, λ^B are the two Lagrange multipliers which represent the *per capita marginal gain in expected votes* with respect to a marginal change in the policy made by candidates. In equilibrium λ^A must be equal to λ^B; namely, the per capita marginal gain in expected votes should be equalized between the two candidates. Suppose it is not; then, a candidate realizes that in changing her policy there is the possibility to obtain more votes than her rival and thus to win elections. As a consequence, there exists an incentive for her to increase transfers towards the generation which assures a greater increase in the expected number of votes; as long as this incentive persists an equilibrium cannot exist. ■

Conditions (17), (18) state that candidates choose tax credits up to the level where the marginal political cost (MPC), which represents the reduction in expected votes of raising an additional dollar, is equalized across cohorts. Hence, the political optimal structure is one which minimizes total political costs and clears the budget constraint. An example of political equilibrium is depicted in Figure 5.

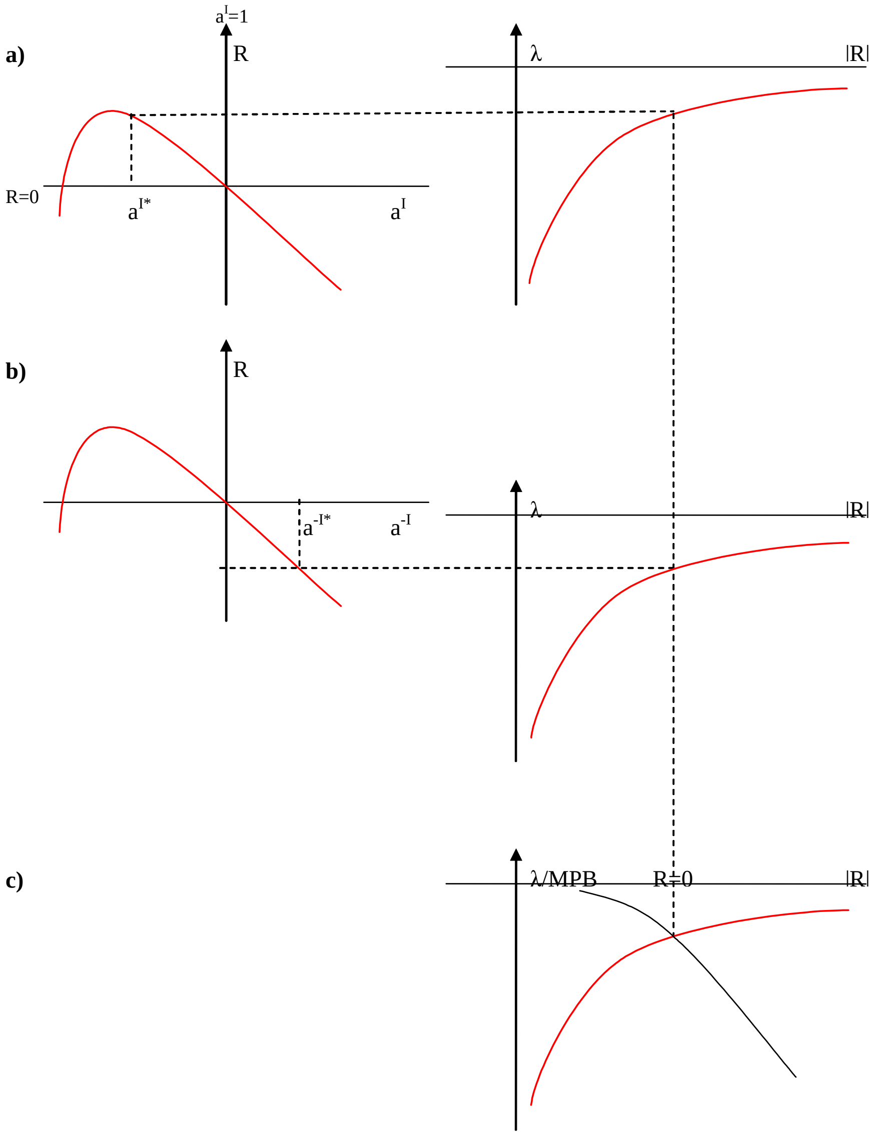

Figure 5 - Tax structure in a political equilibrium

Left-hand side graphs show revenues (vertical axis) as a function of tax credits (horizontal axis). The shape of the function reminds us of the famous "Laffer curve" or rate-revenue relationships. With respect to traditional Laffer curves, these ones have a negative segment; this is not surprising in a pure redistribution model, because if one generation obtains a positive transfer the other one must pay for it. Right-hand side graphs show the relation between Lagrange multipliers (vertical axis) and revenues (horizontal axis). Lambdas measure the intensity with which political tastes react to a change in full income by reducing expected backing. Different preferences for leisure and different economic and political reactions to taxation result in different tax rates. Finally, graph c shows the political equilibrium. The marginal political benefit (MPB) equates the sum of single MPBs expressed per dollar of expenditure. The equilibrium is represented by a point where the budget is cleared, $R_t^t + R_t^{t-1} = 0$, and the marginal political cost is equal to the marginal benefit, $MPC^* = MPB^*$. Nothing can be said about the concavity of p_t^j, due to the difficulties arising in evaluating the sign of the value function's second-order derivative[4].

Otherwise, Figure 6 shows a situation which cannot be an equilibrium.

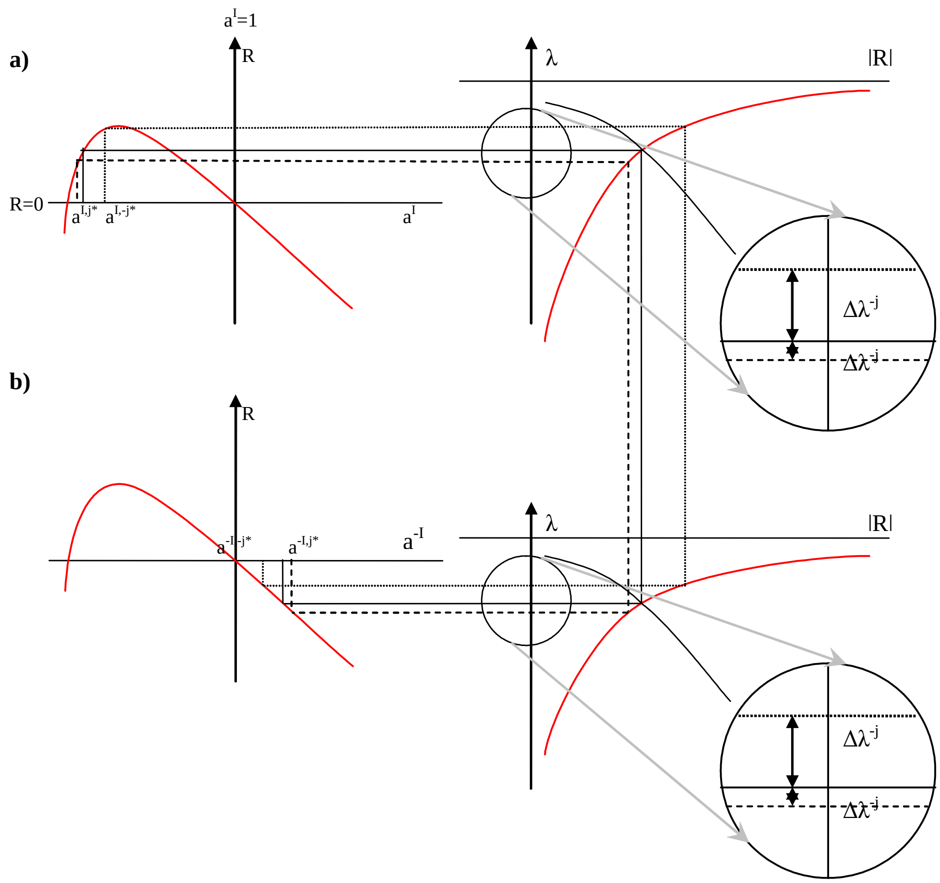

Figure 6 - Non-optimal policies

This time, the two candidates have chosen tax credits such that lambdas are not equalized. With respect to a situation where policies are convergent (solid line), candidate j (dashed line) has chosen a tax policy $\left(a_t^{Ij*}, a_t^{-Ij*}\right)$ and candidate $-j$ (dotted line) a tax policy $\left(a_t^{I-j*}, a_t^{-I-j*}\right)$. The circumscribed area shows that in this situation an increase in lambda by candidate j, $\Delta\lambda^j$, is greater than an increase in lambda by candidate $-j$, $\Delta\lambda^{-j}$; that is $\Delta\lambda^j > \Delta\lambda^{-j}$. But since we defined lambdas as the increase in probability of winning elections, it is clear how such a situation cannot be an equilibrium because j would obtain an increase in the probability of winning. As a consequence, $-j$ would have an incentive to mime j's policy. Therefore, the only possible equilibrium must be one where in moving away from a policy $\Delta\lambda^j = \Delta\lambda^{-j}$.

Corollary 7 *In equilibrium* $\widehat{\zeta} = 0$.

Proof. By proposition 6 the electoral equilibrium is a tie; then the probability of winning must be equal to $\frac{1}{2}$ for every candidate. Since $p_t^j = [\widehat{\zeta} + \frac{1}{2}]$, then $\widehat{\zeta}$ must be equal to zero. ■

In the first stage candidates choose optimal policy vectors which are obtained from the resolution of the maximization problem.

Proposition 8 *A tie in elections may be achieved (i) either if policies converge (ii) or if a policy chosen by one candidate favours one group and a policy chosen by the other candidate favours the other group.*

Proof. From Corollary 7 $\frac{1}{2s}\sum_I s^I[V^I(\mathbf{q}_t^{A*}) - V^I(\mathbf{q}_t^{B*})]$ is equal to zero. This may be achieved only in two ways. Either (i) when policies are convergent, $\mathbf{q}_t^{A*} = \mathbf{q}_t^{B*}$, which entails the equalization of the indirect utility functions $V^I(\mathbf{q}_t^{B*}) = V^I(\mathbf{q}_t^{A*})$; or (ii) when policies are divergent, $\mathbf{q}_t^{A*} \neq \mathbf{q}_t^{B*}$, and in this case the following condition must hold:

$$\frac{s^t}{s}[V^t(\mathbf{q}_t^{A*}) - V^t(\mathbf{q}_t^{B*})] + \frac{s^{t-1}}{s}[V^{t-1}(\mathbf{q}_t^{A*}) - V^{t-1}(\mathbf{q}_t^{B*})] = 0$$

which may be also written as:

$$\frac{s^t}{s}\left[V^t(\mathbf{q}_t^{A*}) - V^t(\mathbf{q}_t^{B*})\right] = \frac{s^{t-1}}{s}\left[V^{t-1}(\mathbf{q}_t^{B*}) - V^{t-1}(\mathbf{q}_t^{A*})\right]$$

■

Notice that:

1. if an equilibrium is achieved via a `policy convergence`, then it must be true that $p_t^{A,t} = p_t^{A,t-1} = p_t^{B,t} = p_t^{B,t-1} = \frac{1}{2}$.

2. if an equilibrium is achieved via a `policy divergence`, one of the following two statements must be true: (i) either $p_t^{A,t} = 1$, $p_t^{A,t-1} = 0, p_t^{B,t} = 0$, $p_t^{B,t-1} = 1$, (ii) or $p_t^{A,t} = 0$, $p_t^{A,t-1} = 1, p_t^{B,t} = 1, p_t^{B,t-1} = 0$.

Proposition 9 *if* $\mathbf{q}_t^A = \mathbf{q}_t^B = \mathbf{q}_t$ *then* $p_t^j(\mathbf{q}_t, \mathbf{q}_t) = \frac{1}{2}$.

Proof. Notice that if $\mathbf{q}_t^A = \mathbf{q}_t^B$, $V^{t-1}(\mathbf{q}_t^A) = V^{t-1}(\mathbf{q}_t^B)$ and $V^t(\mathbf{q}_t^A) = V^t(\mathbf{q}_t^B)$ and thus the probability of winning for the two candidates for generations I is equal to $\frac{1}{2}$. Since every generation has size equal to $\frac{1}{2}$, it must be $p_t^j(\mathbf{q}_t, \mathbf{q}_t) = \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{2}$. ■

The problem is now to evaluate whether the equilibrium of the model is achieved via a convergence or a divergence of policies. I will provide a sufficient (but not necessary) condition which assures that an equilibrium is achieved via policy convergence. Instead, note that the classic Lindbeck and Weibull's `monotonicity condition` for the policy convergence in probabilistic voting models is not applicable and thus cannot be used in this setting. Appendix 1 demonstrates the non-applicability of this condition.

Proposition 10 *In a constant-sum game* $\mathbf{q}_t^{A*} = \mathbf{q}_t^{B*} = \mathbf{q}_t^*$.

Proof. First of all, we have defined Γ as a constant-sum game, since $p_t^B\left(\mathbf{q}_t^A, \mathbf{q}_t^B\right) = 1 - p_t^A\left(\mathbf{q}_t^A, \mathbf{q}_t^B\right)$. Suppose now that the pair $(\mathbf{q}_t^{A\diamond}, \mathbf{q}_t^{B\diamond}) \in \mathfrak{Q} \times \mathfrak{Q}$ is the electoral equilibrium of the game. Suppose also that $\mathbf{q}_t^{A\diamond} \neq \mathbf{q}_t^{B\diamond}$. We know by (9) that $p_t^A\left(\mathbf{q}_t^{B\diamond}, \mathbf{q}_t^{B\diamond}\right) = \frac{1}{2}$. Therefore, by the definition of Nash Equilibrium it must be

$$p_t^A\left(\mathbf{q}_t^{A\diamond}, \mathbf{q}_t^{B\diamond}\right) > p_t^A\left(\mathbf{q}_t^{B\diamond}, \mathbf{q}_t^{B\diamond}\right) = \frac{1}{2} \tag{22}$$

By definition of constant-sum game we also know that $p_t^B\left(\mathbf{q}_t^{A\diamond}, \mathbf{q}_t^{A\diamond}\right) = 1 - p_t^A\left(\mathbf{q}_t^{A\diamond}, \mathbf{q}_t^{A\diamond}\right) = \frac{1}{2}$ and again by definition of Nash Equilibrium, it must be

$$p_t^B\left(\mathbf{q}_t^{B\diamond}, \mathbf{q}_t^{A\diamond}\right) > p_t^B\left(\mathbf{q}_t^{A\diamond}, \mathbf{q}_t^{A\diamond}\right) = \frac{1}{2} \tag{23}$$

Since $p_t^B\left(\mathbf{q}_t^{B\diamond}, \mathbf{q}_t^{A\diamond}\right) = 1 - p_t^A\left(\mathbf{q}_t^{B\diamond}, \mathbf{q}_t^{A\diamond}\right)$, this implies that $p_t^A\left(\mathbf{q}_t^{B\diamond}, \mathbf{q}_t^{A\diamond}\right) < \frac{1}{2}$. By 22, this implies that $p_t^A\left(\mathbf{q}_t^{A\diamond}, \mathbf{q}_t^{B\diamond}\right) > \frac{1}{2}$, a contradiction. Therefore, $\mathbf{q}_t^{B\diamond} = \mathbf{q}_t^{B\diamond}$. ■

Summarising, in this model the equilibrium is achievable via a convergence of policies even if the Lindbeck and Weibull monotonicity condition is not applicable. The Nash equilibrium of the game is

$$\left(\mathbf{q}_t^*, \mathbf{q}_t^*; \frac{1}{2}, \frac{1}{2}\right)$$

The optimal tax credit is a function of the density of both groups, of the nominal marginal tax rate, of the total endowment of time and of preferences of groups for leisure[5]. That is:

$$a_t^{Ij} = a\left(s\left(l\left(\psi^I, \tau\right)\right), s\left(l\left(\psi^{-I}, \tau\right)\right)\tau, T, \psi^I, \psi^{-I}\right)$$

In Mathematical Appendix 2 I will provide a complete resolution to the problem.

Analysing results we may see that this political economy framework suggests that tax rates should be differentiated. Indeed, if a traditional normative approach suggests that a benevolent government *should tax* the poorest groups less, this political economy outline suggests that in real world vote-seeker governments *tax* groups according to their ability to threaten politicians in an electoral competition.

An analytical solution to the maximization problem of the first stage is impossible to find because it is a hard task to understand which shape the value function has. Nevertheless, we know that since Q is a compact set and the value function is continuous in $\left[a_t^{I\min}, a_t^{I\max}\right]$ the `Weierstrass theorem` ensures that a maximum exists. Then, it only remains for us to understand whether the optimum is an interior solution or stands at one (or both) endpoint(s) of the interval.

If the maximum is an interior solution, it must come from the resolution of the first order conditions (see Appendix 2) which finds all the stationary points. Otherwise,

Proposition 11 *If the maximum is an endpoint solution, then the NE is*

$$\max\left\{\left(a_t^{t\min A}, a_t^{t-1\max A}; a_t^{t\min B}, a_t^{t-1\max B}\right), \left(a_t^{t-1\min A}, a_t^{t\max A}; a_t^{t-1\min B}, a_t^{t\max B}\right)\right\}$$

Proof. In order to balance the budget constraint, if the marginal tax rate of a generation is greater than one, the marginal tax rate of other generation must be lower than one; otherwise the sum of two positive tax revenues could never be equal to zero. Since $a_t^{I\min j} < 1$ and $a_t^{I\max j} > 1$, solutions such as $\left(a_t^{t\min A}, a_t^{t-1\min A}; a_t^{t\min B}, a_t^{t-1\min B}\right)$ or $\left(a_t^{t\max A}, a_t^{t-1\max A}; a_t^{t\max B}, a_t^{t-1\max B}\right)$ are not clearly achievable. Therefore, we must conclude that the only possible solution is

$$\max\left\{\left(a_t^{t\min A}, a_t^{t-1\max A}; a_t^{t\min B}, a_t^{t-1\max B}\right), \left(a_t^{t-1\min A}, a_t^{t\max A}; a_t^{t-1\min B}, a_t^{t\max B}\right)\right\}$$ [6]

■

This proposition has an important meaning. It says that, if an internal solution is not achievable, candidates must favour a generation and penalize the other generation as much as they can, by choosing the highest and the lowest tax rates in the set of common strategies.

As I stated before, due to the complexity of the system of equations we cannot rely on an analytical solution to the first stage of the game. Therefore, I introduce some conjectures about the equilibrium of the game and I verify them performing numerical simulations (see Appendix 3).

Conjecture 12 *Tax credits are higher for the older generations.*

Proof. result obtained via numerical simulations. ■

Conjecture 13 *The older generations offer either a very low level of labour or retire at all, depending on the values which parameters assume, whilst the younger generations offer a greater amount of labour.*

Proof. result obtained via numerical simulations. ■

Conjecture 14 *Tax revenues collected via labour taxation of the younger generations are positive, whilst those of the older generations are negative.*

Proof. result obtained via numerical simulations. ■

Analysing results of numerical simulation we may conclude that a fiscal system where self-interested governments maximize the probability of winning induces the old to retire early. In order to facilitate early retirement, revenues raised from the taxation of the old are negative and, on the contrary, revenues raised from the taxation of the young are positive and equal to the amount of transfers that the old receive. Thus, in this model there exists a net transfer of resources from the younger to the older generation, suggesting that the former carry the entire burden of social security systems, whilst the latter are net beneficiaries.

A variant with altruism

The simple model described above is able to explain the very negative phenomenon of early retirement. It depicts an economic environment where politicians are captured by most strongly focused single-minded groups. As long as candidates are self-interested and only aim to win elections, this political failure magnifies labour markets failures. Of course this cannot be optimal for society, especially considering the effects on inter-generational equity: older generations are net beneficiaries, whilst younger generations net payers. Is there any possibility of mitigating this uneven situation? As long as the old are selfish and only aim to maximise their welfare a solution which increases the young's welfare is hardly achievable. Otherwise, I suggest that altruism offers a rationale against early retirement. Altruism is seen as a change in preferences by the old which this time pays attention to the young's needs. This change in preferences should lead to a more equitable equilibrium.

In this chapter I consider a model where the old workers internalise their offspring's wealth. A classical altruistic model considers that households can be represented by a dynasty who perpetuates forever. As a consequence, the old internalize the utility function of the young. Hence, the new utility function of the old may be written as:

$$U^{t-1} = c_t^{t-1} + \psi^{t-1} \log l_t^{t-1} + \sigma U^t \tag{24}$$

where $\sigma \in [0, 1]$ is a parameter which captures the degree of altruism of the old for the young; the higher σ the more the old assign a greater importance to the young's wealth. Under this new framework, we should expect that policies chosen by the government become less burdensome on the young, since the old are now prone to share the onus of the system.

Conjecture 15 *with respect to the basic model, tax credits for the old (young) are lower (higher) and inter-generational transfers from the young to the old are reduced.*

Proof. result obtained via numerical simulations. ■

Conclusions

I introduced a very simple probabilistic voting model where the generation of the old is more-single minded than the generation of the young; that is, the former has greater preferences for leisure than the latter. This enables this group to be more politically powerful in a political competition between two candidates who have to choose effective marginal tax rates on labour in order to maximise the probability of winning elections. The equilibrium of the game is one where the two candidates even out and policies are convergent; this could be achieved via an internal or a corner solution, depending on the concavity/convexity of the value function. Simulations show that the old generation obtains higher tax credits, higher levels of leisure and positive inter-generational transfers. Therefore, the young are worse-off and they bear the burden of social security systems. Altruism may reduce this uneven redistribution scheme, lightening the excess pressure on younger generations. Inter-generational pacts could represent a possible solution to the early retirement problem, forcing the old to internalize the welfare of the young in order to make them share the entire burden of social security systems.

Mathematical Appendix 1

Proposition 16 *(Monotonicity condition) Assume (i) V_t^i is concave in q_t^j (ii) for each group and candidate $\frac{\frac{\partial p_t^j}{\partial a_t^{Ij}}}{\frac{\partial \Upsilon_t^j}{\partial a_t^{Ij}}}$ is strictly monotonic. If $\left(\mathbf{q}_t^{A*}, \mathbf{q}_t^{B*}\right)$ is a pure strategy electoral equilibrium, then $\mathbf{q}_t^{A*} = \mathbf{q}_t^{B*}$.*

Proof. From Proposition 6 we know that λ^j and λ^{-j} must be equal for every generation. This entails that the ratio between the two Lagrange multipliers of different candidates must be equal for every generation as well. I call this ratio $\rho^I := \frac{\lambda^{jI}}{\lambda^{-jI}}$. The problem is to assess whether this condition may be achieved under a divergence or a convergence of policies. To prove this, I start assuming that $\mathbf{q}_t^j \neq \mathbf{q}_t^{-j}$. Since candidates must clear the balanced-budget constraint, there must exist a generation which gets higher tax credits under candidate

j (suppose it is t) and another generation which gets higher tax credits under candidate $-j$ $(t-1)$. We have to assess whether the condition $\rho^t = \rho^{t-1}$ is achievable in such a situation. If it is, then an equilibrium is achieved under divergent policies; otherwise, policies are convergent. Notice that if both the numerator and the denominator are monotonic, the ratio is monotonic. If so, it means that (i) either $\rho^t > 1 > \rho^{t-1}$ or (ii) $\rho^{t-1} < 1 < \rho^t$; therefore, an equilibrium cannot be achieved via divergent policies. ■

In this model, and more in general in models where the direct utility function is quasi-linear in the consumption and leisure, the monotonicity condition cannot be applied to solve the candidate's problem. The failure of the monotonicity condition may have several implications. First of all, the possibility that the equilibrium is not achievable via a convergence of policies. Secondly, and more importantly, the convexity of V_t^i means that a maximum does not exist, since the value function is not concave.

(i) Convexity of V_t

Write the worker problem where the direct utility function is quasi-linear in consumption and leisure:

$$\max_{\{l_t\}} U_t = c_t + \psi \log l_t$$

subject to the budget constraint

$$c_t = \tau \left(1 - a_t^j\right)(T - l_t), l_t > 0$$

The optimal leisure is $l_t^* = \frac{\psi}{1-\tau\left(1-a_t^j\right)}$. Obtain the indirect utility function $V_t = T(1-\tau\left(1-a_t^j\right)) - \psi + \log\left(\frac{\psi}{1-\tau\left(1-a_t^j\right)}\right) = T(1-t) - \psi + \psi\log\psi - \psi\log w - \psi\log\left(1-\tau\left(1-a_t^j\right)\right)$. Define the constant term $A := T - 1 + \psi\log\psi$, substitute and obtain $V_t = A - \tau\left(1-a_t^j\right)T - \psi\log\left(1-\tau\left(1-a_t^j\right)\right)$. Write the first order condition

$$\frac{\partial V_t}{\partial a_t^j} = T\tau - \frac{\psi\tau}{1-\tau+\tau a_t^j} = 0$$

Note that there exist only a stationary point, $a_t^{jo} = 1 - \frac{\psi - T}{\tau\psi}$. Write the second order condition

$$\frac{\partial^2 V_t}{\partial\left(a_t^j\right)^2} = \frac{\psi\tau^2}{\left(1-\tau+\tau a_t^j\right)^2} > 0$$

That is, V_t is a convex function and $a_t^{jo} := \arg\min\left(V_t\right)$.

(ii) Non-monotonicity

Impose the ratio $\rho^I = \frac{\lambda^{jI}}{\lambda^{-jI}}$ to be equal to one, subtract the denominator from the numerator and verify whether the expression has a definite sign. Denoting $z_t = V^I(\mathbf{q}_t^j) - V^I(\mathbf{q}_t^{-j})$ we get the following:

$$\overbrace{\left(\frac{\frac{\partial}{\partial a_t^{tj}}\left(\frac{1}{s}\right)}{\frac{\partial \Upsilon^j}{\partial a_t^{tj}}} - \frac{\frac{\partial}{\partial a_t^{t-j}}\left(\frac{1}{s}\right)}{\frac{\partial \Upsilon^j}{\partial a_t^{t-j}}}\right)\sum_I \frac{1}{2} s^t z_t +}^{A} \tag{25}$$
$$\overbrace{+\frac{1}{s}\left(\left(\frac{\frac{\partial s^t}{\partial a_t^{tj}}}{\frac{\partial \Upsilon^j}{\partial a_t^{tj}}} - \frac{\frac{\partial s^t}{\partial a_t^{t-j}}}{\frac{\partial \Upsilon^j}{\partial a_t^{t-j}}}\right) z_t + \left(\frac{\frac{\partial V_t^t}{\partial a_t^{tj}}}{\frac{\partial \Upsilon^j}{\partial a_t^{tj}}} - \frac{\frac{\partial V_t^t}{\partial a_t^{t-j}}}{\frac{\partial \Upsilon^j}{\partial a_t^{t-j}}}\right) s^t\right)}^{B}$$

for generation t, and

$$\overbrace{-\left(\frac{\frac{\partial}{\partial a_t^{t-1j}}\left(\frac{1}{s}\right)}{\frac{\partial \Upsilon^j}{\partial a_t^{t-1j}}} - \frac{\frac{\partial}{\partial a_t^{t-1-j}}\left(\frac{1}{s}\right)}{\frac{\partial \Upsilon^j}{\partial a_t^{t-1-j}}}\right)\sum_I \frac{1}{2} s^{t-1} z_t +}^{C} \tag{26}$$
$$\overbrace{+\frac{1}{s}\left(\left(\frac{\frac{\partial s^{t-1}}{\partial a_t^{t-1j}}}{\frac{\partial \Upsilon^j}{\partial a_t^{t-1j}}} - \frac{\frac{\partial s^{t-1}}{\partial a_t^{t-1-j}}}{\frac{\partial \Upsilon^j}{\partial a_t^{t-1-j}}}\right)(-z_t) + \left(\frac{\frac{\partial V_t^{t-1}}{\partial a_t^{t-1j}}}{\frac{\partial \Upsilon^j}{\partial a_t^{t-1j}}} - \frac{\frac{\partial V_t^{t-1}}{\partial a_t^{t-1-j}}}{\frac{\partial \Upsilon^j}{\partial a_t^{t-1-j}}}\right) s^{t-1}\right)}^{D}$$

for generation $t-1$.

By the meaning of Proposition 6 A and C are equal to zero. Thus we have to verify that B and D are monotonic. Notice that as demonstrated before $\frac{\partial V_t^t}{\partial a_t^{tj}}$ is not monotonic and that $\frac{\partial \Upsilon^j}{\partial a_t^{tj}}$ is not monotonic either.

$sign(z_t)$ changes according to the interval where tax credits find. Denoting by $a_{|_{V_t=0^+}}$ and $a_{|_{V_t=0^-}}$ points where the V_t intersects the axis[7] (respectively at the right and at the left hand side) representing the tax credit we may easily see that 6 cases to study arise:

$$a_t^{-j+} < a_t^{j+} < a_{|_{V_t=0}} \Longrightarrow z_t < 0$$

$$a_t^{-j+} < a_{|V_t=0} < a_t^{j+} < a_t^{jo} \Longrightarrow z_t < 0$$

$$a_{|_{V_t=0}} < a_t^{-j+} < a_t^{j+} < a_t^{jo} \Longrightarrow z_t > 0$$

$$a_{|V_t=0} < a_t^{-j+} < a_t^{jo} < a_t^{j+} < a_{|V(a^{-j})<V(a^j)} \Longrightarrow z_t > 0$$

$$a_{|V_t=0^+} < a_t^{-j+} < a_t^{jo} < a_{|V(a_t^j)<V(a_t^{-j})} < a_t^{j+} < a_{|V_t=0^-} \Longrightarrow z_t > 0$$

$$a_{|V_t=0^+} < a_t^{-j+} < a_t^{jo} < a_{|V(a_t^j)<V(a_t^{-j})} < a|_{V_t=0^-} < a^{j+} \Longrightarrow z_t > 0$$

We study the sign of expression (25) and (26). Since the $sign\left(z_t\right)$ is discontinuous, $\frac{\partial V_t^j}{\partial a_t^{Ij}}$ and $\frac{\partial \Upsilon_t^j}{\partial a_t^{Ij}}$ are not monotonic, the sign of the expression is not clear and thus we cannot say *a-priori* whether the monotonicity condition holds. As a consequence the Lindbeck and Weibull's monotonicy condition may not be exploited in this model to demonstrate that an equilibrium is only achievable via a convergence of policies.

Mathematical Appendix 2

In this Appendix I provide a complete resolution to the candidates' problem when the equilibrium is internal. The two candidates face exactly the same optimization problem, maximizing the probability of winning.

$$\max_{\left\{a_t^{tj}, a_t^{t-1j}\right\}} p_t^j = \frac{1}{2} + \frac{1}{2s} \sum_{I=t-1}^{t} s^I \left[V^i\left(\mathbf{q}_t^j\right) - V^i\left(\mathbf{q}_t^{-j}\right)\right]$$

$$\Upsilon_t^j := \frac{\tau}{2} \sum_{I=t-1}^{t} \left(T - l_t^I\right)\left(1 - a_t^{Ij}\right) = 0$$

$$a^{\min} \le a_t^{Ij} \le a^{\max}$$

I write the Lagrangian function for candidate j:

$$\mathcal{L}^j = \frac{1}{2} + \frac{1}{2s} \sum_{I=t-1}^{t} s^I \left[V^i\left(\mathbf{q}_t^j\right) - V^i\left(\mathbf{q}_t^{-j}\right)\right] - \lambda^j\left(\Upsilon_t^j\right) - \mu_1^j\left(a^{\min} - a_t^{Ij}\right) - \mu_2^j\left(a_t^{Ij} - a^{\max}\right)$$

Deriving the Lagrangian I obtain Kuhn-Tucker conditions:

$$\begin{cases} (1)\ \frac{\partial \mathcal{L}^j}{\partial a_t^{t-1j}} := \frac{\partial}{2\partial a_t^{t-1j}}\left(\frac{1}{s}\right)\sum_I s^I\left[V^i\left(\mathbf{q}_t^j\right)-V^i\left(\mathbf{q}_t^{-j}\right)\right]+ \\ +\frac{1}{2s}\cdot\frac{\partial s^{t-1}}{\partial l_t^{t-1}}\cdot\frac{\partial l_t^{t-1}}{\partial a_t^{t-1j}}\left(V_t^{t-1j}-V_t^{t-1-j}\right)+\frac{s^{t-1}}{2s}\left(\frac{\partial V_t^{t-1}}{\partial a_t^{t-1j}}\right)+\mu_1^j-\mu_2^j=\lambda^j\left(\frac{\partial \Upsilon_t^j}{\partial a_t^{\tau-1j}}\right) \\ (2)\ \frac{\partial \mathcal{L}^j}{\partial a_t^{tj}} := \frac{\partial}{2\partial a_t^{tj}}\left(\frac{1}{s}\right)\sum_I s^I\left[V^i\left(\mathbf{q}_t^j\right)-V^i\left(\mathbf{q}_t^{-j}\right)\right]+ \\ +\frac{1}{2s}\cdot\frac{\partial s^t}{\partial l_t^t}\cdot\frac{\partial l_t^t}{\partial a_t^{tj}}\left(V_t^{tj}-V_t^{t-j}\right)+\frac{s^t}{2s}\left(\frac{\partial V_t^t}{\partial a_t^{tj}}\right)+\mu_1^j-\mu_2^j=\lambda^j\left(\frac{\partial \Upsilon_t^j}{\partial a_t^{tj}}\right) \\ (3)\ \Upsilon_t^j=0 \\ (4)\ a^{\min}-a_t^{Ij}\le \quad \mu_1^j\ge 0 \quad \mu_1^j\left(a_t^{Ij}-a^{\min}\right)=0 \\ (5)\ a_t^{Ij}-a^{\max}\le 0 \quad \mu_2^j\ge 0 \quad \mu_2^j\left(a_t^{Ij}-a^{\max}\right)=0 \end{cases}$$

By Proposition 10 we know that at an equilibrium $\mathbf{q}_t^A = \mathbf{q}_t^B$, such that first order conditions if the solution is internal may be simplified in the following way:[8]

$$\begin{cases} (1)\ \frac{\partial \mathcal{L}^A}{\partial a_t^{t-1A}} := \frac{s^{t-1}}{2s}\left(\frac{\partial V_t^{t-1}}{\partial a_t^{t-1A}}\big|_{\mathbf{q}_t^A=\mathbf{q}_t^{A*}}\right)=\lambda^A\left(\frac{\partial \Upsilon_t^A}{\partial a_t^{t-1A}}\big|_{\mathbf{q}_t^A=\mathbf{q}_t^{A*}}\right) \\ (2)\ \frac{\partial \mathcal{L}^A}{\partial a_t^{tA}} := \frac{s^t}{2s}\left(\frac{\partial V_t^t}{\partial a_t^{tA}}\big|_{\mathbf{q}_t^A=\mathbf{q}_t^{A*}}\right)=\lambda^A\left(\frac{\partial \Upsilon_t^A}{\partial a_t^{tA}}\big|_{\mathbf{q}_t^A=\mathbf{q}_t^{A*}}\right) \\ (3)\ \Upsilon_t^A=0 \\ (4)\ \mu_1^A=0 \quad \mu_1^A\left(a_t^{IA}-a^{\min}\right)=0 \\ (5)\ \mu_2^A=0 \quad \mu_2^A\left(a_t^{IA}-a^{\max}\right)=0 \\ (6)\ \frac{\partial \mathcal{L}^B}{\partial a_t^{t-1B}} := \frac{s^{t-1}}{2s}\left(\frac{\partial V_t^{t-1}}{\partial a_t^{t-1B}}\big|_{\mathbf{q}_t^B=\mathbf{q}_t^{B*}}\right)=\lambda^B\left(\frac{\partial \Upsilon_t^B}{\partial a_t^{t-1B}}\big|_{\mathbf{q}_t^B=\mathbf{q}_t^{B*}}\right) \\ (7)\ \frac{\partial \mathcal{L}^B}{\partial a_t^{tB}} := \frac{s^t}{2s}\left(\frac{\partial V_t^t}{\partial a_t^{tB}}\big|_{\mathbf{q}_t^B=\mathbf{q}_t^{B*}}\right)=\lambda^B\left(\frac{\partial \Upsilon_t^B}{\partial a_t^{tB}}\big|_{\mathbf{q}_t^B=\mathbf{q}_t^{B*}}\right) \\ (8)\ \Upsilon_t^B=0 \\ (9)\ \mu_1^B=0 \quad \mu_1^B\left(a_t^{IB}-a^{\min}\right)=0 \\ (10)\ \mu_2^B=0 \quad \mu_2^B\left(a_t^{IB}-a^{\max}\right)=0 \\ (11)\ \lambda^A=\lambda^B=\lambda \end{cases}$$

We then obtain the reaction functions:

$$r_t^A = \begin{cases} a_t^{tA} = r\left(a_t^{tB}, a_t^{t-1B}, s\left(l\left(\psi^{t-1},\tau\right)\right), s\left(l\left(\psi^t,\tau\right)\right), \tau, T, \psi^{t-1}, \psi^t\right) \\ a_t^{t-1A} = r\left(a_t^{tB}, a_t^{t-1B}, s\left(l\left(\psi^{t-1},\tau\right)\right), s\left(l\left(\psi^t,\tau\right)\right), \tau, T, \psi^{t-1}, \psi^t\right) \end{cases}$$

$$r_t^B = \begin{cases} a_t^{tB} = r\left(a_t^{tA}, a_t^{t-1A}, s\left(l\left(\psi^{t-1},\tau\right)\right), s\left(l\left(\psi^t,\tau\right)\right), \tau, T, \psi^{t-1}, \psi^t\right) \\ a_t^{t-1B} = r\left(a_t^{tA}, a_t^{t-1A}, s\left(l\left(\psi^{t-1},\tau\right)\right), s\left(l\left(\psi^t,\tau\right)\right), \tau, T, \psi^{t-1}, \psi^t\right) \end{cases}$$

Solving the system we obtain the optimal vector of policies from the set of intersection points Λ_1 :

$a_t^{t-1A} = a(s\left(l\left(\psi^{t-1},\tau\right)\right), s\left(l\left(\psi^t,\tau\right)\right), \tau, T, \psi^{t-1}, \psi^t)$

$a_t^{tA} = a(s\left(l\left(\psi^{t-1},\tau\right)\right), s\left(l\left(\psi^t,\tau\right)\right), \tau, T, \psi^{t-1}, \psi^t)$

$a_t^{t-1B} = a(s\left(l\left(\psi^{t-1},\tau\right)\right), s\left(l\left(\psi^{t},\tau\right)\right), \tau, T, \psi^{t-1}, \psi^{t})$
$a_t^{tB} = a(s\left(l\left(\psi^{t-1},\tau\right)\right), s\left(l\left(\psi^{t},\tau\right)\right), \tau, T, \psi^{t-1}, \psi^{t})$
with $a_t^{t-1A} = a_t^{t-1B}$ and $a_t^{tA} = a_t^{tB}$

With altruism the first order conditions are modified as follows:

$$\begin{cases}
(1)\ \frac{\partial \mathcal{L}^A}{\partial a_t^{t-1A}} := \frac{s^{t-1}}{2s}\left(\frac{\partial V_t^{t-1}}{\partial a_t^{t-1A}}\right) = \lambda^A\left(\frac{\partial \Upsilon_t^A}{\partial a_t^{t-1A}}\right) \\
(2)\ \frac{\partial \mathcal{L}^A}{\partial a_t^{tA}} := \frac{s^{t}}{2s}\left(\frac{\partial V_t^{t}}{\partial a_t^{tA}}\right) + \frac{\sigma s^{t-1}}{2s}\left(\frac{\partial V_t^{t}}{\partial a_t^{t-1A}}\right) = \lambda^A\left(\frac{\partial \Upsilon_t^A}{\partial a_t^{tA}}\right) \\
(3)\ \Upsilon_t^A = 0 \\
(4)\ \mu_1^A = 0 \quad \mu_1^A\left(a_t^{IA} - a^{\min}\right) = 0 \\
(5)\ \mu_2^A = 0 \quad \mu_2^A\left(a_t^{IA} - a^{\max}\right) = 0 \\
(6)\ \frac{\partial \mathcal{L}^B}{\partial a_t^{t-1B}} := \frac{s^{t-1}}{2s}\left(\frac{\partial V_t^{t-1}}{\partial a_t^{t-1B}}\right) = \lambda^B\left(\frac{\partial \Upsilon_t^B}{\partial a_t^{t-1B}}\right) \\
(7)\ \frac{\partial \mathcal{L}^B}{\partial a_t^{tB}} := \frac{s^{t}}{2s}\left(\frac{\partial V_t^{t}}{\partial a_t^{tB}}\right) + \frac{\sigma s^{t-1}}{2s}\left(\frac{\partial V_t^{t}}{\partial a_t^{t-1B}}\right) = \lambda^B \frac{\partial \Upsilon_t^B}{\partial a_t^{tB}} \\
(8)\ \Upsilon_t^B = 0 \\
(9)\ \mu_1^B = 0 \quad \mu_1^B\left(a_t^{IB} - a^{\min}\right) = 0 \\
(10)\ \mu_2^B = 0 \quad \mu_2^B\left(a_t^{IB} - a^{\max}\right) = 0 \\
(11)\ \lambda^A = \lambda^B = \lambda
\end{cases}$$

which gives a new set of intersection points Λ:

$a_t^{t-1A} = a(s\left(l\left(\psi^{t-1},\tau\right)\right), s\left(l\left(\psi^{t},\tau\right)\right), \tau, T, \psi^{t-1}, \psi^{t}, \sigma)$
$a_t^{tA} = a(s\left(l\left(\psi^{t-1},\tau\right)\right), s\left(l\left(\psi^{t},\tau\right)\right), \tau, T, \psi^{t-1}, \psi^{t}, \sigma)$
$a_t^{t-1B} = a(s\left(l\left(\psi^{t-1},\tau\right)\right), s\left(l\left(\psi^{t},\tau\right)\right), \tau, T, \psi^{t-1}, \psi^{t}, \sigma)$
$a_t^{tB} = a(s\left(l\left(\psi^{t-1},\tau\right)\right), s\left(l\left(\psi^{t},\tau\right)\right), \tau, T, \psi^{t-1}, \psi^{t}, \sigma)$
with $a_t^{t-1A} = a_t^{t-1B}$ and $a_t^{tA} = a_t^{tB}$

Appendix 3 - Numerical simulations

Numerical simulations were performed in order to assess the validity of conjectures 12-14, under the assumption that the maximum is an interior solution to the maximization problem. They jointly suppose that the old generation, because more single-minded, obtains more favourable policies from governments. That is, the old obtain higher tax credits (conjecture 12) and positive intergenerational transfers (conjecture 14). Furthermore, the combination of higher preferences for leisure and higher tax credits enables the old to reach higher levels of leisure (conjecture 13). As a consequence the young are worse off, because they obtain lower tax credits and have to endure the entire cost of social security systems[9]. I assume that optimal values are always acceptable, that is

$a_t^{\max}$ is sufficiently high to be always greater than a_t^{Ij*}. Solutions show that only one stationary point exists.

To perform simulations a suitable density function is required. As suggested by Profeta (2002) I will use one with a constant positive elasticity ε

$$s^I = (l^I)^{\varepsilon}$$

with $\varepsilon = 1$ for computational purposes. Table 1 shows results. The nominal marginal tax rate, τ, was set equal to 1 and the total endowment of time, T, equal to 0.9. Simulations were performed using different values of preferences of workers for leisure, under the condition that the parameter of the old is higher the that of the young. Tax credits are always higher for the old but the difference between tax credits of the two generations reduces with respect to a reduction in the difference between preferences. Leisure is always higher for the old and the amount of leisure increases both for the young and for the old from situation 1 to situation 9. Tax revenues are always positive for the generation of the young and negative for the generation of the old, meaning that the young bear the entire burden of social security systems; otherwise, the old get a transfer (i.e. a pension). Notice that the inter-generational redistribution effect is higher the higher is the difference between preference for leisure amongst cohorts. Finally, notice that, even though the sum of preferences for leisure of the old and the young is equal to one, the total level of leisure is not constant. The worst situation for the aggregate level of employment is achieved in situation 9, whilst the reverse is true for situation 1.

	ψ^{t-1}	ψ^t	τ	T	a^{t-1}	a^t	l^{t-1}	l^t	l	T^{t-1}	T^t
1	0.95	0.05	1	0.9	2.144	0.261	0.442	0.19	0.632	−0.261	0.261
2	0.9	0.1	1	0.9	1.915	0.385	0.469	0.259	0.728	−0.196	0.196
3	0.85	0.15	1	0.9	1.739	0.484	0.488	0.309	0.797	−0.152	0.152
4	0.8	0.2	1	0.9	1.592	0.571	0.502	0.35	0.852	−0.117	0.117
5	0.75	0.25	1	0.9	1.465	0.649	0.511	0.384	0.895	−0.09	0.09
6	0.7	0.3	1	0.9	1.352	0.722	0.517	0.415	0.932	−0.067	0.067
7	0.65	0.35	1	0.9	1.25	0.791	0.519	0.442	0.961	−0.047	0.047
8	0.6	0.4	1	0.9	1.159	0.859	0.517	0.465	0.982	−0.03	0.03
9	0.55	0.45	1	0.9	1.076	0.928	0.51	0.484	0.994	−0.014	0.014

Table 1 - Numerical simulation (basic model)

Notice that the result which states that the old enjoy lower effective marginal tax rates than the young contradicts previous results obtained by probabilistic voting models applied to social security systems. In Profeta, the old are taxed more heavily than the young (Proposition 3.1, p. 345); the same result is achieved by Mulligan and Sala-i-Martin (Proposition 8, p.31).

Table 2 shows results of simulations performed with the altruistic model in order to check the validity of conjecture 15. The altruistic parameter was set equal to 0.3. With respect to results obtained with the basic model notice that the old (young) obtain lower (higher) tax credits and that there are less redistributive effects since transfers from the young to the old are reduced. Furthermore, notice that in situation 9 the young obtain a positive transfer, although this is rather small. Leisure increases for the old, meaning that the higher effective marginal tax rate increases the incentive to quit the labour force, whilst leisure of the young reduces. Aggregate leisure reduces as well, except for situations 8, 9 where this is slightly higher than the previous situation.

	ψ^{t-1}	ψ^t	τ	T	σ	a^{t-1}	a^t	l^{t-1}	l^t	l	T^{t-1}	T^t
1	0.95	0.05	1	0.9	0.3	2.104	0.342	0.451	0.145	0.596	-0.247	0.247
2	0.9	0.1	1	0.9	0.3	1.845	0.503	0.487	0.198	0.685	-0.174	0.174
3	0.85	0.15	1	0.9	0.3	1.645	0.625	0.516	0.239	0.755	-0.123	0.123
4	0.8	0.2	1	0.9	0.3	1.479	0.723	0.540	0.276	0.816	-0.086	0.086
5	0.75	0.25	1	0.9	0.3	1.338	0.804	0.56	0.31	0.87	-0.057	0.057
6	0.7	0.3	1	0.9	0.3	1.217	0.873	0.575	0.343	0.918	-0.035	0.035
7	0.65	0.35	1	0.9	0.3	1.111	0.932	0.584	0.375	0.959	-0.017	0.017
8	0.6	0.4	1	0.9	0.3	1.019	0.987	0.588	0.405	0.993	-0.003	0.003
9	0.55	0.45	1	0.9	0.3	0.939	1.040	0.585	0.432	1.017	0.009	-0.009

Table 2 - Numerical simulation (altruistic model)