- Chapter 1 -

CGE MODELS AND
MICROSIMULATION
TECHNIQUES
1. INTRODUCTION

Since the pioneering work by Adelman and Robinson (1978) for South Korea and Lysy and Taylor (1980) for Brazil\textsuperscript{1}, many CGE models for developing countries combine a highly disaggregated representation of the economy within a consistent macroeconomic framework and a description of the distribution of income through a small number of representative households (RH) meant to represent the main sources of heterogeneity in the whole population with respect to the phenomena of the policies under study. Models were initially static and rigorously Walrasian. They now often are dynamic (in the sense of a sequence of temporary equilibria linked by asset accumulation, or recursively dynamic) and often depart from Walrasian assumptions so as to incorporate various macro-economic features or “closures” as well as imperfect competition features.

Several “representative households” are necessary to account for heterogeneity among the main sources of household income (or among the changes in income) due to the phenomena or the policies being studied. Despite the need for variety, the number of RH is generally small in these models, however (usually less than 10). The chosen taxonomy and the level of disaggregation depend critically on the questions that the model is expected to answer: the household account is to be broken down into a number of relatively homogeneous household groups reflecting the socioeconomic characteristics of the country or region under consideration. The degree of homogeneity is crucial in the design of classifications, especially in a classification of household groups, where one would like to identify groups that are relatively homogeneous in terms of income sources and levels and expenditure patterns, and that may be able to reproduce the socioeconomic and structural stratification observed within the society and the economy under study. It is noteworthy that a household classification based on income or expenditure brackets does not satisfy any of these requirements – except perhaps the last one. Indeed, consider for instance the poorest segment of society (say the bottom decile of the income pyramid): it may include very different household heads, such as a landless

\textsuperscript{1} See also the work by Gunning (1983) for Kenya. Other significant examples are represented by the models built in connection with the OECD research program on “Structural Adjustment and Poverty”: Thorbecke (1991) for Indonesia, Morrisson (1991) for Morocco and Bourguignon \textit{et al.} (1991) among others.
agricultural worker and an urban informal sector worker, and policies aimed at improving conditions in the two cases are likely to be very different.

There is no unique (standard) classification scheme or way of disaggregating the household data in a CGE model. The taxonomy used in any given model depends on the prevailing country or region specific characteristics and the objectives of the studies underlying the building of the model. Major criteria and sub-criteria used in the classification and disaggregation of the different household accounts are:

a) location (e.g. rural vs. urban);

b) asset and productive factor ownership (particularly land ownership in the rural areas and human capital in urban areas);

c) characteristics of the head or main earner, such as his/her employment status, occupation, branch of industry and educational attainment, skill level, sex, main language, race (tribal) kinship.

For what concerns the degree of heterogeneity among agents, the CGE/RH framework sometimes explicitly considers that households within a RH group are heterogeneous in a “constant” way. That is, in order to capture within-group inequality, it is often assumed that each RH group represents an aggregation of households in which the distribution of relative income follows an exogenously fixed statistical law. However, if households within a group are different, why should they be affected in the same way by a policy or by a shock? The empirical analyses conducted on household surveys support this doubt: the within-group component of observed changes in income distribution generally is at least as important as the between-group component of these changes. Thus, the RH

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2 See Decaluwé et al. (1999a).

3 For an interesting discussion of the importance of an appropriate households’ taxonomy, see Duchin (1996).

4 For early applications of this type of models, see Adelman and Robinson (1978) and Dervis, de Melo and Robinson (1982), who specified lognormal within-group distributions with exogenous variances. For a survey of CGE models applied to developing countries see Decaluwé and Martens (1988) and Robinson (1989). More recent examples of this kind of models can be found in de Janvry et al. (1991), Chia et al. (1994), Decaluwé et al. (1999a), Colatei and Round (2000) and Agénor et al. (2001).

5 After Mookherjee and Shorrocks’ (1982) study of UK, there are now other examples of “within/between” decomposition analysis of changes in inequality that indicate that changes in overall inequality are usually
approach based on the assumption that relative incomes are constant within household groups may be misleading in several circumstances, and this is especially true when studying poverty. This argument may be better understood by presenting an example: consider a shock which reduces the world price of a specific commodity, say maize; under the small country assumption (that is, the country is price-taker on the world market), a country exporting this good will see a decrease in its exports and a domestic contraction of this sector. After the simulation of the shock with a CGE/RH model of this country, suppose that we find a little change in the mean income of a RH group, say workers in the agricultural sector; however, in this case, poverty might be increasing by much more than suggested by this drop in the income of this group: indeed, in some households there may be individuals that have lost their job after the shock, or there may be some households that encounter more difficulties to diversify their activity or their consumption than others. For these individuals or families, the relative fall in income is necessarily larger than for the whole group, and this fall in their income is not represented by the slight fall in the mean income of the whole group: the RH approach does not allow to catch the effects that a shock or a policy change may have on single individuals or households. Suppose moreover that the initial income of these individual was low; then poverty may be increasing by much more than what predicted by a simple RH model, which is based on the assumption of distribution neutral shocks.

As it is explained in Savard (2003), another significant drawback in linking the intra-group distribution change to a statistical law that is completely exogenous is that no economic behaviour is considered behind this change in within-group distribution. The intra-group distribution change is usually linked to a theoretical statistical relationship between average ($\mu$) and variance ($\sigma^2$) of the lognormal distribution. Savard (2003) also underlines the fact that the average behaviour of a specific group is biased towards the richest in the group. Standard CGE models, indeed, use household groupings that take into account the total income and expenditure of each group and the behavioural parameters which are generally calibrated at the base year. In most of the models these parameters reflect the aggregate and not necessarily the average behaviour. Thus, as the richest of a group are endowed with most of the factors, their behaviour will be dominant in the group. Moreover, keeping in mind that when doing poverty analysis is very important to consider the behaviour around the poverty line,
In order to overcome these problems, recent literature has tried to develop new modelling tools which should be able at the same time to account for heterogeneity and for the possible general equilibrium effects of the policy reform (or the exogenous shock) under study. In view of the fact that most of the available economic models have either a microeconomic or a macroeconomic focus, and they do not address the question adequately, recent literature has focused on the possibility of combining two different types of models. Since most of the economic policies (structural adjustment programs or trade liberalizations, for example) and exogenous shocks commonly analyzed for developing countries (such as fluctuations in the world price of raw materials and agricultural exports) are often macroeconomic phenomena (or may have, at least, some structural effects on the economy), while poverty and inequality are mainly microeconomic issues, this approach, which takes into account important micro-macro linkages, seems to be the right answer to the problem. In particular, some authors have tried to link microsimulation models to CGE models, in order to account simultaneously for structural changes of the economy and general equilibrium effects of economic policies, and for their impacts on households’ welfare, income distribution and poverty. The literature that follows this approach is quite flourishing in recent years: there are, among others, the important contributions by Decaluwé et al. (1999a) and (1999b), Cognneau and Robilliard (2000), Agénor et al. (2001), Cockburn (2001), Cognneau (2001), Bourguignon, Robilliard and Robinson (2003), Boccanfuso et al. (2003a) and Savard (2003).

In this chapter, after a functional introduction to microsimulation modelling techniques, we’ll go into the details of the different approaches used in literature to model the data coming from household surveys into a general equilibrium framework: in particular, we will analyze, respectively, the integrated approach, the Top-Down or Sequential

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7 These models include macro models, microsimulation models, multiplier models and computable general equilibrium (CGE) models.

8 More generally, this current of the literature develops the use of micro-data drawn from household surveys in the context of a general equilibrium setting, which is usually but not necessarily a CGE model.
approach and the approach developed by Savard, known also as Top-Down/Bottom-Up approach. Finally, in the last section, we will see in detail how to build a behavioural microsimulation model, and how to link it to a CGE model.

2. INTRODUCTION TO MICROSIMULATION MODELS

Guy Orcutt is known as the originator of microsimulation as an instrument for economic analysis, but it is only since early 1980s that the use of microsimulation models developed, undoubtedly as a consequence of the increasing availability of large and detailed datasets on individual agents and the continuous increase in, and falling cost of, computing power; in fact, during the last twenty years, this kind of models have been increasingly applied in qualitative and quantitative analysis of public policies. Their field of application ranges among the ones included in the broader area of redistribution policies: indirect and direct taxation, social security system reforms, etc.

Microsimulation (MS) models are tools that allow the simulation of the effects of a policy on a sample of economic agents (individual, households, firms) at the individual level. Usually, MS models are based on two fundamental elements: a micro dataset containing the economic and socio-demographic characteristics of a sample of individuals or households (household surveys), and the rules of the policies to be simulated, and especially their impact on the budget constraint faced by each agent.

Consider for instance a simple MS model which aims at computing the disposable income of a sample of households, given a tax-benefit system; in general, the disposable income of household $h$, which is made up of $m$ individuals at working age, will be computed as follows:

$$ YD_h = \sum_{i=1}^{m} w_{ih} \cdot L_{ih} + y_h^0 + NT\left(w_{ih}L_{ih}, y_h^0, z_h, \tau \right), $$

9 See Orcutt (1957), Orcutt et al. (1961) and Orcutt et al. (1976).

10 For the history and developments of microsimulation in economic analysis see, among others, Harding (1996) and Gupta and Kapur (2000).
where $w_{ih}$ is the wage rate received by individual $i$, $L_{ih}$ is individual $i$’s labour supply, $y_{ih}^0$ is the non-labour income (for example, rent from capital), and $NT(\cdot)$ is the tax-benefit system function, or “net tax” rule, which computes the net taxes to be paid given gross incomes $(w_{ih}, L_{ih}, y_{ih}^0)$. Taxes and benefits depend on the characteristics of the household represented by the vector $z_h$ (which may contain variables such as the number of individuals and the number of kids living in the household, the region/province of residence, the number of dependents, etc.), on the labour and non-labour incomes received by each agent belonging to that household, while $\tau$ stands for the parameters of the tax-benefit system (various tax rates, means-testing of benefits, etc.). In order to see how this function $NT(\cdot)$ may work in practice, consider a very simple tax-benefit system, and a household composed by two adults and a child, in which only one individual ($i$) works, while the other ($j$) is unemployed; the household receives also an income from a capital asset $K_h$, and a cash transfer from the welfare system (an unemployment benefit for the non-working adult, for example):

$$
YD_h = (1 - \sigma) \cdot w_{ih} \cdot L_{ih} + (1 - \delta) \cdot r \cdot K_h - \gamma \cdot [ (1 - \sigma) \cdot w_{ih} \cdot L_{ih} + (1 - \delta) \cdot r \cdot K_h - D_h ] + TF_{jh},
$$

where $\sigma$ is the social contribution rate, $\delta$ is the tax rate on capital income, $r$ is the interest rate, $\gamma$ is the direct income tax rate for the income class to which household $h$’s income level belongs, $D_h$ is the deduction for the presence of two dependents, and $TF_{jh}$ is the unemployment benefit received by the non-working individual (transfers from the government are supposed to be tax free). It is easy to see that, in this simplified world, a reform of the tax-benefit system may influence the disposable income of household $h$ in different ways: a reform of the income tax rates or a reform of the deduction system will directly affect its disposable income, a reform of the social security contributions will influence the labour income, directly or through a change in the labour supply of the individuals, and a reform of capital taxation may affect the non-labour income of the household, while a reform of the unemployment benefit system will affect the amount of transfers received by the household (see Figure 1).

In the real world, there are many possible measures which may have an influence upon the disposable income of the households and individuals in a country, either directly or through a change in the economic behaviour of individuals (labour supply, consumption behaviour, savings and income allocation, tax evasion, etc.): all these policy and fiscal
reforms are thus expected to have an effect on the distribution of income of the population under study, and consequently on poverty and inequality indices. The main task for which most of MS models are built is in fact that of capturing this expected change in income distribution, trying to evaluate it by using the microdata coming from a sample survey of the population, enlightening who are the gainers and losers of the reform, while computing at the same time what are the costs or the gains of the reform in terms of revenue for the government.

\[
YD_h = \sum_{i \in h} (1 - \theta) \cdot w_{ih} \cdot L_{ih} + (1 - \delta) \cdot r \cdot K_h + \sum_{i \in h} TF_{ih} - \gamma \cdot [I_h - D_h]
\]

**Figure 1 – An Arithmetical Model of a Tax-Benefit System**

As said before, models simulating the household sector typically begin with a microdata file. Such a file may be based upon administrative data (such as tax or social security records) or upon a sample survey of the population. In both cases, the microdata usually contain thousands of individual or family records, with a list of variables describing the demographic, labour force, income and other characteristics of each individual or family. These data usually come from a sample of the population; nonetheless, it is possible to obtain also the values for the entire population by using “ad hoc” weights, which allow to know how many households of the population are represented by each observation in the survey. These sampling weights (also known as expansion or raising factors) simply
depend on the selection probabilities of each observation in the sample (and, thus, the sample design\textsuperscript{11}); they are usually reported in the surveys, so that it is always possible to pass from sample to population data.

Different stages are in general included in sampling methodology: typically, they are stratification and sampling. Stratification involves the division of the population into sub-groups, or strata, from which independent samples are taken (there may be a need to adapt these categories according to the local context)\textsuperscript{12}. This ensures that a representative sample will be drawn with respect to the stratifiers (i.e. the proportions of units sampled from any particular stratum will equal the proportion in the population with that characteristic: stratification ensures that proportions of the sample falling into each group reflect those of the population). A separate random sample can thus be selected from each group. Some adjustments should be done for particularly small or large groups: the desired sample size will be determined by the expected variation in the data. The more varied the data are, the larger the sample size needs to be to obtain an adequate level of accuracy in generalizing the results. Using group sampling will ensure a balanced representation of the different household categories. Stratification of a sample can lead to substantial improvements in the precision of survey estimates. Optimal precision is achieved where the factors used as strata are those that correlate most highly with the survey variables\textsuperscript{13}.

\textsuperscript{11} Sample design is about choosing how many households to include in a survey in order to provide a good basis for measuring economic and social phenomena in the whole population.

\textsuperscript{12} For example, the 1997-98 Family Expenditure Survey (FES) for Great Britain is a voluntary sample survey of private households. The FES sample for Great Britain is a multi-stage stratified random sample with clustering. It is drawn from the Small Users file of the Postcode Address File – the Post Office’s list of addresses. Postal sectors (ward size) are the primary sample unit. 672 postal sectors are randomly selected during the year after being arranged in strata defined by standard regions (subdivided into metropolitan and non-metropolitan areas) and two 1991 Census variables – socio-economic group and ownership of cars. See the website of the UK Office for National Statistics \url{www.statistics.gov.uk}, and some documentation about UK Family Expenditure Surveys at \url{www.data-archive.ac.uk/findingData/fesTitles.asp}

\textsuperscript{13} For a more detailed treatment of sampling procedures and on how to build a household survey, one can turn to the Living Standard Measurement Study (LSMS) manual, which is specifically designed to guide the many collaborators involved in planning surveys through the process of planning and implementing an LSMS survey. The manual provides practical information about, among other topics, questionnaire
We can make a very simple example in order to clarify this procedure. Suppose that in a given country there are 10,000 households from census data, and we have to select a representative sample from this population. We can start by stratifying the population into different regional areas, say North-East (2300 households from census data, 23% of household national population), North-West (1000 households, 10% of the whole population), South-East (4000 households, 40%) and South-West (2700 households, 27%). We can further stratify the population by the sub-division of these regions into rural and urban areas, for instance, thus obtaining 8 sub-groups in total. After this stratification, we can select a sample from each of these groups (we will draw a 10% sample from each sub-group, except for the urban North-Western sub-group, from which we take a 30% sample, given the small size of this sub-group with respect to the others). In this way, we get a sample of 1024 households (See Table 1). Once we will have the data (on income, expenditure, etc.) from the survey corresponding to each household in our sample, in order to get the corresponding population values we have to multiply each value by the weight corresponding to that representative household. In this case, to get these weights we must divide each sample size by the total sample size, and multiply this number by the household population. Now, for instance, each household in the first group is representative of 351 households in the whole population, each household in the second group is representative of 859 population households, and so on (see Table 1 and Figure 2). We can thus say that we have a representative sample of the given population.  

The fundamental information contained in the weighting factors is that every family in the survey represents a given number of households of the population; only in this case we can say that we are dealing with a representative sample survey of the population. These particular numbers are usually supplied by the original data providers, and they formatting and development, sample design, and data management. One can also have a look at the LSMS website: www.worldbank.org/lsms.

14 In a real household survey, however, one has to take into account also the fact that some of the households in the selected sample could not be reached, some others will not co-operate, or will not give full response to the questionnaire. Obviously, the weights need to be adjusted to take into account also this kind of problems. For a complete description of the techniques and full methodology used to build a real household survey, see for example the General Household Survey for Great Britain conducted by the Office for National Statistics, http://www.statistics.gov.uk
can be modified by the model builder only to account for possible socio-demographic or economic changes occurred since the time of the survey (this procedure is also known as reweighting or grossing-up procedure, see below).

Apart from this, the data frequently require further amendment by central statistical agencies or microsimulation modellers before they can be used. For example, this may include adjustment for under-reporting or misreporting of income or expenditure, the imputation of missing values, and the adjustment of weights for non-response.

Microsimulation thus involves a set of distinct processes, which include data cleaning and validation, imputation of missing data required for particular policy simulations, updating and re-weighting the data in order that they represent the desired population as closely as possible, applying detailed rule-modelling to simulate different policy regimes, and designing methods of presenting the results.

### Table 1 – Stratification and Sampling: an Example

<table>
<thead>
<tr>
<th>Geographic Area (GA)</th>
<th>Number of Households (population)</th>
<th>% w.r.t. the whole pop.</th>
<th>Region</th>
<th>Number of Households (population)</th>
<th>% w.r.t. Households in GA</th>
<th>Sample</th>
<th>Sample size</th>
<th>Group sample size/Total sample size</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>North-West</td>
<td>1000</td>
<td>10 %</td>
<td>Urban</td>
<td>120</td>
<td>12 %</td>
<td>30</td>
<td>36</td>
<td>0.0352</td>
<td>351.56</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Rural</td>
<td>880</td>
<td>88 %</td>
<td>10</td>
<td>88</td>
<td>0.0859</td>
<td>859.38</td>
</tr>
<tr>
<td>North-East</td>
<td>2300</td>
<td>23 %</td>
<td>Urban</td>
<td>460</td>
<td>20 %</td>
<td>10</td>
<td>46</td>
<td>0.0449</td>
<td>449.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Rural</td>
<td>1840</td>
<td>80 %</td>
<td>10</td>
<td>184</td>
<td>0.1797</td>
<td>1796.88</td>
</tr>
<tr>
<td>South-West</td>
<td>2700</td>
<td>27 %</td>
<td>Urban</td>
<td>1890</td>
<td>70 %</td>
<td>10</td>
<td>189</td>
<td>0.1846</td>
<td>1845.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Rural</td>
<td>810</td>
<td>30 %</td>
<td>10</td>
<td>81</td>
<td>0.0791</td>
<td>791.02</td>
</tr>
<tr>
<td>South-East</td>
<td>4000</td>
<td>40 %</td>
<td>Urban</td>
<td>2400</td>
<td>60 %</td>
<td>10</td>
<td>240</td>
<td>0.2344</td>
<td>2343.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Rural</td>
<td>1600</td>
<td>40 %</td>
<td>10</td>
<td>160</td>
<td>0.1563</td>
<td>1562.50</td>
</tr>
<tr>
<td>Total</td>
<td>10000</td>
<td>100 %</td>
<td></td>
<td>10000</td>
<td></td>
<td>1024</td>
<td>1.0000</td>
<td>10000.00</td>
<td></td>
</tr>
</tbody>
</table>
**Figure 2** – Stratification and Sampling

Uprating and reweighting procedures are also known as static “ageing” techniques\(^\text{15}\); they are defined as methods attempting to align the available micro-data with other known information (such as changes in population aggregates, age distributions or unemployment rates), without modelling the processes that drive these changes (e.g., migration, fertility, or economic downturn), which is instead the intention of dynamic MS models (for more details on the differences between static and dynamic models see sub-section 1.3). There are several reasons why these procedures may be considered as necessary before using the data, but the most common one is that tax-benefit models are used to analyse the effects of social and fiscal policies in a period \(t\), while the most recent data which are available usually come from some earlier period \(t\). The micro-data from some previous period may thus need to be adjusted (“aged”) to approximate the

\(^{15}\) For this part on static data “ageing” techniques and their evaluation, see Immervoll et al. (2005).
population in period $t^*$. It is of course possible that, for the purpose of achieving a “good” approximation of taxes and benefits payable in period $t^*$, data from period $t$ are already sufficiently close to period $t^*$ population. But it is useful to ask whether we can improve on this to any meaningful extent by further enhancing the degree to which data available for modelling describe the target population in period $t^*$.

In thinking about the correlation between period $t$ data and period $t^*$ population, it is useful to separate the factors that determine how representative sample $S_t$ (taken in period $t$) will be of population $P_{t^*}$.

One can, for this purpose, look separately at:

1. the degree to which $S_t$ is representative of $P_t$, and
2. the processes causing $P_t$ to differ from $P_{t^*}$.

With regard to the first point, we have already mentioned the fact that each observation in the sample represents a certain number of population members in the sense that the variable values recorded for a given observation approximate the characteristics of a certain fraction of the population. We will denote the group in the population represented by observation $i$ in sample $S_t$ as $G_{ti}$. The changes in the population (point 2) can then be broken down further into:

2a. Processes altering the average value of a variable in group $G_{ti}$: given the conceptualization of each observation $i$ in the sample as representing $G_{ti}$, this translates into changing the value of the relevant variable of observation $i$ in sample $S_t$.

2b. Processes causing the composition of $G_{ti}$ to change. That is, some population members who have fitted into group $G_{ti}$ may, due to changes in the population structure, be better represented by another group in population $P_{t^*}$ (or may no longer be part of the population in period $t^*$ at all). Similarly, group $G_{t^*i}$ may encompass population members which were not part of $G_{ti}$ before.

Each of these factors will be discussed in turn.

2a. Adjusting variable values of individual observations (“uprating”)

A common reason why we would consider a procedure of “ageing” (that is, an adjustment to the original data) as necessary is that our tax-benefit model will be used to analyse the effects of a fiscal reform in period $t^*$, while the most recent data that are
available come from some earlier period \( t \). For instance, let us suppose that we observe in some national statistics referred to period \( t^* \) that the average value \( \bar{v} \) of a variable observed in a given group \( G_{i,t} \) has changed between \( t \) and \( t^* \). Then, for observation \( i \) to still be representative of \( G_{i,t^*} \) in period \( t^* \), this change will need to be reflected in the variable value recorded for that observation. For monetary variables, this can be achieved by “uprating” (i.e. inflating or deflating) each value by an appropriate index (such as price indices or indices based on income growth) describing how the value of the variable, averaged across the population group represented by \( i \), has behaved between \( t \) and \( t^* \). In doing so, it is important to separate changes in the average value of the variable averaged across members of \( G_{i,t} \) from changes in the number of population members with certain variable values. To illustrate, let us consider an example. Let us suppose that in our sample \( S_t \), which is representative of population \( P_t \), in region “South” we have a certain number of observations and for all of them the registered wage level is zero, with the exception of observation \( i \), whose wage level is 100. If we observe in the statistics for period \( t^* \) that the average wage level in this region has raised of 10% and, at the same time, the occupational level between \( t \) and \( t^* \) has been stable in that region, then we would want the wage of observation \( i \) to be “uprated” by the observed change in the average wage (+10%), rather than an “uprating” of all the wages earned in region “South”.

Of course, indices capturing the change in variable values separately for each group \( G_{i,t} \) will often not be available. One will, for instance, usually see more than one observation with non-zero wages in a given region and if there is only one index of average earnings available for the region as a whole, then the same index will need to be applied to all wage earners of that region in the sample. In other words, we cannot hope to perfectly replicate the distribution of all relevant variable changes occurring between \( t \) and \( t^* \). Moreover, it is often a common choice among MS modellers to assume that the distribution of the variables among households remains constant, and therefore they just “uprate” average variables by multiplying them by a constant term (such as the inflation rate or the real growth rate of income).
2b. Adjusting the relative sizes of sub-populations (“re-weighting”)

“Re-weighting” (that is, altering the “weights” of different observations in the data) can be used to align weighted frequencies of subgroups in sample $S_t$ with external control totals (the true number of population members with certain characteristics) related to time period $t^*$; that is, the original weights can be forced to correspond to these new numbers. While the process of “uprating” discussed above aims at correcting the information of observations in sample $S_t$, so that they are still approximately representative of equivalent population members in $P_t^*$, re-weighting sample $S_t$ can be used to correct for the difference in probabilities of drawing an observation $i$ (which is already part of $S_t$) if another sample were to be taken of population $P_t^*$. When moving from period $t$ to $t^*$, it is possible and, indeed, likely that both the probability of drawing observation $i$ and the average values of variables in the group represented by this observation will have changed. To exploit all available information, it will generally be desirable to use both uprating and re-weighting.

Clearly, there are many ways in which a sample could be weighted in order to match a given set of control totals. How then should the weights be re-computed? Since no exact solution exists to the re-weighting problem and since the original weights provided in the dataset prior to any re-weighting contain a great deal of information about the population, a natural approach is to achieve the control totals by changing the existing weights as little as possible.

Indeed, adding information about the target population by altering the statistical weights in a dataset comes at the cost of potentially distorting information that the original weights represent. The likelihood of such distortions grows with the number of dimensions used for re-weighting as well as with the magnitude of the change along each individual dimension. If the size or number of relevant changes becomes very sizable, then forcing the data to correspond to the observed values in the target period can compromise the representativity along several dimensions. In such a situation, ageing techniques do not provide a reliable approximation of the population of interest (clearly, large changes will also render the “unadjusted” data non-representative of the target population).
For what concerns the process of validation, it includes a range of internal and external checks on the reliability of model inputs, model procedures, and model outputs\textsuperscript{16}:

- **Model inputs**: validation of the reliability of the underlying microdata, which could involve internal checks such as an assessment of the degree of the estimation and imputation in responses to individual questions, or external checks such as the comparison of grossed up aggregates or distributions with official data.

- **Model procedures**: validation of the reliability of the simulations; this could involve internal case-by-case testing of simulated entitlements and liabilities against legally correct outcomes, or it could involve external comparisons which compare the taxes and benefits simulated for the same individuals and families by two models of the same country.

- **Model outputs**: the results of the simulations could be validated internally by comparison with recorded entitlement or liability taken directly from the microdata, or they could be validated externally. This could entail a comparison of the simulated aggregates and distributions for actual policy in force with official statistics or forecasts; alternatively, validation of model outputs could involve a study of the effect of sampling error on the reliability of outputs, or sensitivity testing of key assumptions\textsuperscript{17}.

However, there are several reasons why one should not expect model outputs to match exactly estimates from alternative sources. For instance, there may be structural changes between the survey year and the modelled year that are not captured by the methods that are used to gross up survey respondents to represent the population as a whole in the modelled year. There may, for example, be a change in unemployment rates or in patterns of households’ expenditures, or structural changes in the volume of some income streams. However sophisticated the grossing-up technique that is used, the method will be unable to capture all the complexity of structural shifts that occur. Other reasons for which model outputs may be in disagreement with other sources may depend on the quality and completeness (miscoding and misreporting of information is always possible) of collected data, and on how much representative they are.

\textsuperscript{16} For this part on validation techniques see Redmond et al. (1998), chapter 9.

\textsuperscript{17} For a discussion about validating procedures of model outputs, see, among others, Pudney and Sutherland (1994) and Lambert et al. (1994).
MS models are called *behavioural* when they include a theoretical model of the behavioural response of agents to changes in their budget constraint\(^{18}\): these models allow individuals to adjust their behaviour in response to the simulated policy change. The behaviours which are most frequently taken into account are consumption and labour supply. In order to compute the optimal consumption and labour supply of each agent, a model of consumption and labour supply must be estimated (or “calibrated”) and incorporated into the microsimulation framework. There is then a choice between the popular “reduced form” approach and the more challenging and problematic structural approach. The latter requires making assumptions about the functional forms of preferences and specifying constraints facing households and individuals carefully, in a world where these steps may be arbitrary and difficult. We will focus on all these procedures and, more in general, on the setting up of a behavioural MS model in the following sub-section.

MS models that do not include behavioural responses are called *arithmetical* (or accounting) models, because they simply derive in an arithmetical way the disposable income and net tax payments of each agent, given the rules for the computation of taxes and benefits in the simulated policy: for each household in the database, information on income, expenditure and personal and family characteristics are used to perform the arithmetic necessary to calculate liability for personal taxes and entitlement to social security benefits under a given tax-benefit system (or some other default policy)\(^{19}\). These are contrasted with parallel sets of calculations for an alternative regime. From these calculations, the distribution of changes in income resulting from the alternative policy regime can be established. From the government’s point of view, the sum of these changes represents the impact of the alternative policy on revenue. Viewed from the

\(^{18}\) Behavioural responses that may be quite relevant when dealing with redistributritional issues are, for example, labour supply, savings and household family composition (i.e. marriage, fertility, ...). We will see in more detail how to build a behavioural model that allows for labour supply responses in the next sub-section.

\(^{19}\) There is an extensive literature on the application of arithmetical MS models to the analysis of reforms of tax-benefit systems. Atkinson and Sutherland (1988), Harding (1996), Sutherland (1998) and Gupta and Kapur (2000), among others, offer surveys of MS models and their use in Europe and United States. On tax incidence and on the incidence of public spending in areas like education or health see for instance Creedy (1999) and Demery (2003).
perspective of households, instead, it is the impact on the total tax burden, net of social security benefits. Throughout all these calculations, no change in behaviour is modelled following a policy change: the estimates computed are of the immediate (or “morning after”) effect of the change, before individuals, households and the economy adjust in response. Thus, in this kind of models, one takes detailed account of taxes and transfers to model household income distribution and consumption, leaving household behaviour exogenous. Better said, this kind of models is limited to “first round effects” and disregards second round effects due to the behavioural responses of agents. This approach is particularly useful for the analysis of the “morning after” effects of a policy change; that is, when the individual behaviour is assumed to remain the same as before the change.

For a practical application of a simple arithmetical MS model to a household survey, see in particular the section dedicated to the description of the Top-Down approach (the equations of the MS model are reported in Table 10, while the description of the model is at page 41; one can find instead the household survey on which the model is based in the preceding sub-section, Table 2, page 34) in the next paragraph about the linking methods of MS and CGE models.

The primary outputs of tax-benefit models are estimates of the revenue impact of a policy reform and of the distribution of associated income gains and losses across households. 

Arithmetical MS models that are representative of the population allow the revenue and distributional impacts to be estimated together, taking full account of the interactions of different policy components and, at the same time, considering the range of all possible

20 For a detailed description of an arithmetical tax-benefit model, see for instance POLIMOD, an arithmetical MS model for UK, which is described in detail in Redmond et al. (1998). A lot of material about this topic can also be found on the website of EUROMOD, a tax-benefit MS model that estimates the effects of changes in social and fiscal policies on measures of personal income and household welfare; it is an integrated model covering all 15 European Union countries. See: http://www.econ.cam.ac.uk/dae/mu/emod.htm. Another good description of an arithmetical model is given in Oliver and Spadaro (2004), who describe GLADHISPANIA, a MS model for the study of the effects of the 1999-2003 Spanish income tax reforms and other hypothetical scenarios based on the adoption of proportional tax rates. See also the website: http://www.gladhispania.es
circumstances in which families find themselves. The simultaneous generation of these estimates provides a powerful aid to policy design, allowing the policy maker to consider how expenditure aimed at achieving a particular distributional objective is to be paid for, or alternatively, how the impact of a measure aimed at raising a particular amount of revenue is distributed and how unintended losers might be compensated.

However, taking into account behavioural responses may be of great importance for poverty incidence analysis, since they may increase or, on the contrary, mitigate the first round effects revealed by the accounting approach. The difficulty of course is to identify this behavioural response and to understand its determinants properly in order to integrate it into the analysis. To have an idea of the possible applications of behavioural MS models, consider for example the “conditional cash transfers programs”, a policy adopted in several developing countries: it consists of a cash transfer to households whose income per capita is below a certain threshold, conditionally on their effective keeping to some particular behaviour, such as sending their kids to school or carrying out regular visits to health care facilities.

2.1. Behavioural Models

As arithmetical models, behavioural MS models rely on micro household databases. Nevertheless, they add an important component to the analysis: the point is not only to count how much more, or less, everyone is receiving or paying because of a reform in his/her budget constraints, but to take into account the behavioural response of the agents to this change in the budget constraint. This may be done through the estimation of a structural econometric model on the cross-section of households available in the survey being used, and/or through the calibration of a behavioural model with some

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21 See Redmond et al. (1998).
22 For an application of this type, see for instance Bourguignon et al. (2003c) with their simulations of the Bolsa Escola program in Brazil. To have an idea of the potential of behavioural MS models for the evaluation of public policies, see also Labeaga et al. (2005).
23 The source from which part of this paragraph is taken is Bourguignon and Spadaro (2006).
predetermined structure so as to make it consistent with behaviour actually observed in the survey, and meant to correspond to the status quo (see below).

Tax-benefit models with labour supply response are the archetypical example of behavioural MS models. Because of the recurring importance placed on labour supply responses to tax and benefit reforms\footnote{Here we would like to remark the fact that labour supply strongly depends on structural factors and institutional features, and not only on individual choices and preferences.}, we will mainly focus our attention on the modelling of labour supply behaviour, the understanding of which continues nowadays to attract considerable research interest\footnote{Blundell and MaCurdy (2000) present a detailed state of the art.}. Changes in the tax-benefit system in these models affect the budget constraint of households. They modify their disposable income with unchanged labour supply, but through the corresponding income effects, and also through the changes in the after tax price of labour, they also modify labour supply decisions. By how much is determined through simulating a model of labour supply behaviour and factor.

The behavioural MS model approach comprises three main steps: specifying the logical economic structure of the model being used, estimating or calibrating the model and simulating it with alternative reforms of the tax-benefit system.

There are two main trends in the literature on behavioural models of labour supply: the traditional continuous approach pioneered by Hausman (1980 and 1985), and the discrete choice approach, developed by Van Soest (1995) and Aaberge \textit{et al.} (1995).

\textit{The standard continuous approach}

The logical economic structure is that of the textbook maximizing utility of the consumers. An economic agent, $i$, with characteristics $z_i$ chooses his/her volume of consumption, $c_i$, and his/her labour supply, $L_i$, so as to maximize his/her preferences represented by the utility function $u(\cdot)$ under a budget constraint that incorporates the whole tax-benefit system\footnote{For simplicity, we suppose that no savings are possible and all the disposable (after tax) income is spent for consumption.}. Formally, this is represented by:
In the budget constraint, $y_0$ stands for (exogenous) non-labour income, $w_i$ for the wage rate and $NT(·)$ for the tax-benefit or “net tax” schedule, which represents the way in which the tax-benefit system transforms gross income into disposable income. This function actually stands for a fairly complex set of rules for the computation of taxes and benefits, which depend on the characteristics of the agent $z_i$, his/her non-labour income and his/her labour income, $w_i L_i$. It may also depend directly on the quantity of labour being supplied, as in workfare programs. $\gamma$ stands for the parameters of the tax-benefit system (various tax rates, means-testing of benefits, etc.). Likewise, $\beta$ and $\epsilon_i$ are coefficients that parameterise preferences, the latter being idiosyncratic. The solution of that program yields the following labour supply function:

$$L_i = F(w_i, y_0; z_i; \beta, \epsilon_i; \gamma)$$

This function is non-linear. In particular, it is equal to zero in some subset of the space of its arguments, i.e. the non-participation solution.

Suppose now that a sample of agents is observed in some household survey. The problem is to estimate the function $F(·)$ above, or, equivalently, the preference parameters, $\beta$ and $\epsilon_i$, since all the other individual-specific variables or tax-benefit parameters are actually observed. To do so, preference parameters are broken down into a set of coefficients $\beta$ common to all agents, and a set $\epsilon_i$ that is idiosyncratic. The latter plays the usual role of the random term in standard regressions.

Estimation proceeds as with standard models, minimizing the role of the idiosyncratic preference term in explaining cross-sectional differences in labour supply. This leads to a set of estimates $\hat{\beta}$ for the common preference parameters and $\hat{\epsilon}_i$ for the idiosyncratic preference terms. By definition of the latter, it is true for each observation in the sample that:

$$L_i = F(w_i, y_0; z_i; \hat{\beta}, \hat{\epsilon}_i; \gamma)$$

It is now possible to simulate alternative tax-benefit systems. This simply requires modifying the set of parameters $\gamma$. In absence of general equilibrium effects on wages
and labour demand, the change in labour supply due to moving to the set of parameters $\gamma^*$ is given by:

$$L_i' - L_i = F\left(w_i, y_{0i}; \hat{\beta}, \hat{\delta}; \gamma^*\right) - F\left(w_i, y_{0i}; \beta, \delta; \gamma\right)$$

The change in the disposable income (which, in our case, corresponds to the consumption level, as we do not have the possibility of saving, see note 26) may also be computed for each agent. It is given by:

$$C_i' - C_i = w_i \left(L_i' - L_i\right) + NT\left(w, L_i', L_i; y_{0i}; \gamma^*\right) - NT\left(w, L_i, L_i, y_{0i}; z_i; \gamma\right).$$

Then, one may also derive changes in any measure of individual welfare.

Several difficulties in the preceding model must be emphasized. Its estimation generally is uneasy. It is highly non-linear because of the non-linearity of the budget constraint and possibly its non-convexity due to the tax-benefit schedule, $NT(\cdot)$, and corner solutions at $L_i=0$. Functional forms must be chosen for preferences without specifying a functional form for preferences, we might still be able to characterize the optimal solution as a function of $w_i$ and $y_{0i}$: $L_i^* = F^{NT}\left(w_i, y_{0i}; z_i; \beta, \delta; \gamma\right)$ and estimate $F^{NT}(\cdot)$. However, $F^{NT}(\cdot)$ depends on the current tax-benefit rule $NT(\cdot)$ and therefore it cannot be used to simulate policies that introduce a different tax rule, say $NT'(\cdot)$. The problem is that the behavioural function $F^{NT}(\cdot)$ in general mixes up preferences and constraints. More generally, the opportunity set might be defined by complicated budget and quantity constraints that do not even allow recovering a closed form solution for $L_i^*$. What we really need is the estimate of the utility function $u(c_i, L_i)$ itself. Once preferences are estimated, in principle we are able to simulate the effect of any policy by solving $\text{Max } u(c_i, L_i)$ subject to the appropriate constraints.

27 Without specifying a functional form for preferences, we might still be able to characterize the optimal solution as a function of $w_i$ and $y_{0i}$: $L_i^* = F^{NT}\left(w_i, y_{0i}; z_i; \beta, \delta; \gamma\right)$ and estimate $F^{NT}(\cdot)$. However, $F^{NT}(\cdot)$ depends on the current tax-benefit rule $NT(\cdot)$ and therefore it cannot be used to simulate policies that introduce a different tax rule, say $NT'(\cdot)$. The problem is that the behavioural function $F^{NT}(\cdot)$ in general mixes up preferences and constraints. More generally, the opportunity set might be defined by complicated budget and quantity constraints that do not even allow recovering a closed form solution for $L_i^*$. What we really need is the estimate of the utility function $u(c_i, L_i)$ itself. Once preferences are estimated, in principle we are able to simulate the effect of any policy by solving $\text{Max } u(c_i, L_i)$ subject to the appropriate constraints.
functions (see MaCurdy et al., 1990). However, the principal inconvenience of using this methodology is that the behavioural restrictions it imposes are too strong, requiring that the labour supply function globally satisfies the Slutsky conditions. As a result, the estimation results suffer from a lack of robustness, which reduces their usefulness for policy evaluation (see MaCurdy et al., 1990, and MaCurdy, 1992).

Such weaknesses have pushed researchers towards the estimation of total income elasticities or the estimation of direct utility functions by a discretisation of the labour supply alternatives (Van Soest, 1995, Aaberge et al., 1995, Hoynes, 1996, Keane and Moffit, 1998, and Blundell et al., 2000). This second approach has been heavily employed in the recent analysis of tax reforms. Since behavioural changes probably occur at the corner or kink points of the labour supply function, this method has the advantage of capturing them, providing the analyst with an estimation of the elasticity at the extensive margin. Moreover, this methodology allows us to avoid the computational and analytical difficulties associated with utility maximization under non-linear and non-convex budget constraints. This is because the budget constraint is now directly modelled in the utility function. It also enables to consider fixed costs, simultaneous participation and the intensity of work choices, as well as spouses’ joint labour supply decisions.

\[ \frac{\partial h_l(p,u)}{\partial p_k} = \frac{\partial x_l(p,w)}{\partial p_k} + \frac{\partial x_l(p,w)}{\partial w} \cdot x_k(p,w), \quad \text{for all } l, k \text{ (goods)}, \]

where \( h(p,u) \) is the Hicksian demand function, which is derived from the dual problem of utility maximization, i.e. the expenditure minimization problem: \( \{ \min p \cdot x \text{ s.t. } u(x) \leq \bar{u} \} \), where \( \bar{u} \) is a given level of utility, while \( x(p, w) \) is the Marshallian demand function, derived from the utility maximization problem (see Note 30), and \( w > 0 \) is the wealth level of individual \( i \). The satisfaction of the Slutsky equation requires a continuous utility function \( u(\cdot) \) representing a locally nonsatiated and strictly convex preference relation.

Other problems presented by this approach are the lack of identification of the responses of hours to marginal changes in taxes (see, for instance, Van Soest, 1995), and the under-identification of wage effects due to misspecification of dynamic components (see MaCurdy, 1992).

An excellent application of behavioural MS based on discrete choice models, which illustrates very well the potential of this approach, is that of Blundell et al. (2000), which evaluates the likely effect of the
Discrete choice models of labour supply

Specifications used in recent work consider labour supply as a discrete variable that may take only a few alternative values, and evaluate the utility of the agent for each of these values and the corresponding disposable income given by the budget constraint. As before, the behavioural rule is then simply that agents choose the value that leads to the highest level of utility. However, the utility function may be specified in a very general way, with practically no restriction. Such a representation is therefore as close as possible to what is revealed by the data.

The approach essentially consists in representing the budget set with a set of discrete “points”. Let \([0, D]\) be the (continuous) range of possible values for hours of work \(D_j\). Let us pick \(J\) points \(d_1, d_2, \ldots, d_J\) to represent \([0, D]\). The utility level attained by individual \(i\) at point \(j\) is \(U_i^{j}(c_i^j, d_j)\), where \(c_i^j\) is obtained through some budget rule such as in (1).

Formally, a specification that generalizes what is most often found in the recent tax and benefit labour supply literature is the following:

\[
L_i = d_j \quad \text{if} \quad U_i^{j} = f(z_i; w_i, c_i^j; \beta^j, \varepsilon_i^j) \geq U_i^{k} = f(z_i; w_i, c_i^k; \beta^k, \varepsilon_i^k) \quad \text{for all } k \neq j, \quad (3)
\]

where \(d_j\) is the duration of work in the \(j^{th}\) alternative and \(U_i^{j}\) the utility associated with that alternative, \(c_i^j\) being the disposable income given by the budget constraint in (1):

\[
c_i^j = y_{i0} + w_iL_i + NT(w_i d_j, d_j, y_{i0}; z_i; \gamma).
\]

This problem is discretized in the sense that the choice of working hours is supposed to be made between few alternatives. This approach is computationally very convenient when compared to the continuous one, since it does not require going through complicated Kuhn-Tucker conditions involving derivatives of the utility function and of the budget constraint. As a consequence, it is not affected by how complex it is the rule that defines the budget set or by how many goods are contained in the utility function. When the function \(f(\cdot)\) is linear with respect to its common preference parameters, \(\beta^j\), additive with respect to the idiosyncratic terms, \(\varepsilon_i^j\), and when those terms are iid with a

introduction of the Working Families Tax Credit (WTFC) in the UK. They estimate, separately, a discrete labour supply model for married couples and single parents on a sample of UK households in the Family Resources Survey for 1995 and 1996.
double exponential distribution, this model is the standard multinomial logit. It may also be noted that it encompasses the initial model (1). It is sufficient to make the following substitution:

\[ f(z_i; w_i, c_i^j; \beta^j, \varepsilon_i^j) = u(e_i^j, d_j; z_i; \beta^j, \varepsilon_i^j). \]  

(4)

This specification, which involves restrictions across the various duration alternatives, is actually the one that is most often used. The idea is that there generally are commonly agreed durations of work in the labour market – full-time, ¾ full-time, half-time, etc. – so that employees have indeed a limited finite set of options, including the possibility not to work at all. Thus, the set of alternatives \( j = 1, 2, \ldots, J \) now corresponds to \( J \) work durations or to \( J \) combinations of spouses’ labour supplies in the case of joint decisions by a couple\(^{31}\).

Within the literature adopting this approach there are however two potentially important issues. A first issue concerns the procedure by which the discrete alternatives are chosen. For example, Van Soest (1995) and Blundell et al. (2000) choose (non probabilistically) a set of fixed points identical for every individual. This is by far the most widely adopted method. By contrast, Aaberge et al. (1995) adopt a sampling procedure and also assume that the choice set may differ across the households.

A second issue concerns the availability of the alternatives. Most authors assume all the values of hours-of-work in \([0, D]\) are equally available. At the other extreme, some authors assume only two or three alternatives (e.g. non-participation, part-time and full-time) are available for everyone. Aaberge et al. (1995) assume instead that not all the hour opportunities in \([0, D]\) are equally available to everyone; they specify a probability density function of opportunities for each individual and the discrete choice set used in the estimation is built by sampling from that individual-specific density function\(^{32}\).

Even under its more general form, the preceding specification of discrete choice models might be still found to be restrictive because it relies on some utility maximizing assumption. Two remarks are important at this respect. First, it must be clear that ex-ante incidence analysis of tax-benefit systems cannot dispense with such a basic assumption:

\[^{31}\] For an extensive discussion of these specifications, see Bargain (2005).

\[^{32}\] For more details on the implications of alternative methods of representing the choice set within the discrete choice approach, see Aaberge et al. (2006).
the ex-ante nature of the analysis requires some assumption to be made about the way agents choose between alternatives. The assumption that agents maximize some criterion defined in the most flexible way across alternatives is not really restrictive. Second, it must be clear that, if no restriction is imposed across alternatives, then the utility maximizing assumption is compatible with the most flexible representation of the way in which labour supply choices observed in a survey are related to individual characteristics, including the wage rate and the disposable income defined by the tax-benefit system, $NT(\cdot)$.

One important thing is that model (3) fits the data as closely as possible: the only restriction with respect to that objective in the general expression (3) is the assumption that the utility associated with each alternative depends on the wage rate and on the non-labour income of an individual only through $c^j$, that is the disposable income given by the budget constraint and the tax-benefit schedule, $NT(\cdot)$

It is also necessary to check that utility is monotonically increasing with disposable income for this general specification to make any sense.
Conclusions about the behavioural approach

The role of the idiosyncratic terms, $\hat{\epsilon}_i$ or $\hat{\epsilon}_{ij}$ in the whole approach must not be downplayed: they represent the unobserved heterogeneity of agents’ labour supply behaviour. Thus, they may be responsible for some heterogeneity in response to a reform of taxes and benefits. It may be seen in (4) that agents who are otherwise identical might react differently to a change in disposable incomes, despite the fact that these changes are the same for all of them. For this, it is sufficient for the idiosyncratic terms, $\hat{\epsilon}_i$, to be different enough.

Estimates of the idiosyncratic terms result directly from the econometric estimation of the common preference parameters, $\hat{\beta}$ in the continuous specification (2) or $\hat{\beta}_{ij}$ in the discrete model (3). These are standard regression residuals in the former case and so-called “pseudo-residuals” in the latter. However, one may also opt for a “calibration” rather than an econometric estimation approach. With the former, some of the coefficients are not estimated but given arbitrary values deemed reasonable by the analyst. Then, as in the standard estimation procedure, estimates of the idiosyncratic terms are obtained by imposing that predicted choices, under the status quo, coincide with actual choices.

It is important to emphasize that there is some ambiguity about who the “agents” behind the standard labour supply model (1) should be. Traditionally, the literature considers individual agents, even though the welfare implications of the analysis concern households. Extending the model to households requires considering simultaneously the labour supply decision of all members at working age. This makes the analysis more complex. It becomes practically intractable with the continuous representation (see, for instance, Hausman and Ruud, 1994) but only lengthens computation time with the discrete approach.

Applications of the preceding models now are numerous. They are surveyed in Blundell and MaCurdy (2000) and in Creedy and Duncan (2002a). The discrete approach underlined above is best illustrated by Van Soest (1995), Hoynes (1996) or Keane and

In addition to labour supply and consumption patterns, there are other dimensions of household behaviour mattering from a welfare point of view and that may be affected by tax-benefit systems. *Oportunidades* in Mexico, *Bolsa Familia* in Brazil and similar “conditional cash transfer programs” in several other countries, offer a clear example of policies in developing countries that can be evaluated ex ante by behavioural microsimulation models.

However, some limitations of the behavioural approach to MS modelling must be stressed. First, it has to be recognized that this approach is difficult to implement because it generally requires the estimation of an original behavioural model that fits the policy to be evaluated or designed, and of course the corresponding micro data. Because of this, it is unlikely that an analysis conducted in a given country for a particular policy can be applied without substantial modification to another country or in the same country to another type of policy. The methodological investment behind this approach may thus be important. This justifies applying first a pure arithmetical MS approach or a simpler behavioural model based on calibration. Second, the fact that the behavioural approach relies necessarily on a structural model that requires some minimal set of assumptions is to be emphasized. In general, there is no way these assumptions may be tested. In the labour supply model with a discrete choice representation, the basic assumption is that wage and non-labour income variables matter for occupational decisions only through the net disposable income they command, as given by the tax-benefit system. On the contrary, a reduced form model would be based independently on wage and non-labour income. Econometrically, the difference may be tenuous, but the implications in terms of

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34 An interesting example of discrete choice models of labour supply applied to the Spanish tax reforms can also be found in Labeaga et al. (2005). In particular, it can be of great interest the comparison between the results obtained from an arithmetical model such as GLADHISPANIA (see note 19), and those from a behavioural model applied to the same country and simulating a similar scenario.


36 For more detailed information, see the website of the International Food Policy Research Institute (IFPRI): http://www.ifpri.org/themes/progresa.htm

37 See Bourguignon et al. (2003c).
microsimulation results of specific policies may be huge. Finally, the strongest assumption is that cross-sectional income effects, as estimated on the basis of a standard household survey, coincide with the income effects that will be produced by the program or the reforms under study. In other words, time income effects for a given agent are assumed to coincide with observed cross-sectional income differences. Here again, this is an hypothesis that is hard to test, and yet absolutely necessary for ex-ante analysis: nothing is possible without it. The only test one can think of would be to combine ex-ante and ex-post analysis: coincidence between the results obtained in the two evaluations of a given program would support the assumption that cross-sectional and time individual specific income effects are identical\textsuperscript{38}.

Some other general issues about the modelling of behavioural responses may be taken into account. First of all, it is important to have in mind the fact that modelling human behaviour is extraordinary difficult: there are too many dimensions in which rationality may exist, too many interrelated factors involved for the task to be straightforward. Nor is it clear that the effort involved is justified by an improvement in reliability\textsuperscript{39}. Furthermore, the advantages of transparency should not be forgotten. The greater the choice of inputs in the form of estimates of behavioural effects, the larger the scope for manipulation of model results.

Because of some possibly strong assumptions there unavoidably is some uncertainty about the prediction that comes out of ex-ante incidence analysis based on behavioural MS models. This being said, such a tool is absolutely necessary in order to reflect on the optimal design of policies that are most likely to generate strong behavioural responses. However, modelling the labour-supply response to policy changes within a model that only addresses the household sector raises the question of the supply of jobs and how this is affected, in the first place by the policy change and in the second place by the shift in labour supply. Similar problems arise in the detailed modelling of other household

\textsuperscript{38} Rather satisfactory results have been obtained in that direction by Todd and Wolpin (2002) and Attanasio \textit{et al.} (2003).

\textsuperscript{39} For example, Pudney and Sutherland (1996) show that incorporating a typical simulation of female labour supply into POLIMOD, an arithmetical microsimulation model for UK (see Redmond \textit{et al.}, 1998), can greatly increase the uncertainty with which some of POLIMOD’s estimates are made.
responses to policy changes. Modelling the full or equilibrium effect of any policy change requires a model of the whole economy.

**Behavioural MS models and applied optimal redistribution theory**

Including behavioural response in a MS framework allows for an explicit analysis of the equity-efficiency trade-off in the spirit of standard optimal redistribution analysis. In arithmetical models, that analysis could be performed only in a very indirect way, for instance comparing social welfare indicators and the distribution of marginal effective rates across alternative tax-benefit systems, the latter being taken as an indicator of the disincentives and distortions caused by these systems. A more rigorous treatment can be used once a behavioural model has been specified. This is discussed below in the case where the behaviour of interest is labour supply.

The specification of labour supply behaviour implicitly refers to preferences represented by some utility function, as in model (1) above. With the same notations, let $V(w_i, y_{0i}; z_i; \beta, \epsilon_i; \gamma)$ be the corresponding indirect utility function for individual $i$. The social welfare function corresponding to a tax-benefit system with parameters $\gamma$ may then be defined as:

$$SWF(\gamma) = \sum_{i=1}^{n} G[V(w_i, y_{0i}; z_i; \beta, \epsilon_i; \gamma)],$$

where $n$ is the number of agents in the population and $G[\cdot]$ is the social valuation of individual welfare. $G[\cdot]$ is an increasing and concave function, its concavity being an indicator of the level of aversion towards inequality of the redistribution authority.

Following a methodology proposed by King (1983), it is often convenient to replace the indirect utility function $V(\cdot)$ by a money metric, $y_e$, defined as the non-labour income that must be given to the agent in some benchmark situation to raise his/her utility to the level actually achieved with a given policy. More precisely, one may use as a benchmark the

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40 Given the utility maximization problem budget constraint for individual $i$, under his/her budget constraint, as: $\{\text{Max } u(x) \text{ s.t. } px \leq w\}$, where $w > 0$ is the wealth level of individual $i$ and $p > 0$ the vector of good prices, the solution to this problem is the Marshallian demand function $x(p, w)$, which is a function of prices and wealth level. For each $(p, w) > 0$, the utility value of the problem is denoted $v(p, w) \in \mathbb{R}$, and it is equal to $u(x^*)$ for any $x^* \in x(p, w)$. The function $v(p, w)$ is called the indirect utility function.
case where the individual does not work because his/her productivity is too low, say zero, and the tax-benefit system is defined by the set of parameters $\gamma^0$. Let $V_i = V(w_i, y_0; z_i; \beta, \epsilon_i; \gamma)$ be the utility actually achieved by individual $i$ when the parameters of the tax-benefit system are $\gamma$. Then, a money metric $y_e(V_i)$ of $V_i$ using the tax-benefit system $\gamma^0$ and the case $w_i = 0$ as a benchmark, is given by the solution to the equation:

$$V[0, y_e(V_i); z_i; \beta, \epsilon_i; \gamma^0] = V_i.$$  

(5)

The social welfare function may then be defined on the money metric of utility, rather than on the utilities themselves:

$$SWF(\gamma) = \sum_{i=1}^{n} \Gamma\{y_e[V(w_i, y_0; z_i; \beta, \epsilon_i; \gamma)]\},$$

where $\Gamma(\cdot)$ may now be given the usual interpretation of the social utility of individual “income”. The obvious advantage of that transformation of the initial expression of social welfare is that it does not depend any more on the cardinalisation of the utility function used to represent individual preferences.

Within such a framework, it is possible to perform comparative social evaluation of alternative redistribution policies, as summarized by sets of parameters $\gamma^A$ and $\gamma^B$. This only requires being able to compute the indirect utility functions for each individual $i$ in the population, inverting it as in (5) thanks to some numerical algorithm, and evaluating the social welfare function associated to each system.

Behavioural MS models and the computation of social welfare according to the equations above make possible some simple application of the optimal taxation literature. The simplest application consists of comparing two tax-benefit systems, as characterized for instance by two sets of parameters, $\gamma^A$ and $\gamma^B$, and to determine which system leads to the highest level of social welfare. Of course, the comparison makes sense only if the budget of the redistribution authority is the same in the two systems, that is if tax receipts net of transfers are the same with $\gamma^A$ and $\gamma^B$ 41. This corresponds to the standard “government budget constraint” in optimal taxation models.

A very similar type of application consists of investigating the effects of modifying some subset of the parameters, $\gamma$, of a tax-benefit system and to see whether this improves the

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41 An example of this approach is provided by Spadaro (2005), where the 1995 French and British tax-benefit systems are micro-simulated respectively on samples of French and UK households in order to find which system is the “best” for a given level of social aversion to inequality and for each population.
social welfare function, allowing, of course, for constant government budget. If this exercise is repeated for a broad enough set of alternative definitions of the social welfare function, this is equivalent to investigating “Pareto-improving” reforms of the initial tax-benefit system\(^\text{42}\).

### 2.2. Static vs Dynamic Models

The data on which MS models usually rely on are based upon administrative data or upon a sample survey of the population. In both cases, the microdata usually contain thousands of individual or family records, with a host of variables describing the demographic, labour force, income and other characteristics of each individual or family. As a consequence of the fact that there is frequently a lag of several years between the collection of microdata and its public release by an agency, the data have to be “aged” to simulate the impact of current (or future) government policy. Whether the data are aged “statically” or “dynamically” is a major difference between the various types of MS models\(^\text{43}\).

In a static framework, the size and demographic characteristics of the population are fixed; these models are most frequently used to provide estimates of the immediate distributional impact of policy changes. On the other hand, dynamic settings consider in an endogenous way the demographic phenomena that affect the original population, such as changes in the mortality and fertility rates, in the intertemporal consumption allocation, in retirement or in the time taken out for education\(^\text{44}\).

The ageing in static MS models involves two basic steps: reweighting and uprating. The first one, sometimes also called grossing-up, as outlined previously, implies changing the weight attached to each individual record in the microdata, to reflect economic and social change since the data were collected. For example, when a survey was originally

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\(^{42}\) Ahmad and Stern (1984) have pioneered this type of application of MS models in the case of indirect taxation.

\(^{43}\) See the introductory chapter to Harding’s (1996) book on MS.

\(^{44}\) For a detailed literature review on dynamic MS modelling see O’Donoghue (2001).
conducted, the central statistical agency might have decided that there were 400 people in a particular country with similar characteristics to the first person on a particular microdata file. Four years later, due to an increase in, say, unemployment or the incidence of sole parenthood, more recent data might suggest that there are now 450 people with similar characteristics to the first person in the microdata file. The weight of the first person would thus be increased to reflect this, while the weights of other records in the file might be decreased. Reweighting has typically been used to age sample surveys by a few years, in order to bring them up to date, but in general they have not been used for exploring some decades into the future.

The principal aim of the other key technique, uprating, is instead to adjust monetary values within the original microdata to account for estimated movements since the time of the survey. For example, earnings or rent paid are typically increased to account for growth since the survey, although there are different possible approaches of uprating procedures.

After these two steps, static modellers typically impute the receipt of social security and other benefits and/or income tax or other liabilities, by applying the rules for eligibility or liability to each of the microunits. At this point, a baseline data file has been generated; most static models allow then the analyst to vary these rules, and produce output showing the gains or losses, and the cost to revenues and the government budget from the policy change. In this case we have an arithmetical or accounting model as described before; when instead the modeller attempts to simulate the changes in the behaviour of the individuals directly affected by a policy shock (for example, by allowing the labour supply and consumption patterns to vary in response to a tax change), we have a behavioural model.

The time dimension of MS models depends on the object of the analysis and the kind of behavioural response that is incorporated in the model. For instance, evaluating the effects of a reform of the income tax that would modify the treatment of children will have little effects on household composition in the short-run; however, long-run effects require simulating the impact on fertility decisions of the tax reform, and a dynamic framework may then become necessary. Likewise, changes in the parameters of the tax-benefit system that affect intertemporal consumption allocation, retirement, training, schooling, etc., must be analysed with dynamic MS models rather than static models.
Dynamic MS models applied to economics were first introduced in the United States at the end of the 1960’s. Since the 1980’s, they have been rapidly developing due to the increase in computational capabilities and to the availability of longitudinal data (or “panel data”). However, such data frequently are not easily accessible and, moreover, they necessarily are historically dated and consequently may not be of very much relevance for simulating the forward-looking effects of a change in policy. Rather than relying on actual panel data, thus, dynamic MS models often start from the same cross-section sample surveys as static models. However, the individuals within the original microdata are then progressively moved forward through time by making major life events – such as death, marriage, divorce, fertility, education, labour force participation, etc. – happen to each individual according to the probabilities of such events happening to real people within a particular country. In such a way, the characteristics of each individual are recalculated for each time period. Transition probabilities themselves are obtained from different sources – for example from comprehensive longitudinal or panel surveys which would allow one to set such probabilities with some confidence; they are assumed to be constant, so that the society is supposed to be in some kind of steady state, and they are supposed to be independent of the policy being analyzed. Thus, this kind of models age each person in the microdata file from one year to the next by probabilistically deciding whether or not that person will get married, get divorced, have a child, drop out of school, get a job, change jobs, become unemployed, retire, or die.

There are two major types of dynamic MS models. Dynamic population models involve ageing a sample of an entire population, and typically begin with a cross-section sample survey for a particular point in time. Such dynamic models have been used for different purposes, such as the analysis of retirement incomes, future health status, the long-term impact of social security amendments, and the lifetime redistributive impact of the social security system.

Dynamic cohort models use exactly the same type of ageing procedures, but usually age only one cohort rather than the many cohorts represented in an entire population. Typically, one cohort is aged from birth to death, so that the entire lifecycle is simulated.

In a dynamic behavioural MS model, transition probabilities should partly become endogenous and reactive to the intertemporal budget constraint faced by agents. Browning et al. (1999) and Blundell and MaCurdy (2000) contain an excellent discussion about these problems.
For some applications, such models are more cost-efficient than ageing an entire population in terms of computational costs. Such models have been used to analyse lifetime income redistribution, lifetime rates of return to education, and repayment patterns for student income-contingent loans.\textsuperscript{46}

\textbf{2.3. Conclusions}

One of the peculiarities of MS models is that they allow to identify precisely who are the gainers and losers of a reform, as they provide information on the way every individual or household in a sample is affected by the reform. However, in order to obtain some significant information at the policy level, the changes in disposable income due to the reform are usually given for groups, which are derived from the aggregation of individuals or households according to their socio-demographic characteristics. Most models also provide changes in several welfare indicators computed on the whole population: these include, among others, the mean disposable income per adult equivalent, a number of inequality indices (Gini, Theil and Atkinson’s measures), several poverty indicators (FGT indices\textsuperscript{47}, for instance), and the application of relative or absolute Lorenz dominance criteria\textsuperscript{48}.

The importance and usefulness of microsimulation techniques in the analysis of public policies come essentially from two aspects: first of all, microsimulation models allow the explicit accounting for the heterogeneity of economic agents as they are observed in micro-data sets; the second aspect concerns the possibility of accurately evaluating the aggregate financial cost/benefit of a reform: indeed, the results obtained with a MS model at the level of individual agents can be aggregated at the macro level allowing the analyst to evaluate the effect of the policy on the government budget constraint. Because

\textsuperscript{46} For instance, Baldini (1997) analyses the redistributive impact of the Italian tax-benefit system over the life cycle utilizing a dynamic cohort model.

\textsuperscript{47} It’s the commonly used Foster-Greer-Thorbecke (FGT) class of indices for the measurement of poverty. See Foster, Greer and Thorbecke (1984).

\textsuperscript{48} For a complete survey on welfare dominance theory, see Lambert (1993).
of these strong advantages over the representative agent approach, and also because of continuing progresses in data availability and computing facility, the microsimulation approach to economic policy analysis is bound to intensify and to deepen in next future. One of the principal limits of MS models is that they are usually partial equilibrium models with a particular focus on the household side of the economy: the explanatory power of MS models generally ends with the determination of changes to disposable income. They do not simulate the response of the production side of the economy and further economic activity thereby generated, missing out a significant part of the economic process: the possible general equilibrium effects of a policy reform. In particular, when dealing with substantial policy changes, it is essential to take into account their macroeconomic effects because they are likely to influence the microeconomic outcomes. This is one of the reasons why, in last years, a growing group of works is focusing on the introduction of heterogeneous consumers and workers, whose characteristics are specified by reference to micro data (and especially to household surveys), in general equilibrium models.

3. LINKING MICROSIMULATION AND CGE MODELS

The idea of linking the two approaches appeared for the first time in the work by Dervis, de Melo and Robinson (1982). Nonetheless, their idea had to wait until the end of the 1990s to see its first realizations by Decaluwé et al. (1999b) and Cogneau (1999). It’s only from then on that the literature on this subject has been flourishing. The aim of

49 For example, consider the case of a reform of the tax system which generates large enough labour supply effects: then, changes in the structure of wages and prices may be expected to take place.

50 Parts of this paragraph draw on Savard (2003) and on Bourguignon et al. (2003b).

51 See for instance Bourguignon et al. (2003b) with their model for Indonesia, Cogneau and Robilliard (2004) who built a model for Madagascar, and, among the most recent works, see Hérault (2005) with a
combining these models is to exploit the advantages of CGE and MS models and to offset their main drawbacks, which are essentially the limitations arising from the representative household assumptions for CGE models and the lack of general equilibrium effects for MS models. The existing literature on this subject has followed different ways in the attempt of linking the two types of models\(^{52}\). We present an analysis of the advantages and drawbacks of the various approaches that are nowadays implemented in the literature. The main advantages of all the approaches, compared to the RH approach, are that they allow for intra-group distribution analysis and that it is not necessary to adopt any prior grouping of households or individuals. This way, all the approaches leave the modeller free from pre-selecting households grouping or aggregation, and thus able to investigate the sensitivity of results to different policy reforms.

### 3.1. The Integrated Approach

The first and most immediate possibility of linking CGE and MS models consists in moving from representative to “real” households within the CGE approach: it suffices to replace the small number of RHs by the full sample in the household survey in order to capture the heterogeneity of households’ characteristics\(^ {53}\). With such a model, one can explore how household heterogeneity combines with market mechanisms to produce more or less inequality in economic welfare as a consequence of shocks or policy changes. With the fast development of computing efficiency over the last few years,

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\(^{52}\) See Savard (2003 and 2004), Davies (2004) and Cororaton and Cockburn (2005) for a more extensive survey of the different approaches.

\(^{53}\) The first attempt in this direction was made by Decaluwé \textit{et al.} (1999b). Among the models following this approach there are the works by Cockburn (2001) for Nepal, by Cogneau and Robilliard (2001) for Malagasy economy, by Boccanfuso \textit{et al.} (2003a) for Senegal, and the more recent work by Cororaton and Cockburn (2005), who studied the case of Philippine economy.
these so-called integrated CGE-MS models can incorporate as many households as found in household surveys. The logic according to which this approach of linking a household survey with a CGE model does work is illustrated in Figure 3.

![Figure 3 – Integrated CGE-MS Models](image)

The main disadvantages of this approach are the limits it imposes in terms of microeconomic behaviour\(^{54}\) and the fact that the size of the model can quickly become problematic, thus making the data reconciliation process relatively difficult. Indeed, a procedure for reconciling household survey data (incomes and expenditures) and their adjustment with the social accounting matrix (SAM) must be adopted to balance out both accounts. The literature on data reconciliation offers different alternatives. One may keep the structure of the SAM and adjust the household survey. This method has the advantage to save the structure of the economy but it is likely to change the structure of income and expenditure in the household survey. The other alternative is to adjust the SAM to meet the totals of the household survey, loosing in this way some information.

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\(^{54}\) Certain types of equations that are commonly included in a behavioural model, and especially switching regime equations, are not easily modelled within standard CGE modelling softwares (to this regard, see Savard’s discussion about the limits and advantages of the various approaches of linking, 2003), so that CGE-MS models that follow the fully integrated approach are not always able to capture the behavioural responses of the agents to the policy reforms that are implemented.
contained in the structure of national accounts\textsuperscript{55}. Another alternative may be that of using an intermediate approach. For example, one may keep the initial structure of consumption from the household survey and adjust the corresponding accounts in the SAM, and with regards to income one can adjust the household survey to meet the national data based on the SAM\textsuperscript{56}. Whatever the method used, however, it is clearly best to adjust least those estimates in which researchers have greatest confidence.

After these changes in the initial SAM, one has also the problem of re-balancing it (row totals must be equal to column totals). In order to do this, one can use a SAM balancing program designed for this purpose. These programs can be based on different principles, such as on the “Row and Sum”, or RAS, method (see Bacharach, 1971), on a least squares minimization principle, known also as Stone-Byron method\textsuperscript{57}, or on the cross-entropy approach proposed by Robinson et al. (2001) and Robilliard and Robinson (2003).

A practical example may be useful to understand better the problem of data reconciliation. We will consider a very simple survey with a sample of five households, who are supposed to be representative of the whole population. Of course, this is only a simplified example; household surveys are usually made up of samples containing thousands of observations. Moreover, as this is a simplified example, we do not consider the procedure of stratification which is normally used to draw a sample (this procedure is anyway described in detail in section 2). The population is composed of 1000 families, in which live 2500 adults and 2900 children as a whole. The household survey is reported in Table 2. The economy considered here is a very simple one: households’ income is obtained by the employment of two factors, labour and capital, plus some public transfers, and the only tax levied by government is a direct income tax; the monetary data may be, for instance, in ten thousands of a given monetary unit. Now, in order to carry the micro-data (which are referred to a single household) forward to population data, we have to use an equivalence scale. Indeed, it is obvious that a family composed of one

\textsuperscript{55} The first alternative of modifying the structure of the household survey may be preferred to the latter in some cases, due to the fact that one will often find some under or over reporting for items in the household surveys.

\textsuperscript{56} See for example Annabi et al. (2005).

\textsuperscript{57} See Stone (1977) and Byron (1978).
individual who perceives an income of one thousand monetary units has not the same purchasing power as could have a family receiving the same income with four children dependent. One solution could be that of dividing the household income by the number of family members. However, one should also consider the fact that some scale economies are likely to arise when people live in the same house (for instance, the electricity consumed by a two-members family is not double of that consumed by one individual), and in special way when there are some children living in the family (in the common meaning, children are considered to consume less than what can spend an adult). There are different measures one can adopt. We have chosen the following equivalence scale:

\[
ES = 1 + 0.7 \cdot (Adults - 1) + 0.5 \cdot Children
\]

Thus, given our population, we can build an “average” family with 2.5 adults and 2.9 children:

\[
ES = 1 + 0.7 \cdot (2.5 - 1) + 0.5 \cdot (2.9) = 3.5 \text{ Adult Equivalents}
\]

Thus, in our economy there are in total 3500 adult equivalents. We can now compute the sample weights \(\omega_h\) for each of the families in the survey: after having calculated the number of adult equivalents for each household (for example, in \(H_1\) there are 2.2 adult equivalents), it is sufficient to divide it by the total number of adult equivalents of our representative sample (15.2). The results are reported in Table 3 below. Now, we know that the adult equivalents of the first household in the survey, \(H_1\), represent the 14.47% of all the adult equivalents in the population, the 22.37% of population adult equivalents are represented by the adult equivalents in the second family of the sample, \(H_2\), and so on. This way, by multiplying these weights by the total number of adult equivalents in the population, we obtain the number of population adult equivalents living in that type of representative household (\(H_1, H_2\), etc.). Thus, after having computed the per-capita per adult equivalent values of the variables in the survey (for this, it is sufficient to divide income, consumption expenditure, labour and capital income, public transfers, etc. of each household in the survey by the corresponding number of sample adult equivalents; see Table 4), and multiplying them by the number of population adult equivalents, we can find the population values corresponding to each household type. The results for our sample are reported in Table 5.
Table 2 – Household Survey, an Example

<table>
<thead>
<tr>
<th></th>
<th>$CE_1$</th>
<th>$CE_2$</th>
<th>$S$</th>
<th>$Y$</th>
<th>$LY$</th>
<th>$KY$</th>
<th>$TF$</th>
<th>$TY$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>0.4343</td>
<td>0.7817</td>
<td>-</td>
<td>1.2160</td>
<td>0.5646</td>
<td>0.4343</td>
<td>0.2171</td>
<td>-</td>
</tr>
<tr>
<td>$H_2$</td>
<td>1.0423</td>
<td>1.0857</td>
<td>0.0751</td>
<td>2.5623</td>
<td>1.3029</td>
<td>1.1726</td>
<td>0.0869</td>
<td>0.3592</td>
</tr>
<tr>
<td>$H_3$</td>
<td>0.8686</td>
<td>1.1726</td>
<td>0.0560</td>
<td>2.3866</td>
<td>1.1726</td>
<td>1.0857</td>
<td>0.1303</td>
<td>0.3348</td>
</tr>
<tr>
<td>$H_4$</td>
<td>1.3029</td>
<td>2.1714</td>
<td>0.8056</td>
<td>5.8629</td>
<td>2.1714</td>
<td>3.6914</td>
<td>-</td>
<td>1.5830</td>
</tr>
<tr>
<td>$H_5$</td>
<td>1.7371</td>
<td>2.1714</td>
<td>0.4095</td>
<td>5.2983</td>
<td>3.0400</td>
<td>2.2583</td>
<td>-</td>
<td>0.9802</td>
</tr>
</tbody>
</table>

$CE_i$: consumption expenditure for commodity $i$; $S$: savings; $Y$: income; $LY$: labour income; $KY$: income from capital; $TF$: transfers received from government; $TY$: amount of income taxes paid to government (direct taxes).

Table 3 – Adult Equivalents and Sample Weights

<table>
<thead>
<tr>
<th>Number of Adults</th>
<th>Number of Children</th>
<th>Sample Adult Equivalents</th>
<th>Sample Weights ($\omega$)</th>
<th>Population Adult Equivalents</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>2</td>
<td>1</td>
<td>2.2</td>
<td>14.474%</td>
</tr>
<tr>
<td>$H_2$</td>
<td>3</td>
<td>2</td>
<td>3.4</td>
<td>22.368%</td>
</tr>
<tr>
<td>$H_3$</td>
<td>1</td>
<td>3</td>
<td>2.5</td>
<td>16.447%</td>
</tr>
<tr>
<td>$H_4$</td>
<td>2</td>
<td>3</td>
<td>3.2</td>
<td>21.053%</td>
</tr>
<tr>
<td>$H_5$</td>
<td>3</td>
<td>3</td>
<td>3.9</td>
<td>25.658%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>11</strong></td>
<td><strong>12</strong></td>
<td><strong>15.2</strong></td>
<td><strong>100.000%</strong></td>
</tr>
</tbody>
</table>

Table 4 – Per-capita Per Adult Equivalent Values

<table>
<thead>
<tr>
<th>$CE_1$</th>
<th>$CE_2$</th>
<th>$S$</th>
<th>$Y$</th>
<th>$LY$</th>
<th>$KY$</th>
<th>$TF$</th>
<th>$TY$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>0.1974</td>
<td>0.3553</td>
<td>-</td>
<td>0.5527</td>
<td>0.2566</td>
<td>0.1974</td>
<td>0.0987</td>
</tr>
<tr>
<td>$H_2$</td>
<td>0.3066</td>
<td>0.3193</td>
<td>0.0221</td>
<td>0.7536</td>
<td>0.3832</td>
<td>0.3449</td>
<td>0.0255</td>
</tr>
<tr>
<td>$H_3$</td>
<td>0.3474</td>
<td>0.4690</td>
<td>0.0224</td>
<td>0.9554</td>
<td>0.4690</td>
<td>0.4343</td>
<td>0.0521</td>
</tr>
<tr>
<td>$H_4$</td>
<td>0.4071</td>
<td>0.6786</td>
<td>0.2518</td>
<td>1.3832</td>
<td>0.6786</td>
<td>1.1536</td>
<td>-</td>
</tr>
<tr>
<td>$H_5$</td>
<td>0.4454</td>
<td>0.5568</td>
<td>0.1050</td>
<td>1.3585</td>
<td>0.7795</td>
<td>0.5790</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5 – Weighted (Population Values) Household Survey

<table>
<thead>
<tr>
<th>CBUD</th>
<th>$CE_1$</th>
<th>$CE_2$</th>
<th>$S$</th>
<th>$YD$</th>
<th>$Y$</th>
<th>$LY$</th>
<th>$KY$</th>
<th>$TF$</th>
<th>$TY$</th>
<th>$ty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>280</td>
<td>100</td>
<td>180</td>
<td>0.0</td>
<td>280.0</td>
<td>280</td>
<td>130</td>
<td>100</td>
<td>50</td>
<td>0.0</td>
</tr>
<tr>
<td>$H_2$</td>
<td>490</td>
<td>240</td>
<td>250</td>
<td>17.3</td>
<td>507.3</td>
<td>590</td>
<td>300</td>
<td>270</td>
<td>20</td>
<td>82.7</td>
</tr>
<tr>
<td>$H_3$</td>
<td>470</td>
<td>200</td>
<td>270</td>
<td>12.9</td>
<td>472.9</td>
<td>550</td>
<td>250</td>
<td>250</td>
<td>30</td>
<td>77.1</td>
</tr>
<tr>
<td>$H_4$</td>
<td>800</td>
<td>300</td>
<td>500</td>
<td>185.5</td>
<td>985.5</td>
<td>1350</td>
<td>500</td>
<td>850</td>
<td>0</td>
<td>364.5</td>
</tr>
<tr>
<td>$H_5$</td>
<td>900</td>
<td>400</td>
<td>500</td>
<td>94.3</td>
<td>994.3</td>
<td>1220</td>
<td>700</td>
<td>520</td>
<td>0</td>
<td>225.7</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>2940</strong></td>
<td><strong>1240</strong></td>
<td><strong>1700</strong></td>
<td><strong>310.0</strong></td>
<td><strong>3240.0</strong></td>
<td><strong>3990</strong></td>
<td><strong>1900</strong></td>
<td><strong>1990</strong></td>
<td><strong>100</strong></td>
<td><strong>750.0</strong></td>
</tr>
</tbody>
</table>

See Table 2. *CBUD*: consumption expenditure; *YD*: disposable income; *ty*: direct income tax rate.
The data presented in Table 5 can now be compared to the national accounts that we observe in the SAM for this economy, in Table 6. As we can easily see, however, the aggregated data observed in the survey do not coincide with those presented in the SAM. One may think that there is some under or over reporting in survey data and adjust them to national accounts, or choose to save the structure of the household survey and adjust the national data of the SAM. Here, we want to keep fixed the consumption and income data from the survey; thus, we run an appropriate program that minimizes least squares in order to re-balance the SAM, after having introduced into it the data from the survey, and in particular the five household accounts. This way, of course, we will loose part of the original structure of the national accounts. The new balanced SAM is reported in Table 7.

Thus, it must be stressed as one of the main disadvantages of the integrated approach the fact that the data reconciliation process will necessarily lead to changes in structure of either the micro-data on income and expenditures of the household survey, or the national accounts’ data contained in the SAM.

Another difficulty of this approach is the problem of identifying the heterogeneity of factor endowments or preferences at the level of a single household or individual. Indeed, as this approach treats with every single household observed in the survey, it automatically loses any possibility of characterization of the socio-economic structure in the model.
<table>
<thead>
<tr>
<th></th>
<th>C₁</th>
<th>C₂</th>
<th>S₁</th>
<th>S₂</th>
<th>K</th>
<th>L</th>
<th>G</th>
<th>H₁</th>
<th>H₂</th>
<th>H₃</th>
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<th>H₅</th>
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<td></td>
</tr>
<tr>
<td>Total</td>
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<td>3888.662</td>
<td>2835.418</td>
<td>3888.662</td>
<td>1990.0</td>
<td>1900.0</td>
<td>745.921</td>
<td>280.0</td>
<td>589.995</td>
<td>551.275</td>
<td>1350.000</td>
<td>1220.000</td>
<td>305.349</td>
<td>499.300</td>
<td></td>
</tr>
</tbody>
</table>

Table 7 – Re-Balanced SAM with Five Households
We can build a little CGE model for our archetypical economy, using the new SAM containing the data from the household survey (Table 7), in order to fully understand how this approach really works in practice. The CGE designed for this aim has the following characteristics:

- five households with a Cobb-Douglas utility function;
- two commodities, used in production and consumption;
- four production factors: capital, labour and both commodities;
- two firms with Leontief technology in value added and intermediate aggregate inputs;
- Cobb-Douglas aggregator function for capital and labour;
- Leontief aggregator function in intermediate inputs;
- capital and labour are mobile among sectors and exogenously fixed;
- public sector: government maximizes a Cobb-Douglas utility function, buys consumption goods, uses labour and capital, raises taxes on income and pays transfers to households;
- savings and investments (investments are savings-driven);
- open economy, with Armington assumption for the composite good aggregation, and exports demand depending on the world price.

The equations relative to this CGE model, called \( CGE_{HH} \), are presented in Table 8.

With this model calibrated on the SAM with five households (Table 7), we simulate an exogenous shock: a 30% increase in the price of imports of the good imported by sector 1. We report in Tables 10 and 11 the resulting changes in some of the variables and the inequality indices (Gini, Atkinson’s, Theil, etc.) computed after the shock. We used disposable income per Adult Equivalent (the variable \( YD_h \) in the model divided by the number of Adult Equivalents) as reference variable for these computations.
<table>
<thead>
<tr>
<th>Equation Description</th>
<th>Mathematical Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Consumption demand</strong></td>
<td>$P_i \cdot C_{hi} = \alpha H_{hi} \cdot CBUD_{ih}$ for $i = 1,2$ and $h = 1,2,3,4,5$</td>
</tr>
<tr>
<td><strong>Savings</strong></td>
<td>$S_h = mps_h \cdot (1 - t_h) \cdot Y_h$</td>
</tr>
<tr>
<td><strong>Production function</strong></td>
<td>$XD_i = aF_i \cdot K_i^{\alpha_f} \cdot L_i^{(1-\alpha_f)}$</td>
</tr>
<tr>
<td><strong>Tangency condition</strong></td>
<td>$PK = \frac{\alpha F_i}{PL} \cdot L_i \cdot K_i$</td>
</tr>
<tr>
<td><strong>Investment demand</strong></td>
<td>$P_i \cdot I_i = \alpha I_i \cdot \sum_{h=1}^{5} S_h$</td>
</tr>
<tr>
<td><strong>Price of exports (local currency)</strong></td>
<td>$PE_i = PWE_i \cdot ER$</td>
</tr>
<tr>
<td><strong>Price of imports (local currency)</strong></td>
<td>$PM_i = PWM_i \cdot ER$</td>
</tr>
<tr>
<td><strong>Armington function</strong></td>
<td>$X_i = aA_i \cdot \left[ \gamma A_i \cdot M_i^{\alpha_i \frac{1}{\alpha_i}} + (1 - \gamma A_i) \cdot XDD_i^{\alpha_i \frac{1}{\alpha_i}} \right]$</td>
</tr>
<tr>
<td><strong>Imports demand</strong></td>
<td>$M_i = \frac{X_i}{aA_i} \cdot \left( \frac{\gamma A_i}{PWE_i} \right)^{\frac{1}{\alpha_i}} \cdot \left[ \gamma A_i^{\alpha_i} \cdot M_i^{(1-\alpha_i)} + (1 - \gamma A_i)^{\alpha_i} \cdot XDD_i^{(1-\alpha_i)} \right]^{\frac{1}{1-\alpha_i}}$</td>
</tr>
<tr>
<td><strong>Exports demand</strong></td>
<td>$E_i = EZ_i \cdot \left( \frac{PWE_i}{PWEZ_i} \right)^{\eta_i}$</td>
</tr>
<tr>
<td><strong>Market clearing condition for labour</strong></td>
<td>$\sum_{i=1}^{2} L_i + LG = \sum_{h=1}^{5} LS_h$</td>
</tr>
<tr>
<td><strong>Market clearing condition for capital</strong></td>
<td>$\sum_{i=1}^{2} K_i + KG = \sum_{h=1}^{5} KS_h$</td>
</tr>
<tr>
<td><strong>Market clearing condition for commodity $i$</strong></td>
<td>$XD_i + M_i = \sum_{j=1}^{2} io_{ij} \cdot XD_j + CG_i + \sum_{h=1}^{5} C_{hi} + I_i + E_i$</td>
</tr>
<tr>
<td><strong>Income definition</strong></td>
<td>$Y_h = PK \cdot KS_h + PL \cdot LS_h + PL \cdot TF_h$</td>
</tr>
<tr>
<td><strong>Disposable income</strong></td>
<td>$CBUD_h = (1 - \gamma_h) \cdot Y_h - S_h$</td>
</tr>
<tr>
<td><strong>Zero profit condition in production</strong></td>
<td>$PD_i \cdot XD_i = PK \cdot K_i + PL \cdot L_i + \sum_{j=1}^{2} io_{ij} \cdot XD_j \cdot PD_j$</td>
</tr>
<tr>
<td><strong>Zero profit condition in Armington function</strong></td>
<td>$P_i \cdot X_i = PM_i \cdot M_i + PD_i \cdot XDD_i$</td>
</tr>
<tr>
<td><strong>Zero profit condition in exports supply</strong></td>
<td>$PD_i \cdot XD_i = PE_i \cdot E_i + PD_i \cdot XDD_i$</td>
</tr>
<tr>
<td><strong>Demand of commodity $i$ by government</strong></td>
<td>$P_i \cdot CG_i = \alpha CG_i \cdot \left( TAXREV - PL \cdot \sum_{h=1}^{5} TF_h \right)$</td>
</tr>
<tr>
<td>Demand of capital by government</td>
<td>[ PK \cdot KG = \alpha KG \cdot \left( TAXREV - PL \cdot \sum_{h=1}^{s} TF_h \right) ]</td>
</tr>
<tr>
<td>Demand of labour by government</td>
<td>[ PL \cdot LG = \alpha LG \cdot \left( TAXREV - PL \cdot \sum_{h=1}^{s} TF_h \right) ]</td>
</tr>
<tr>
<td>Tax revenues</td>
<td>[ TAXREV = \sum_{h=1}^{s} t_y_h \cdot Y_h ]</td>
</tr>
<tr>
<td>Number of variables: 76</td>
<td>Exogenous variables:</td>
</tr>
<tr>
<td>Number of equations: 56</td>
<td>- exogenous capital endowment (KS_h)</td>
</tr>
<tr>
<td>Number of exogenous variables: 20</td>
<td>- exogenous labour supply (LS_h)</td>
</tr>
<tr>
<td>Walras’ Law is satisfied</td>
<td>- exogenous public transfers (TF_h)</td>
</tr>
<tr>
<td>Walras’ Law is satisfied</td>
<td>- exogenous world prices (PWE_i, PWM_i)</td>
</tr>
<tr>
<td>Walras’ Law is satisfied</td>
<td>- fixing the numeraire (wage rate, PL)</td>
</tr>
</tbody>
</table>

### Variables:
- PK: return to capital
- PL: wage rate
- ER: exchange rate
- P_i: Armington composite good prices
- PE_i: imports prices (local currency)
- PM_i: exports prices (local currency)
- KS_h: capital endowment (exogenous)
- LS_h: labour endowment (exogenous)
- XD_i: gross domestic output
- X_i: sales on the domestic market
- E_i: exports
- M_i: imports
- K_i: capital demand by firms
- KG: capital demand by government
- L_i: labour demand by firms
- LG: labour demand by government
- I_i: investment demand
- C_m: consumer commodity demand
- CG_i: government commodity demand
- Y_h: household h’s income
- S_h: household h’s savings
- PWE_i: imports world prices (exogenous)
- PWM_i: exports world prices (exogenous)
- XDD_i: internal production for the domestic market
- CBUD_h: disposable income of household h
- TAXREV: tax revenues
- TF_h: real public transfers to household h

### Parameters:
- ty_h: direct income tax rate for household h
- mps_h: marginal propensity to save of household h
- io_{ij}: technical coefficients
- aF_{ij}: efficiency parameter in production function of firm i
- aF_i: C-D power of capital in production function of firm i
- aL_i: C-D power in bank’s utility function
- aH_{hi}: C-D power of commodity i in household h utility f.
- aCG_i: C-D power of commodity i in government utility f.
- aKG: C-D power of capital in government utility function
- aLG: C-D power of labour in government utility function
- aA_i: efficiency parameter in Armington function
- γA_i: share parameter in Armington function
- αA_i: elasticity of substitution in Armington function
- η: price elasticity of exports demand

With respect to the limit to microeconomic behaviour, note that CGE modelling imposes that behavioural functions respect certain conditions: for example, modelling switching regimes is not easy to introduce with current CGE modelling softwares, as the equation system of the model cannot change as the iteration process moves along. Indeed, integrated models often rely on relatively simple microsimulation models focusing on only one or two dimensions of household (or individual) behaviour. Yet, it is not clear
that this type of model may be convincingly used to describe the full complexity of household income inequality and the way it may be affected by macroeconomic policies. To this extent, micro-econometric modelling provides much more flexibility in terms of the modelling structure used.

Moreover, when the size of the model becomes problematic, the modeller may be forced to impose some simplifications either on the complexity of microeconomic household behaviours or on the size of the CGE model in terms of the number of sectors and factors of production.

3.2. The Top-Down Approach

The idea of this approach is to develop separately a MS model and then to run the simulation on the basis of changes in consumer/producer prices, wages, and sectoral employment levels as predicted by some macro model, a CGE model in this case. This approach does not try to integrate the two models, but uses instead a CGE and a MS model in a sequential way: first, the policy reform is simulated with the CGE model, and the second step consists of passing the simulated changes in some variables (usually prices, wage rates, self-employment incomes and possibly employment levels58) down to

---

58 When the assumption of imperfect labour market is adopted, or when the presence of a formal and an informal sector is predicted, the rationing in the labour market is usually carried out in the macro or CGE model, while the main use of the MS module is to select those households or individuals who will actually be barred out of, or let in, employment, or the formal sector.
the MS module\textsuperscript{59}. The logical scheme followed by this approach is illustrated in Figure 4\textsuperscript{60}.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{The Top-Down Approach}
\end{figure}

\textsuperscript{59} This CGE-MS approach was developed by Bourguignon \textit{et al.} (2001) in a model that simulated the effects of the 1997 crisis in Indonesia. However, there are preceding examples of models designed in this two-layered fashion: see for instance Meagher (1993): a dynamic applied general equilibrium model (the well-known MONASH model) for Australia is used together with the 1990 Australian income survey to bear on a forecast of the incomes of various groups of individuals. Another example of linkage between a macro model and a MS model is the analysis of the distributional consequences of China’s accession to WTO by Chen and Ravallion (2003).

\textsuperscript{60} In principle, one should also mention the possibility of a “Bottom-Up” approach. That is, a framework in which the link between the two models goes in the opposite direction: from the micro to the macro level of analysis. For instance, one could think of implementing a reform of the tax-benefit system with the microsimulation model, then to pass the changes in some relevant variables (such as labour supply, disposable income or consumption levels, for instance) onto the CGE model, and finally to run the CGE model to check the general equilibrium effect of the reform. Anyway, the use of a Top-Down approach is more common, at least in the literature on developing countries.
The basic difficulty of this approach is to ensure consistency between the micro and macro levels of analysis. For this reason, one may introduce a system of micro-macro consistency equations that ensure the achievement of consistency between the two levels. Thus, what happens in the MS module can be made consistent with the CGE modelling by judiciously adjusting parameters in the MS model, but, from a theoretical point of view, it would be more satisfying to obtain consistency by modelling behaviour identically in the two models.

We will build a simple CGE-MS model following this approach, in order to make clear the passages of variable changes from one model to the other and the problem of the so-called consistency equations. Let’s consider the economy described in the previous section: the CGE model will be very similar to that one, except for the number of households. In this case, in fact, we will have only one representative household in the CGE model, while the household survey of Table 2 will be used to set up an arithmetical (accounting) MS framework, and will stay out of the CGE model. The SAM on which the CGE is calibrated is the one presented in Table 6, while the equations are the same equations described in Table 8 for the previous model, except for the index $h$, referred to households, which disappears in this model (there is only one representative household).

We have now 36 equations and 44 variables; having fixed 8 exogenous variables, we have that the model is fully determined and a redundant equation according to Walras’ law.

The MS module is very simple, and it is derived from the micro-data on income and expenditure observed in the survey (Table 2): households’ income is obtained by the employment of two factors, labour and capital, plus some public transfers; the only tax levied by government is a direct income tax (to make the model more realistic, one could add also indirect taxes on consumption, social security contributions, capital taxation, etc.). The equations of the MS model are reported in Table 9; they are simple

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61 This way, at the micro level, one adjusts all individual wage rates and all self-employment incomes by the same percentage as obtained in the CGE simulation, and, similarly, the utility from working or being self-employed is adjusted in such a way as to produce employment changes in the MS module equal to those found in the CGE calculations. When the functions involved are not linear, the parameter adjustments needed to achieve consistency with the CGE results are more complicated.
arithmetical computations, as the only behavioural parameters in the model are the marginal propensity to save, $mps_h$, and the shares of consumption expenditure, $\eta_{hi}$, which are fixed on the basis of survey data, and kept constant after simulations. We also assume for simplicity that both prices, the wage rate and the return on capital are all equal to one in the base year, in both the CGE and MS models. This way, in the MS model, we have the same amounts to indicate initial monetary values, $CE_{hi}$, $LY_h$, $KY_h$, and quantities, respectively, $C_{hi}$, $L_h$, $K_h$.

**Table 9 – MS Model Equations**

<table>
<thead>
<tr>
<th>Exogenous variables:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TF_h$: public transfers received by household $h$</td>
</tr>
<tr>
<td>$L_h$: labour supply of household $h$</td>
</tr>
<tr>
<td>$K_h$: capital endowment of household $h$</td>
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</table>

<table>
<thead>
<tr>
<th>Parameters derived from the survey:</th>
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<tbody>
<tr>
<td>Share of consumption expenditure for commodity $i$ $\eta_{hi} = CE_{hi}/CBUD_h$</td>
</tr>
<tr>
<td>Income tax rate for household $h$ $ty_h = TY_h/Y_h$</td>
</tr>
<tr>
<td>Marginal propensity to save of household $h$ $mps_h = S_h/YD_h$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Equations of the model:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household $h$’s income $Y_h = LY_h + KY_h + TF_h$</td>
</tr>
<tr>
<td>Household $h$’s disposable income $YD_h = (1 - ty_h) \cdot Y_h$</td>
</tr>
<tr>
<td>Household $h$’s savings $S_h = mps_h \cdot YD_h$</td>
</tr>
<tr>
<td>Household $h$’s consumption expenditure $CBUD_h = YD_h - S_h$</td>
</tr>
<tr>
<td>Consumption expenditure for commodity $i$ $CE_{hi} = \eta_{hi} \cdot CBUD_h$</td>
</tr>
<tr>
<td>Amount of taxes paid to government $TY_h = ty_h \cdot Y_h$</td>
</tr>
<tr>
<td>Consumption levels $C_{hi} = \frac{CE_{hi}}{(1 + \Delta P_i)}$</td>
</tr>
<tr>
<td>Labour supply of household $h$ $L_h = \frac{LY_h}{(1 + \Delta PL)}$</td>
</tr>
<tr>
<td>Capital endowment of household $h$ $K_h = \frac{KY_h}{(1 + \Delta PK)}$</td>
</tr>
</tbody>
</table>
We will apply to this model the same policy simulation described in the previous section: a positive shock to the price of the good imported by sector 1. First, we run the CGE and obtain some percentage changes in all the variables of the model; however, we are interested only in the variation that we observe in prices, wage rate and capital return (if the model were more complicated, we could also be interested in the variation of employment levels, for instance, or we could observe different wage rates if the model showed the presence of a segmented labour market). In our model we have chosen the wage rate as the numeraire, so that we observe a change only in the two prices, \( P_i \), and in the capital return, \( PK \):

\[
\Delta P_1^{\text{CGE}} = 3.058\% , \\
\Delta P_2^{\text{CGE}} = -0.054\% , \\
\Delta PK^{\text{CGE}} = 0.559\% .
\]

We take these percentage changes and pass them onto the MS module; we can do this easily, as the MS schedule exhibits exogenous prices and return to capital (equal to one in the base year). Things get a bit more complicated when the variables in the household model are endogenous, and especially when the MS model is a behavioural one. For example, consider a MS model with labour supply response, with the possibility of non-participation (i.e. unemployment). In this case, the unemployment level in the household model is no longer an exogenous variable, but it is determined by the labour supply function; however, changes in the number of workers in the MS model must match those same changes in the CGE model. Thus, a choice may be to impose these variable changes from the CGE model onto the micro level of analysis (and this is obtained by modifying some specific coefficients of the labour supply function). This particular choice implies that the MS model is allowed to determine which individuals, amongst the entire population, will fill the need for more workers if their number is to increase in the CGE model (on the contrary, if the number of workers is found to decrease, then the MS model will choose the individuals with the highest probability to lose their job). For a more detailed description of these methods, see Bourguignon et al. (2003b) and Hérault (2005). We will describe this procedure in more detail in the second chapter, where we will link a behavioural MS model to a CGE.
In order to transfer these changes onto the MS framework, we need the following “linking equations”:

\[
\begin{align*}
C_{hi}^* &= \frac{CE_{hi}}{1 + \Delta P_{i}^{\text{CGE}}} \\
LY_h^* &= L_h \\
KY_h^* &= K_h \cdot (1 + \Delta PK^{\text{CGE}})
\end{align*}
\]

Notes:  
- the wage rate is the numeraire in our CGE model  
- the variables with the star are the values referred to the simulation runs

At this point, we can run the MS model (as our model is quite simple and just an arithmetical one, we use an Excel sheet with the appropriate computations to run the model) and obtain a new vector of disposable incomes, from which we compute the inequality indices that are reported in Table 11 (computed on disposable income per Adult Equivalent). The results for the main macroeconomic variables resulting from the CGE model are instead reported in Table 10.

One more thing about the sequential Top-Down approach must be said: with this approach, possible economy-wide feedback effects of the distributional consequences of a given policy are not taken into account. There is indeed complete absence of feedback from the micro to the macro level. «…The cost of adopting this approach is that the causal chain from macroeconomic policies to poverty is in one direction only: we do not capture the feedback effect of changes in the composition of demand (due to shifts in the distribution of income) on macroeconomic balances…»\(^{62}\).

The main advantage of this approach is instead that it provides richness in household behaviour modelling, while remaining extremely flexible in terms of specific behaviours that can be modelled.

It is also true that, by emphasizing changes in relative prices and in the sectoral structure of the economy, this approach is more adapted to developing than developed countries.

\(^{62}\) From Devarajan and Go (2003).
3.3. The Top-Down/Bottom-Up (TD/BU) Approach

This method has been recently developed by Savard (2003). It allows to overcome the problem of the lack of consistency between the micro and macro levels of the “top-down” approach by introducing a bi-directional link between the two models: this is the reason why this approach is also called “Top-Down/Bottom-Up”. According to this method, indeed, aggregate results from the MS model (for example consumption levels) are incorporated into the CGE model, and a loop is used to run both models iteratively until the two produce convergent results. However, the existence of a converging solution is not guaranteed.

The value added of this approach comes from the fact that feedback effects provided by the MS model do not correspond to the aggregate behaviours of the representative households used in the CGE model. It is interesting to take these feedback effects of the MS model back in the CGE to insure coherence between the two models. The main difficulty in this type of exercise is related to aggregation and coherence between the two models. A scheme of the way in which this approach of linking the two models works is illustrated in Figure 5.
A vector $W_1$ of changes in:
- Prices, wage and interest rates
- Quantities (for ex. employment levels)

Aggregate and weight

Variable at the household level (ex. consumption or labour supply)

Aggregated vector $AV_1$ of levels by sector

A vector $W_2$ of changes in:
- Prices, wage and interest rates
- Quantities (for ex. employment levels)

Aggregate and weight

Variable at the household level (ex. consumption or labour supply)

Aggregated vector $AV_2$ of levels by sector

...and so on with as many iterations as are necessary to obtain convergence (to a given number of decimals) of the aggregate variable level in the two models

**Figure 5** – The Top-Down/Bottom-Up Approach
1st run

CGE model

output

input

A vector $W_1$ of changes in:
- Prices, wage and interest rates
- Quantities

Variables at the household level (ex. consumption or labour supply)

Aggregate and weight

Aggregated vector $AV_1$ of levels by sector

MS model

output

Aggregate and weight

1st iter.

CGE model

output

input

A vector $W_2$ of changes in:
- Prices, wage and interest rates
- Quantities

Variable at the household level (ex. consumption or labour supply)

Aggregate and weight

Aggregated vector $AV_2$ of levels by sector

MS model

output

input

...and so on with as many iterations as are necessary to obtain convergence (to a given number of decimals) of the aggregate variable level in the two models

2nd iter.

CGE model

output

input

Figure 6 – The Top-Down/Bottom-Up Approach
We will build a little MS-CGE model for our archetypical economy following this approach, believing that illustrating it through a practical example is the most direct way of understanding quickly how it works. The CGE and MS models we will use are the ones we have already described for the sequential Top-Down approach, and again we will run the same policy simulation previously described (increase in the price of the imported good 1). We will follow the indications proposed by Savard (2003)\(^63\). Anyway, as our simulation is a macroeconomic one, we can run directly the CGE model at first. This means that our model will follow a simplified scheme with respect to the one presented in Figure 5. The new simplified scheme is shown in Figure 6. We follow this scheme because our simulation (shock to the price of the imported good for sector 1) cannot be run in the microsimulation framework only.

So, we will start running first the CGE model. Thus, we will pass the resulting changes in prices \(\Delta P^1\), and in capital return, \(\Delta PK^1\) onto the microsimulation model, as it is described in the previous section. After this, running the microsimulation model, we can compute the new consumption levels, \(C^1_h\);\(^64\) then, in order to obtain the corresponding aggregate consumption levels, \(AC^1_i\), each of these values has first to be divided by the number of adult equivalents in that household (see Table 3), then weighted for its relative sample weight, \(\omega_h\), and multiplied by the total number of adult equivalents in the population (3500), and finally aggregated by sector:

\[
AC^1_i = \sum_{h=1}^{s} \omega_h \cdot C^1_h \cdot 3500 .
\]

Now, the aggregate consumption vector obtained from the MS model, \(AC^1_i\), is imported into the CGE model. To do this, we have to change the hypothesis of the model to allow it to be fully determined, as now we have exogenous consumption levels for the

\(^{63}\) Savard will use household consumption as communicating variable from the micro to the macro level. There are other approaches using different variables: for instance, Müller (2004) in his model for Switzerland, and the CGE-microsimulation model for Germany described in Arntz et al. (2006), use the labour supply level resulting from the microsimulation model as communicating variable from the micro to the macro level.

\(^{64}\) This allows us to obtain a matrix of two goods by five households; aggregating over all the households, produces a single vector (2x1) of aggregate consumption levels.
representative household. Therefore, we need to change some of the equations and
exogenous variables of the model: first, we will remove the equations determining
consumption demand by the representative household, substituting it with the following:

\[ CBUD = \sum_{i=1}^{2} P_i \cdot C_i. \]

In the initial hypothesis (endogenous consumption) we had 2 endogenous variables \((C_i)\)
and 2 equations. Now we have 2 exogenous variables and one equation. As we need to
insure the balancing of the household’s budget constraint, a variable needs then to be
endogenized in the following equation:

\[ CBUD = (1 - mps) \cdot (1 - ty) \cdot (PK \cdot KS + PL \cdot LS + TF). \]

Following Savard, we choose to endogenize the marginal propensity to save, \(mps\), which
is now a variable that changes in order to satisfy the budget constraint.

From this CGE model we will obtain other variations in commodity prices and in capital
return, \(\Delta P^2_i\) and \(\Delta PK^2\). Using the consistency equations described for the sequential
model in the previous section, we can introduce these changes into the MS module. This
way, we obtain other consumption levels \(C_{hi}^2\) for each household, and in the same way as
before we can compute the aggregate consumption levels, \(AC_{i}^2\) to be again imported into
the CGE model, from which we will get \(\Delta P^3_i\) and \(\Delta PK^3\). In the same way, we take these
changes and introduce them into the MS model through the consistency equations,
obtaining the vector \(AC_{i}^3\).

We go on in this way until the two models will produce the same values in the aggregate
consumption vector. We obtain convergence at 3 decimals at the 3\(^{rd}\) iteration (4\(^{th}\) run of
the model).

We can now compute the inequality indices for disposable income levels obtained in the
last run of the MS model (see Table 11). The macroeconomic variables are reported in
Table 10.
The main advantages of this approach are:

1. there is no obligation of scaling the household data to national accounts and no need to balance income and expenditure. Consequently, it allows the modeller to use the exact income and expenditure structure found in the household surveys;
2. there is no limit to the level of desegregation in terms of production sectors or number of factors of production and households to be included in the model;
3. the degree of freedom in choices of functional forms used to reflect micro-economic household behaviour is much higher in this approach;
4. the converging solution, if it exists, produces a numerical validation of the coherence between the CGE and the MS models.

It is however important to note that nothing guarantees a converging solution to be found; therefore, it must be validated and numerically checked for the introduction of each new hypothesis.

3.4. Conclusions

Observing the results of the previous models that are reported in Table 11, we can see that there is no substantial difference in the indices calculated from the same simulation with the three types of models. However, it must be taken into account the fact that these models are really simple, and that there are only five households in the survey. Imagine what would happen with thousands of households surveyed in a sample, and if the model is complicated by the introduction of unemployment, of other fundamental variables such as savings and investments, or of other agents such as the foreign sector, or the introduction of the hypothesis of imperfect competition, and so on. Moreover, in a real economy the taxation system is much more complex than the very simple one that we have implemented in the previous models.

However, even under the extreme minimalism of the three models we have implemented, one can notice that there are some slight differences in the resulting indices reported in Table 11. We can observe, indeed, that the first model, the one following the integrated approach, is the one that leads to the greatest change under the same simulation scenario,
and that it brings about the highest (even if not very different from that of the other models) reduction in all the inequality indices.

However, in general, we can see that the three models lead to very similar results in the values of all inequality indices. The reason for this is to be sought mainly in the fact that the MS model is an arithmetical one, that is, all the variables of the model are derived only by simply computing some arithmetical relations, without providing for a reaction in the behaviour of the agents. This way, the results obtained through such a MS model are not that different from those one can obtain by using a standard CGE model. We will see in the next section that things may change with the use of a behavioural MS model, that is, a model that assumes the possibility of a change in the behaviour of the agents following a policy change. The main reason for this is that some of the behavioural responses that could be modelled into a MS framework cannot be included at all into a CGE model. For instance, it is very difficult to model switching regimes such as occupational choices with current CGE modelling softwares, as the equation system of the model cannot change as the iteration process moves along. For this reason, integrated models often rely on relatively simple microsimulation models focusing on only one or two dimensions of household (or individual) behaviour, while the so-called layered approaches (the Top-Down and the TD/BU ones) are able to include more complex equations in their MS module. To this extent, micro-econometric behavioural modelling provides much more flexibility in terms of the modelling structure used, and is much more suitable to describe the complexity of household and individual behaviour, and the way it may be affected by macroeconomic policies.
Table 10 – Changes in Some Macroeconomic Variables

<table>
<thead>
<tr>
<th></th>
<th>Integrated Model</th>
<th>Top-Down Model</th>
<th>TD/BU Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return to capital</td>
<td>0.530</td>
<td>0.559</td>
<td>0.848</td>
</tr>
<tr>
<td>Consumer price index</td>
<td>1.236</td>
<td>1.222</td>
<td>1.402</td>
</tr>
<tr>
<td>Labour demand by gov.</td>
<td>0.327</td>
<td>0.318</td>
<td>0.483</td>
</tr>
<tr>
<td>Capital demand by gov.</td>
<td>-0.202</td>
<td>-0.239</td>
<td>-0.362</td>
</tr>
<tr>
<td>Tax revenues</td>
<td>0.282</td>
<td>0.276</td>
<td>0.419</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>-5.852</td>
<td>-6.070</td>
<td>-5.711</td>
</tr>
<tr>
<td>Income*</td>
<td>0.246</td>
<td>0.276</td>
<td>0.419</td>
</tr>
<tr>
<td>Disposable income*</td>
<td>0.246</td>
<td>0.276</td>
<td>0.419</td>
</tr>
<tr>
<td>Consumption expenditure*</td>
<td>0.246</td>
<td>0.276</td>
<td>1.846</td>
</tr>
<tr>
<td>Savings*</td>
<td>0.261</td>
<td>0.276</td>
<td>-13.380</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>S_1</th>
<th>S_2</th>
<th>S_1</th>
<th>S_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commodity prices</td>
<td>3.033</td>
<td>-0.055</td>
<td>3.058</td>
<td>-0.054</td>
</tr>
<tr>
<td>Domestic sales</td>
<td>-1.520</td>
<td>0.823</td>
<td>-1.490</td>
<td>0.852</td>
</tr>
<tr>
<td>Domestic production</td>
<td>0.255</td>
<td>-0.279</td>
<td>0.274</td>
<td>-0.290</td>
</tr>
<tr>
<td>Labour demand by firms</td>
<td>0.731</td>
<td>-0.158</td>
<td>0.775</td>
<td>-0.162</td>
</tr>
<tr>
<td>Capital demand by firms</td>
<td>0.200</td>
<td>-0.684</td>
<td>0.215</td>
<td>-0.717</td>
</tr>
<tr>
<td>Consumption*</td>
<td>-2.704</td>
<td>0.302</td>
<td>-2.699</td>
<td>0.330</td>
</tr>
<tr>
<td>Investments</td>
<td>-2.660</td>
<td>0.347</td>
<td>-2.699</td>
<td>0.330</td>
</tr>
<tr>
<td>Price of imports (local currency)</td>
<td>22.392</td>
<td>-5.852</td>
<td>22.109</td>
<td>-6.070</td>
</tr>
<tr>
<td>Price of exports (local currency)</td>
<td>-5.852</td>
<td>-5.852</td>
<td>-6.070</td>
<td>-6.070</td>
</tr>
</tbody>
</table>

* For the integrated model these values are computed as average percentage changes.

Table 11 – Some Inequality Indices on Disposable Income per Adult Equivalent

<table>
<thead>
<tr>
<th></th>
<th>Benchmark Situation</th>
<th>After Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Integrated Model</td>
<td>Top-Down Model</td>
</tr>
<tr>
<td>Gini index</td>
<td>18.17</td>
<td>-0.63%</td>
</tr>
<tr>
<td>Atkinson’s index, ε = 0.5</td>
<td>2.62</td>
<td>-1.23%</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
<td>32.61</td>
<td>-0.91%</td>
</tr>
<tr>
<td>Generalized entropy measures:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I(c), c = 2</td>
<td>5.32</td>
<td>-1.82%</td>
</tr>
<tr>
<td>Mean logarithmic deviation, I(0)</td>
<td>5.34</td>
<td>-1.04%</td>
</tr>
<tr>
<td>Theil coefficient, I(1)</td>
<td>5.26</td>
<td>-1.43%</td>
</tr>
</tbody>
</table>

* Percentage changes with respect to the benchmark situation.
4. CONCLUSION

In all the applications presented here, we have worked mainly with fictitious data and small samples of observations. However, the choice of the modelling structure usually depends on the data that are available for the economy under study, then on the objective of the study one is willing to implement, and thus on the kind of policy simulation to be realized.

For what concerns the different types of possible linkages between microsimulation and CGE models, we have seen that most of the differences in the results coming out from the three main approaches arise when working with layered models (Top-Down and TD/BU approaches) rather than with integrated models.

Indeed, if we observe the results reported in Tables 10 and 11, we find that at a macroeconomic level (changes in the main macro variables, Table 10) all the models show very similar results (especially the integrated model and the Top-Down model predict almost identical results), making an exception for the TD/BU approach, in which we obtained different results for what concerns consumption and savings levels. The reason for this lays in the fact that we changed some of the initial assumptions of the CGE model in order to be able to introduce an exogenous vector of consumption levels from the microsimulation model (see section 3.3 for more details).

However, if we take a look at the change in inequality (Table 11, indices computed on disposable income per Adult Equivalent), we can see that the integrated model predicts inequality to decrease, while according to the two layered models inequality is observed to increase, even if of a very small amount. This difference could be of great importance when modelling real economies and it is probably due to the fact that with a layered model we are able to develop separately the microsimulation model, so that we can achieve a higher precision and a more detailed framework for the computations of the tax-benefit system.

We will see in more detail in the next chapter that the possibility of including behavioural responses into the microsimulation framework can lead to even stronger differences in the microeconomic results (and especially for what concerns the changes in poverty).
Bourguignon et al. (2001) and Bourguignon et al. (2003b) also provide strong arguments for working with layered rather than integrated models. These arguments are most persuasive when, as in their work for Indonesia, it is regarded as very important to simulate realistically variation in labour supply and occupational choice responses to changing prices, wages and employment conditions.

A reasonable conclusion may be that integrated models are best for some purposes and layered models for others. The integrated models, indeed, appear cleaner and more transparent, and they show a better reliability under the point of view of the theoretical consistency between the two levels of analysis. They may have however the drawback of not being able to fully capture even the direction and the relative magnitude of distributional and of other effects in terms of a full microeconomic analysis.

Layered models, in contrast, perhaps have an advantage where the concern is about short-term distributional impacts in a setting where realism is at a premium and theoretical niceties are not so important. In analyzing the impacts of a serious crisis, as in Indonesia, a layered approach may get the job done best.